Higgs thermal and hybrid inflation with low energy supersymmetry

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Scalars 2013 Warsaw

Anders Basbøll, MH, TJ (2011); MH, TJ (2012); MH, TJ (2013)

Outline

Scalars in the Standard Model and cosmology

- The Standard Model now complete
  - Higgs $m_H = \begin{cases} 126.0 \pm 0.4 \text{ (stat)} \pm 0.4 \text{ (sys)} & \text{ATLAS} \\ 125.3 \pm 0.4 \text{ (stat)} \pm 0.5 \text{ (sys)} & \text{CMS} \end{cases}$
- The very early universe: inflation (Planck + WMAP)
  $\mathcal{P}_s(k_0) = (2.196^{+0.051}_{-0.060}) \times 10^{-9}, \; n_s = 0.9603 \pm 0.0073$
- Good evidence for two scalars: inflaton and Higgs
- Reheating: energy Inflaton(s) $\rightarrow$ Standard Model
- How is Inflation sector coupled to SM?

  SM $\leftarrow$? $\rightarrow$ Inflation

Model-building ideology

- Standard Model and inflation share problems of naturalness
- Supersymmetry controls radiative corrections for both
- $M_{\rm EW} \lll m_{\rm Pl}$: Standard Model has renormalisable couplings only
- $M_{\rm Inf} \ll m_{\rm Pl}$ motivates trying to decouple Planck scle physics
  - renormalisable couplings for inflaton
  - “Small-field” inflation ($|\Delta s| \ll m_{\rm Pl}$)
- SUSY + small-field inflation $\rightarrow$ SUSY hybrid inflation[(1)]
- “Minimal” model-building ideology here:
  1. The field content of the MSSM and minimal F-term inflation.
  2. The symmetries of the MSSM and minimal F-term inflation.
  3. Renormalisable couplings only.
  4. An inflaton-sector $U(1)'$ gauge symmetry coupled to the MSSM

[(1)] Arguments: Dine, Pack (2011)

Minimal Hybrid Inflationary Supersymmetric Standard Model

- Low energy MSSM (with massive neutrinos)
- Dynamical explanation of μ-term and RH neutrino masses
- SUSY dark matter (neutralino or gravitino)
- Leptogenesis from RH neutrino decays (if $M_{N_1} \lesssim 10^9$ GeV)
- Baryogenesis (if electroweak phase transition is 1st order)
- F-term hybrid inflation $T_{\rm rh} \sim 10^{15}$ GeV
- Second period of Higgs-driven “thermal” inflation $T_{\rm rh} \sim 10^9$ GeV
- Reduced amount of F-term inflation: $n_s \simeq 0.976$ (from 0.983)
- Cosmic strings, $G\mu_{\rm cs} \simeq 10^{-7}$, consistent with CMB

Fields and symmetries

- (ν)MSSM sector: two-parameter family of anomaly free U(1)$'$

	Q	U	D	L	E	N	H_1	H_2
Y'	$-\frac{1}{3}q_L$	$-q_E - \frac{2}{3}q_L$	$q_E + \frac{4}{3}q_L$	q_L	q_E	$-2q_L - q_E$	$-q_E - q_L$	$q_E + q_L$
R	1	1	1	1	1	1	0	0

- Y: $(q_L, q_E) = (-1, 2)$. $B - L$: $(q_L, q_E) = (-1, 1)$
- Inflation sector:

	Φ	$\bar{\Phi}$	S
Y'	$4q_L + 2q_E$	$-4q_L - 2q_E$	0
R	0	0	2

Coupling the MSSM to F-term inflation

- Superpotential: $W = W_A + W_X + W_I$

- MSSM part: $W_A = H_2 Q Y_U U + H_1 Q Y_D D + H_1 L Y_E E + H_2 L Y_N N$
- Pure F-term inflation part: $W_I = \lambda_1 \Phi \overline{\Phi} S - M^2 S$
- Coupling part : $W_X = \frac{1}{2}\lambda_2 N N \Phi - \lambda_3 S H_1 H_2$

- μ_h-term from $\langle s \rangle \sim M_{\rm SUSY}$
- RH neutrino masses from $\langle \phi \rangle \sim M \gg M_{\rm SUSY}$
- All other renormalisable terms forbidden by symmetries
- All B-violating operators forbidden by $\mathrm{U}(1)'$ and R

Related models

- $Y' = B - L$ [2]. No inflaton coupling to Higgs.
- $U(1)' \to SU(2)_R$ [3]. Higgs thermal inflation not noticed.
- F_D inflation[4]. $\langle s \rangle$ gives both μ_h and (TeV-scale) N masses.
- Alchemical inflation[5]. Higgs and inflaton mix. Non-renormalisable.

[2] Shafi et al. (2005-12); Buchmüller, Domcke, Schmitz, Vertongen (2010-2012); Pallis, Shafi (2013)

[3] Dvali, Lazarides, Shafi (1997)

[4] Garbrecht, Pallis, Pilaftsis (2006)

[5] Nakayama, Takahashi (2012)

The scalar potential

Study $s, \phi, \overline{\phi}, h_1, h_2$ subspace for U(1)′ and SU(2)×U(1) breaking

- $V(s, \phi, \overline{\phi}, h_1, h_2) = V_F + V_D + V_{\text{soft}} + \hbar \Delta V_1$
- $V_F = \left[\lambda_1^2(|\phi|^2 + |\overline{\phi}|^2) + \lambda_3^2(|h_1|^2 + |h_2|^2)\right] |s|^2 \quad + |\lambda_1 \phi\overline{\phi} - \lambda_3 h_1 h_2 - M^2|^2$
- $V_D = \frac{1}{2} g'^2 \left(q_\Phi(\phi^* \phi - \overline{\phi}^* \overline{\phi}) + q_H(h_1^\dagger h_1 - h_2^\dagger h_2) \right)^2 + \frac{1}{8} g_2^2 \sum_a (h_1^\dagger \sigma^a h_1 + h_2^\dagger \sigma^a h_2)^2 + \frac{1}{8} g_1^2 (h_1^\dagger h_1 - h_2^\dagger h_2)^2$
- No FI-term - unproblematic embedding in supergravity [6]

[6] Komargodski & Seiberg (2009)

The minimum and its geometry

- $V_D = 0$ when $|\phi| = |\overline{\phi}|$, $|h_1| = |h_2|$, $h_1^\dagger h_2 = 0$.
- $V_F = 0$ when $s = 0$, $\lambda_1 \phi\overline{\phi} - \lambda_3 h_1 h_2 = M^2$
- Minimum parameters: angles $\chi \in [0, \pi/2]$, $\varphi \in [0, 2\pi]$, SU(2):

$$\phi = \frac{M}{\sqrt{\lambda_1}} \sin\chi e^{i\varphi}, \quad h_1^T = \left(\frac{M}{\sqrt{\lambda_3}} \cos\chi, 0 \right)$$

- Angle φ is associated with U(1)″ gauge symmetry generated by $Y'' = Y' - (q_L + q_E)Y$
- Unbroken symmetries :

$\chi = 0$	$U(1)_{em} \times U(1)''$	h-vacuum
$0 < \chi < \pi/2$	$U(1)_{em}$	
$\chi = \pi/2$	$SU(2) \times U(1)_Y$	ϕ-vacuum (electroweak)

U(1)′ breaking and the see-saw mechanism

- Suppose we are in the ϕ-vacuum: $\langle|\phi|\rangle \simeq \langle|\overline{\phi}|\rangle = M\sqrt{\lambda_1}$
- U(1)′-breaking preserves supersymmetry (neglecting soft terms):
  - $F_s = \lambda_1 \phi\overline{\phi} - M^2 = 0$
  - $F_\phi = \lambda_1 \overline{\phi} s + \frac{1}{2}\lambda_2 nn = 0$
  - $F_{\overline{\phi}} = \lambda_1 \overline{\phi} s = 0$
- The large ϕ vev gives RH neutrino masses from the superpotential term $\lambda_2 NN\Phi$ (type-I see-saw mechanism)
- Low energy theory is identical to the MSSM: S, Φ, $\overline{\Phi}$, A' get large supersymmetric masses from the $\phi, \overline{\phi}$ vevs.

SUSY-breaking

$$V_{\text{soft}} = m_\phi^2|\phi|^2 + m_{\bar\phi}^2|\bar\phi|^2 + m_{h_1}^2|h_1|^2 + m_{h_2}^2|h_2|^2 + m_s^2|s|^2 + \rho M^2 M_{\text{SUSY}}(s+s^*) + (h_{\lambda_1}\phi\bar\phi s + h_{\lambda_3} h_1 \cdot h_2 s + c.c.).$$

- gives μ_h-term
- solves AMSB tachyonic slepton problem
- lifts degeneracy between ϕ- and h-vacua

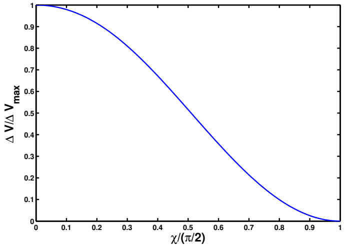
The potential between h and ϕ vacua

Between vacua: $\langle|\phi|\rangle = (\frac{M}{\sqrt{\lambda_1}})\sin\chi, \quad \langle|h_{1,2}|\rangle = (\frac{M}{\sqrt{\lambda_3}})\cos\chi$

$$V(\chi) \simeq -\frac{M^2}{2}\frac{\left(\tilde{h}_{\lambda_1}\sin^2\chi + \tilde{h}_{\lambda_3}\cos^2\chi\right)^2}{\lambda_1\sin^2\chi + \lambda_3\cos^2\chi} + M^2\left(\frac{\bar{m}_\phi^2}{\lambda_1}\sin^2\chi + \frac{\bar{m}_h^2}{\lambda_3}\cos^2\chi\right),$$

where

$$\tilde{h}_{\lambda_1} = \frac{h_{\lambda_1}}{\lambda_1}$$
$$\tilde{h}_{\lambda_3} = \frac{h_{\lambda_3}}{\lambda_3}$$
$$\bar{m}_\phi^2 = m_\phi^2 + m_{\bar{\phi}}^2$$
$$\bar{m}_h^2 = m_{h_1}^2 + m_{h_2}^2$$



MHISSM summary

Mass scales:

- M U(1)$'$ breaking scale
- $M_{\rm SUSY}$ supersymmetry breaking scale

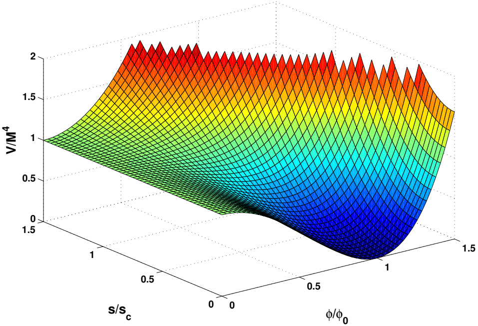
Physics:

- Scale M:
  - flat direction in s, $V = M^4$.
  - after U(1)$'$ breaking, S, Φ, $\bar{\Phi}$ have SUSY masses O(M)
  - Majorana masses for RH neutrinos
- Scale $\sqrt{MM_{\rm SUSY}}$: potential along $H_1 H_2$ “flat" direction
- Scale $M_{\rm SUSY}$: MSSM

F-term hybrid inflation

$$V_F = \left[\lambda_1^2(|\phi|^2 + |\bar{\phi}|^2) + \lambda_3^2(|h_1|^2 + |h_2|^2)\right]|s|^2 + |\lambda_1\phi\bar{\phi} - \lambda_3 h_1 h_2 - M^2|^2$$

- Flat direction s when $|\phi| = |\bar{\phi}| = |h_1| = |h_2| = 0$, with $V_F = M^4$
- Classically stable if $|s| > s_c = \max(M/\sqrt{\lambda_1}, M/\sqrt{\lambda_3})$
- Unstable towards ϕ- or h-vacua if $|s| < s_c$



Predictions from F-term inflation

- Flat direction tilted by radiative corrections:
  $V(s) \simeq M^4 \left[1 + \alpha \ln \frac{2s^2}{s_c^2}\right], \quad \alpha = \frac{\lambda^2}{16\pi^2}, \quad \lambda = \sqrt{\lambda_1^2 + 2\lambda_3^2}$
- Predictions: $\mathcal{P}_\mathrm{s}(k) \simeq \frac{4N_k}{3}\left(\frac{s_c}{m_p}\right)^4, \quad n_s \simeq \left(1 - \frac{1}{N_k}\right).$ (7)
- WMAP9:
  $\mathcal{P}_\mathrm{s}(k_0) = (2.43 \pm 0.11) \times 10^{-9}, n_s = 0.9624 \pm 0.0075$,
- Normalisation: $\frac{s_c}{m_p} \simeq 2.9 \times 10^{-3}\left(\frac{27}{N_{k_0}}\right)^{\frac{1}{4}}, \quad N_{k_0} = 27^{+13}_{-7}$.
- 2σ discrepancy with Hot Big Bang: $N_{k_0} \simeq 58 + \ln(T_\mathrm{rh}/10^{15}\ \mathrm{GeV})$
- Improved fit: linear soft terms;(8) non-renormalisable terms(9)

(7) N_k is number of e-foldings of inflation after $aH = k$
(8) Pallis, Shafi (2013)
(9) Battye, Garbrecht, Moss (2006)

End of inflation and (p)reheating

- $M^2_{\phi,\overline{\phi}} = \lambda_1(\lambda_1|s|^2 \pm M^2)$, $M^2_{h_1,h_2} = \lambda_3(\lambda_3|s|^2 \pm M^2)$
- $\lambda_1 < \lambda_3$:
  - inflation ends at $s^2_{c1} = M^2/\lambda_1$
  - Transition to ϕ-vacuum ($\langle|\phi|^2\rangle = M^2/\lambda_1$)
  - U(1)$''$ symmetry broken: cosmic strings formed
- $\lambda_3 < \lambda_1$:
  - inflation ends at $s^2_{c3} = M^2/\lambda_3$
  - Transition to h-vacuum ($\langle|h_{1,2}|^2\rangle = M^2/\lambda_3$)
  - U(1)$'$ symmetry preserved: no strings formed (yet)
- Reheating time: M^{-1}. Much smaller than expansion time H^{-1}.
- All potential energy $V = M^4$ goes into thermal energy.
- Reheat temperature $T_{\rm rh1} \simeq 2.2\sqrt{\lambda} \times 10^{15}$ GeV, $\lambda = \sqrt{\lambda_1^2 + 2\lambda_3^2}$
- Thermal potential keeps universe in false h-vacuum

Cosmic string constraints on F-term inflation

- U(1)′ symmetry breaking → Nielsen-Olesen vortices:
- Cosmic strings[10] with mass per unit length $\mu_{\rm cs}$
- CMB limit $G\mu_{\rm cs} < 3.2 \times 10^{-7}$ (Planck 95%CL)
- Recall $V_F = |\lambda_1 \phi\bar{\phi} - \lambda_3 h_1 \cdot h_2 - M^2|^2$
- At core of string $\phi = \bar{\phi} = 0$: Higgs condensate preferred
- Reduces $\mu_{\rm cs}$: $G\mu_{\rm cs} \simeq 10^{-7}$ ($\sim 3 \times 10^{-6}$ without condensate)
- CMB limit satisfied

[10] Kibble (1976)

Higgs thermal inflation

- Assume $\Delta V = V_h - V_\phi$ positive, $\lambda_3 < \lambda_1$
- F-term inflation exits to h-vacuum: $\langle |h_{1,2}| \rangle \sim M/\sqrt{\lambda_1}$
- Rapid reheat to $T \sim O(M)$
- Potential energy dominates at $T \sim O(\Delta V)^{\frac{1}{4}} \sim O(\sqrt{MM_{\text{SUSY}}})$, ie $T \sim 10^9$ GeV
- Soft terms drive a second period of Higgs Thermal Inflation[11]
- Higgs trapped by thermal potential until $T \sim M_{\text{SUSY}}$
- Rapid reheating from Higgs oscillations to $T \sim 10^9$ GeV
  - Cosmic strings not formed until second transition
  - Gravitinos created from first inflation era diluted
  - Need $\sim$ 15 fewer e-foldings at high scale: $n_s \simeq 0.976$

[11] c.f. Yamamoto (1985); Binetruy & Gaillard (1986); Lazarides, Panagiotakopoulos, Shafi (1986); Lyth & Stewart (1996)

Gravitino problem(s) of SUSY cosmology

There are two issues

- Too much Dark Matter
  A gravitino LSP, or if it is not the LSP then the LSP density created by its decay, may mean too much Dark Matter.
  $\Omega_{\rm LSP} h^2 \simeq 6 \times 10^{-3} \frac{m_{\rm LSP}}{100\ {\rm GeV}} \left(\frac{T_{\rm rh}}{10^9\ {\rm GeV}} \right) \simeq 0.12$ (WMAP/Planck) [12]
- Big Bang Nucleosynthesis
  Unstable gravitino decaying during/after BBN, $\rightarrow$ photo-dissociation of light elements.
  Lifetime $\tau_{\frac{3}{2}} \sim m_{\rm Pl}^2 / m_{\frac{3}{2}}^3 \sim 10^3 ({\rm TeV}/m_{\frac{3}{2}})^3$ sec
- Gravitino must be stable, heavy, or rare
- Stable gravitino means constraints on NLSP

[12] see e.g. Kawasaki et al (2008)

Gravitino problem becomes gravitino solution

- During θ inflation: Universe cools from $T_{\mathrm{i}} \sim \sqrt{v_\phi M_{\mathrm{SUSY}}}$ to $T_{\mathrm{c}} \sim M_{\mathrm{SUSY}}$.
- e-foldings: $N_\theta \simeq \frac{1}{2} \ln\left(\frac{v_\phi}{M_{\mathrm{SUSY}}}\right) \simeq 15 - \ln\left(\frac{M_{\mathrm{SUSY}}}{1\ \mathrm{TeV}}\right)$
- Gravitinos from high-scale inflation diluted
- Gravitinos regenerated by reheating to $T_{\mathrm{rh2}} \simeq 10^9 \left(\frac{M_{\mathrm{SUSY}}}{1\ \mathrm{TeV}}\right)^{\frac{1}{2}}$ GeV
- Gravitinos are either LSP, or decay into LSP
- $\Omega_{\mathrm{LSP}} h^2 \simeq 6 \times 10^{-3} \frac{m_{\mathrm{LSP}}}{100\ \mathrm{GeV}} \left(\frac{T_{\mathrm{rh2}}}{10^9\ \mathrm{GeV}}\right)$
- If not LSP, require $m_{\frac{3}{2}} \gtrsim$ few TeV to maintain BBN

Summary

- Simplest way of coupling F-term inflation and MSSM gives . . .
- . . . minimal hybrid inflationary supersymmetric standard model (MHISSM)
- Mechanisms for Higgs μ_h-term and RH neutrino masses
- Two phases of inflation:
  - F-term at 10^{15} GeV (42 e-foldings)
  - Higgs thermal at 10^{9} GeV (15 e-foldings)
- 10^{9} GeV reheat temperature high enough for leptogenesis
- DM can be from decays of thermally produced gravitinos
- Cosmic string $G\mu_{\rm cs} \simeq 10^{-7}$ satisfies CMB constraints
- Reduced e-foldings of F-term inflation: $n_s \simeq 0.976$ (2σ)

Concluding comments

- SUSY gravitino problem needs reheat temperature ($\lesssim 10^9$ GeV)
  - very weakly coupled inflaton
  - intermediate scale inflation
- Coupling of inflation to NN also possible (e.g F_D term inflation)
- Cosmic strings common in SUSY GUT hybrid inflation models[13]
  - other observational signatures (e.g. CMB tensors)
- One more $d = 4$ operator to consider: $RH_1 \cdot H_2$
  $\rightarrow$ SUSY Higgs inflation in $h_1 \cdot h_2$ flat direction[14]

(13) Rocher Sakellariadou
(14) Einhorn & Jones 2010,12

SUSY-breaking and the Higgs μ_h-term

- $V_F = \lambda_1(|\phi|^2 + |\overline{\phi}|^2)|s|^2 + \cdots$
- $V_{\rm soft} = m_s^2|s|^2 + \rho M^2 M_{\rm SUSY}(s + s^*) + (h_{\lambda_1}\phi\overline{\phi}s + c.c) + \cdots$
- μ_h-term: $\langle s \rangle \simeq -\frac{h_{\lambda_1}}{\lambda_1^2} - \frac{M_{\rm SUSY}\rho}{\lambda_1}$ (in ϕ-vacuum).
- Recall
$$V_D = \frac{1}{2}g'^2\left(q_\Phi(|\phi|^2 - |\overline{\phi}|^2) + \cdots\right)^2$$
- Using $F_s = 0$, can show $\langle\phi\rangle^2 - \langle\overline{\phi}\rangle^2 = 2\frac{m_{\overline{\phi}}^2 - m_\phi^2}{g'^2 q_\phi^2}$ (in ϕ-vacuum)

SUSY-breaking and AMSB tachyonic slepton

- $V_F = |\lambda_1 \phi\bar{\phi} - M^2|^2 + \cdots$
- $V_{\rm soft} = m_\phi^2|\phi|^2 + m_{\bar{\phi}}^2|\bar{\phi}|^2 + m_L^2|L|^2 + \cdots$
- In AMSB m_L^2 is negative
- $V_D = \frac{1}{2}g'^2\left(q_\Phi(|\phi|^2 - |\bar{\phi}|^2) + \sum q_L|L|^2\cdots\right)^2$
- Given $\langle\phi\rangle^2 - \langle\bar{\phi}\rangle^2 = 2\frac{m_{\bar{\phi}}^2 - m_\phi^2}{g'^2 q_\phi^2}$ (in ϕ-vacuum)
- Hence slepton mass term is $\left(m_L^2 + 2\frac{q_L q_\Phi}{q_\Phi^2}(m_{\bar{\phi}}^2 - m_\phi^2)\right)|L|^2$
- Can choose sign of q_L to make slepton mass positive

SUSY-breaking lifts the vacuum degeneracy

- ϕ-vacuum:
  - Recall: $\langle|\phi|\rangle \simeq \langle|\bar{\phi}|\rangle \simeq M/\sqrt{\lambda_1}$, $\langle|s|\rangle \simeq h_{\lambda_1}/\lambda_1^2$
  - Recall: $V_{\text{soft}} = m_\phi^2|\phi|^2 + m_{\bar{\phi}^2}|\bar{\phi}|^2 + h_{\lambda_1} s\phi\bar{\phi} + c.c.$, and V_F, $V_D \simeq 0$
  - ϕ-vacuum energy: $V_\phi = \frac{M^2}{\lambda_1}\left(m_\phi^2 + m_{\bar{\phi}}^2 - \frac{h_{\lambda_1}^2}{2\lambda_1^2}\right)$
- h-vacuum similarly:
  - h-vacuum energy: $V_h = \frac{M^2}{\lambda_3}\left(m_{h_1}^2 + m_{h_2}^2 - \frac{h_{\lambda_3}^2}{2\lambda_3^2}\right)$
- Must have: $V_h > V_\phi$, ϕ-vacuum a minimum, and h-vacuum a maximum
- Constraint on SUSY-breaking scenarios
- NB V_h, V_ϕ are both $O(M^2 M_{\text{SUSY}}^2)$

Constraints in 3 common SUSY-breaking scenarios

Requiring the ϕ-vacuum be lowest energy leads to:

- Anomaly-mediated Supersymmetry-breaking (AMSB)
  - $\lambda_1 \left(\frac{q_H}{q_\Phi}\right)^2 \lesssim \lambda_3.$
- Gauge-mediated Supersymmetry-breaking (GMSB)
  - $\lambda_1 \left(\frac{q_H}{q_\Phi}\right)^2 \gtrsim \lambda_3.$
- Constrained MSSM (CMSSM)
  - Universal scalar soft mass m_0, universal trilinear coupling A
  - [$2m_0^2 - A^2/2 > 0$ and $\lambda_1 > \lambda_3$] or [$2m_0^2 - A^2/2 < 0$ and $\lambda_1 < \lambda_3$]

SUSY spectrum for AMSB

$m_{\frac{3}{2}}$	40TeV	80TeV	140TeV
(L, e)	$(0, 0.18)$	$(0, 0.72)$	(0,1.96)
$\tilde{g}$	899	1684	2801
$\tilde{t}_2$	536	1001	1629
$\tilde{\tau}_1$	111	280	532
$\tilde{\tau}_2$	223	405	683
$\tilde{\nu}_e$	190	393	689
$\chi_1^{0,\pm}$	132	265	460
h	116	121	125
$H^{\pm}$	419	734	1129
μ_h	603	1111	1852
δa_μ	60×10^{-10}	16×10^{-10}	5.4×10^{-10}

Table: sAMSB spectra (GeV) and δa_μ for $m_t = 172.9$ GeV and $\tan\beta = 16$. $m_{\frac{3}{2}}$: gravitino mass; (L, e): effective leptonic D-terms (TeV). Note $\delta a_\mu^{\mathrm{exp}} = 29.5(8.8) \times 10^{-10}$

The loop-corrected potential

$$V = M^4 + \Delta V_1,$$

$$\begin{aligned}\Delta V_1 &= \frac{1}{32\pi^2}\left[(\lambda_1^2 s^2 + \lambda_1 M^2)^2 \ln\left(\frac{\lambda_1^2 s^2 + \lambda_1 M^2}{\mu^2}\right)\right. \\ &+ (\lambda_1^2 s^2 - \lambda_1 M^2)^2 \ln\left(\frac{\lambda_1^2 s^2 - \lambda_1 M^2}{\mu^2}\right) \\ &- \left. 2\lambda_1^4 s^4 \ln\left(\frac{\lambda_1^2 s^2}{\mu^2}\right)\right] + 2 \times (\lambda_1 \to \lambda_3)\end{aligned}$$

For $\lambda_{1,3} s^2 \gg M^2$

$$V(s) \simeq M^4 \left[1 + \alpha \ln \frac{2s^2}{s_c^2}\right], \quad \alpha = \frac{\lambda^2}{16\pi^2}, \quad \lambda = \sqrt{\lambda_1^2 + 2\lambda_3^2}, \quad s_c^2 = M^2/\lambda.$$

Thermal inflation

Yamamoto, Binetruy and Gaillard, Lazarides et al

- Supersymmetric flat direction X
- Lifted by thermal corrections, supersymmetry-breaking and non-renormalisable terms
- $V(X) = V_0 + m_X^2|X|^2 + cT^2|X|^2 + \cdots$
  - NB X-vacuum unstable; $m_X^2 < 0$
- Universe trapped at $|X| = 0$ until $T_c = \sqrt{-m_X^2/c}$.
- Inflation (θ-inflation) (re)starts if potential dominates energy density: $V_0 > \rho(T)$
- Result: $N_\theta \simeq 15$ e-foldings of thermal inflation
- Reduces high scale inflation $N_{k_0} \simeq 42$, so $n_s \simeq 0.976$

Gravitino constraints and supersymmetry-breaking

$$\Omega_{\rm LSP} h^2 \simeq 6 \times 10^{-3} \omega_{\tilde{G}} \frac{m_{\rm LSP}}{100\ {\rm GeV}} \left(\frac{M_{\rm SUSY}}{1\ {\rm TeV}}\right)^{\frac{1}{2}} \simeq 0.11 \text{ (WMAP)}$$

$$\begin{aligned} \Delta V &= v_\phi^2 M_{\rm SUSY}^2 = M^2 \left(\frac{\tilde{h}^2_{\lambda_1}}{2\lambda_1} - \frac{\tilde{h}^2_{\lambda_3}}{2\lambda_3} - \frac{\bar{m}^2_\phi}{\lambda_1} + \frac{\bar{m}^2_h}{\lambda_3} \right) \\ &= v_\phi^2 \left(\frac{1}{4} \left[\tilde{h}^2_{\lambda_1} - \tilde{h}^2_{\lambda_3} \frac{\lambda_1}{\lambda_3} \right] - \frac{1}{2} \left[\bar{m}^2_\phi - \bar{m}^2_h \frac{\lambda_1}{\lambda_3} \right] \right). \end{aligned}$$

In the three benchmark supersymmetry-breaking schemes:

- $$M_{\rm SUSY}^2 \simeq \begin{cases} m^2_{\frac{3}{2}} \left(\frac{g'^2}{16\pi^2}\right)^2 {\rm Tr}(Y'^2) \left[q_\phi^2 - q_H^2 \left(\frac{\lambda_1}{\lambda_3}\right)\right], & \text{(AMSB)}, \\ \Lambda_s^2 \frac{\lambda_1}{\lambda_3} \left(\frac{g'^2}{16\pi^2}\right)^2 \left[q_H^2 \left(\frac{\lambda_1}{\lambda_3}\right) - q_\phi^2 \right] & \text{(GMSB)}, \\ \frac{1}{2}(m_0^2 - A^2/4) \left[\frac{\lambda_1}{\lambda_3} - 1\right] & \text{(CMSSM)}. \end{cases}$$

Gravitino constraints (AMSB)

$$\Omega_{\rm LSP} h^2 \simeq 6 \times 10^{-3} \frac{m_{\rm LSP}}{100\ \rm GeV} \left(\frac{M_{\rm SUSY}}{1\ \rm TeV}\right)^{\frac{1}{2}} \simeq 0.11$$

- AMSB empirical formula: $m_{\rm LSP} \simeq 3.3 \times 10^{-3} m_{\frac{3}{2}}$ [15]
- $M^2_{\rm SUSY} \simeq m^2_{\frac{3}{2}} \left(\frac{g'^2}{16\pi^2}\right)^2 \mathrm{Tr}(Y'^2) \left[q^2_\phi - q^2_H \left(\frac{\lambda_1}{\lambda_3}\right)\right]$
- Correct μ_h-parameter (EW vacuum): $q^2_\phi g'^2 \simeq \frac{\lambda_3}{\lambda_1}$
- Hence $m_{\frac{3}{2}} \simeq 350 \left(\frac{q_\phi}{\sqrt{\mathrm{Tr}(Y'^2)}} \frac{\lambda_3}{\lambda_1}\right)^{\frac{1}{3}}$ TeV
- Allowed range: $150\ \ \mathrm{TeV} \lesssim m_{\frac{3}{2}} \lesssim 240\ \ \mathrm{TeV}$
- Higgs mass 125 GeV with $m_{\frac{3}{2}} = 140$ TeV

[15] Hindmarsh, Jones (2012)

Gravitino constraints (GMSB)

$$\Omega_{\rm LSP} h^2 \simeq 6 \times 10^{-3} \frac{m_{\rm LSP}}{100 \text{ GeV}} \left(\frac{M_{\rm SUSY}}{1 \text{ TeV}}\right)^{\frac{1}{2}} \simeq 0.11$$

- Lightest supersymmetric particle is gravitino
- $M_{\rm SUSY}$ is controlled by gaugino masses
- Gravitino constraint: $\left(\frac{m_{\frac{3}{2}}}{1 \text{ TeV}}\right)^2 \simeq 5\sqrt{N_{\rm mi}}\sqrt{\frac{\lambda_3}{\lambda_1}}\left(\frac{M_2}{1 \text{ TeV}}\right)^{-1}$
- (SU(2) gaugino mass M_2, messenger index $N_{\rm mi}$)
- OK for e.g. $m_{\frac{3}{2}} \sim 1$ TeV, $M_{\rm gaugino} \sim$ few TeV.
- **But** messenger mass constrained by NLSP decays during BBN[16]
- $\left(\frac{m_{\frac{3}{2}}}{1 \text{ TeV}}\right)^3 \lesssim 0.5\sqrt{N_{\rm mi}}\sqrt{\frac{\lambda_3}{\lambda_1}}$

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[16] Gherghetta, Giudice, Riotto (1998)

Gravitino constraints (CMSSM)

$$\Omega_{\rm LSP} h^2 \simeq 6 \times 10^{-3} \frac{m_{\rm LSP}}{100\ \rm GeV} \left(\frac{M_{\rm SUSY}}{1\ \rm TeV}\right)^{\frac{1}{2}} \lesssim 0.11$$

- Dark matter can be neutralino produced by standard freeze-out
- Density constraint on thermally-produced gravitinos:

$$\left(m_0^2 - A^2/4\right)^{\frac{1}{2}} \lesssim 5 \times 10^2 \left(\frac{\lambda_1}{\lambda_3} - 1\right)^{-\frac{1}{2}} \left(\frac{m_{\rm LSP}}{100\ \rm GeV}\right)^{-2}\ \rm TeV.$$

- $T_{\rm rh2} \simeq 10^9$ GeV means $m_{\frac{3}{2}} \gtrsim$ (few) TeV to avoid BBN constraints