Gravitino DM Relic Abundance and e/n EDM due to Heavy Scalars and a 125 GeV Higgs in (D3-D7) μ - Split SUSY

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Phenomenological Model

- In $\mathcal{N}=1$ supergravity in $\mathbb{R}^{1,3}$ obtained by compactifying type IIB supergravity on a complex three-fold('s orientifold) :

$$K_{\rm Pheno} = -ln\left[-i(\tau-\bar{\tau})\right] - ln\left(-i\int_{CY_3}\Omega\wedge\bar{\Omega}\right)$$
$$-2\,ln[a_B(\sigma_B+\bar{\sigma}_B-\gamma K_{\rm geom})^{\frac{3}{2}} - (\sum_i a_{S,i}(\sigma_{S,i}+\bar{\sigma}_{S,i}-\gamma K_{\rm geom}))^{\frac{3}{2}} + \mathcal{O}(1)\mathcal{V}]$$

where the divisor volumes σ_α are expressible in terms of "Kähler" coordinates $T_\alpha, \mathcal{M}_\mathcal{I}$

$$\sigma_\alpha \sim T_\alpha - \left[i\mathcal{K}_{\alpha bc}c^b\mathcal{B}^c + iC_\alpha^{\mathcal{M}_\mathcal{I}\bar{\mathcal{M}}_{\bar{\mathcal{J}}}}(\mathcal{V})Tr\left(\mathcal{M}_\mathcal{I}\mathcal{M}^\dagger_{\bar{\mathcal{J}}}\right)\right],$$

$\alpha=(B,\{S,i\})$ and $\mathcal{M}_\mathcal{I}\equiv SU(3_c)\times SU(2)_L$ bifundamental matter field $a_{\mathcal{I}=2}$, $SU(3_c)\times U(1)_R$ bifundamental matter field $a_{\mathcal{I}=4}$, $SU(2)_L\times U(1)_L$ bifundamental matter field $a_{\mathcal{I}=1}$, $U(1)_L\times U(1)_R$ bifundamental matter field $a_{\mathcal{I}=3}$, $SU(2)_L\times U(1)_L$ bifundamental $\tilde{z}_{1,2}$.

- The intersection matrix: $C_\alpha^{a_I \bar{a}_{\bar{J}}} \sim \delta_\alpha^B C_\alpha^{I\bar{J}}, C_\alpha^{a_I \bar{\bar{z}}_{\bar{\jmath}}} = 0$, $\rho_{S,B}, \mathcal{G}^a = c^a - \tau b^a$ being complex axionic fields (α, a running over the real dimensionalities of mutually orthogonal real sub-spaces of the internal manifold's cohomology complex).

- The phenomenological superpotential is given as under:

$$W_{\text{Pheno}} \sim \left(z_1^{18} + z_2^{18}\right)^{n^s} e^{-n^s vol(\Sigma_S) - (\alpha_S z_1^2 + \beta_S z_2^2 + \gamma_S z_1 z_2)},$$

where the bi-fundamental $\tilde{z}_i$ in K will be equivalent to the $z_{1,2} \in \mathbb{C}$ in W. It is expected that $\mathcal{M}_{\mathcal{I}}, T_{S,B}, \mathcal{G}^a$ will constitute the $\mathcal{N} = 1$ chiral coordinates. The intersection matrix elements $\kappa_{S/Bab}$ and the volume-dependent $C_\alpha^{\mathcal{M}_{\mathcal{I}} \bar{\mathcal{M}}_{\bar{\mathcal{J}}}}(\mathcal{V})$, are chosen in such a way that at a local (meta-stable) minimum:

$$\langle \sigma_S \rangle \sim \langle (T_S + \bar{T}_S) \rangle - iC^{\tilde{z}_i \bar{\tilde{z}}_{\bar{j}}}(\mathcal{V}) Tr\left(\langle \tilde{z}_i \rangle \langle \bar{\tilde{z}}_{\bar{j}} \rangle\right) \sim \mathcal{O}(1)$$

$$\langle \sigma_B \rangle \sim \langle (T_B + \bar{T}_B) \rangle - iC^{\tilde{z}_i \bar{\tilde{z}}_{\bar{j}}}(\mathcal{V}) Tr\left(\langle \tilde{z}_i \rangle \langle \bar{\tilde{z}}_{\bar{j}} \rangle\right) - iC^{a_I \bar{a}_{\bar{J}}}(\mathcal{V}) Tr\left(\langle a_I \rangle \langle \bar{a}_{\bar{J}} \rangle\right)$$
$$\sim e^{f \langle \sigma_S \rangle},$$

$f \lesssim 1$, and the stabilized values of T_α around the meta-stable local minimum:

$$\langle \Re e T_S \rangle, \langle \Re e T_B \rangle \sim \mathcal{O}(1).$$

- In the context of $\mathcal{N}=1$ type IIB orientifolds, α, a index respectively involutively even, odd sectors of $h^{1,1}(CY_3)$ under a holomorphic, isometric involution. If the volume $\mathcal{V}$ of the internal manifold is large in string length units, one sees that one obtains a hierarchy between the stabilized values $\langle \Re e \tau_{S,B} \rangle$ but not $\langle \Re e T_{S,B} \rangle$.
- To realize the above phenomenological model, *locally*, in string theory consider type IIB compactified on the orientifold (involving a *local large-volume holomorphic isometric involution*) of a Swiss-Cheese Calabi-Yau in the large volume limit that includes perturbative Balasubramanian et al [2005] and non-perturbative AM, P. Shukla [2007, 2010]; M.Dhuria, AM [2012] α' corrections and non-perturbative instanton-corrections.

- For this purpose, we will consider a space-time filling $D3$-brane and multiple fluxed stacks of space-time filling $D7$-branes wrapping a single four-cycle, the big divisor, with different choice of small two-form fluxes turned on the different two-cycles homologously non-trivial from the point of view of this four-cycle's Homology (for the purpose of decomposing initially adjoint-valued matter fields to bi-fundamental matter fields, for generating the SM gauge groups and to effect gauge-coupling unification at the string scale). Then $z_{1,2}$ get identified with the $D3$-brane's position moduli, τ is the axion-dilaton modulus and $\mathcal{G}^a$ are NS-NS and RR two-form axions complexified by the axion-dilaton modulus.

- We will assume that near (but not globally): $|z_1| \sim \mathcal{V}^{\frac{1}{36}}$, $|z_2| \sim \mathcal{V}^{\frac{1}{36}}$, $|z_3| \sim \mathcal{V}^{\frac{1}{6}}$, the Calabi-Yau is diffeomorphic to the Swiss-Cheese $\mathbb{WCP}^4_{1,1,1,6,9}[18]$. The defining hypersurface for the same is: $u_1^{18} + u_2^{18} + u_3^{18} + u_4^3 + u_5^2 - 18\psi \prod_{i=1}^5 u_i - 3\phi(u_1u_2u_3)^6 = 0$.
- Corresponds to a hypersurface in an ambient complex four-fold: $P(x_1, ..., x_5; \xi) = 0$ after $\mathbb{Z}_3$-singularity resolution with the toric data for the same J.Louis et al [2012]:

	x_1	x_2	x_3	x_4	x_5	ξ
Q^1	1	1	1	6	0	9
Q^2	0	0	0	1	1	2

.

In $x_2 \neq 0$(i.e. away from the $\mathbb{Z}_3$-singular $(0,0,0,x_4,x_5)$ in $\mathbb{WCP}_{1,1,1,6,9}[18]$), $\xi \neq 0$, the following are the gauge-invariant coordinates: $z_1 = \frac{x_1}{x_2}, z_2 = \frac{x_3}{x_2}, z_3 = \frac{x_4^2}{x_2^3 \xi}, z_4 = \frac{x_5^2 x_2^9}{\xi}$.

- We henceforth assume the Calabi-Yau hypersurface to be written in this coordinate patch as: $z_1^{18}+z_2^{18}+\mathcal{P}(z_{1,2,3,4};\psi,\phi)=0$. The divisor $\{x_5=0\}\cap\{P(x_{1,2,3,4,5};\xi)=0\}$ is rigid with $h^{0,0}=1$ satisfying the Witten's unit-arithmetic genus condition, and that the Calabi-Yau volume can be written in the Swiss-Cheese form as

$$\mathrm{vol}(CY_3)=\frac{\tau_4^{\frac{3}{2}}}{18}-\frac{\sqrt{2}\tau_5^{\frac{3}{2}}}{9},$$

implying that the 'small divisor' Σ_s is

$$\{x_5=0\}\cap\left\{z_1^{18}+z_2^{18}+\mathcal{P}(z_{1,2,3},z_4=0;\psi,\phi)=0\right\}$$

and the 'big divisor' Σ_B is

$$\{x_4=0\}\cap\left\{z_1^{18}+z_2^{18}+\mathcal{P}(z_{1,2,4},z_3=0;\psi,\phi)=0\right\}.$$

- Near

$$C_3 : |z_1| \sim \mathcal{V}^{\frac{1}{36}},\ |z_2| \sim \mathcal{V}^{\frac{1}{36}},\ |z_3| \sim \mathcal{V}^{\frac{1}{6}}$$

the Calabi-Yau can be thought of, locally, as a complex three-fold $\mathcal{M}_3$ which is a T^3 (swept out by ($argz_1$,$argz_2$, $argz_3$)-fibration over a large base $(|z_1|, |z_2|, |z_3|)$) - precisely apt for application of mirror symmetry as three T-dualities a la S(trominger) Y(au) Z(aslow); C_3 is almost a s(pecial) Lag(rangian) sub-manifold M. Dhuria, AM [2012] because it satisfies (using the large volume estimate of $K_{\rm geom}$ using Donaldson's Algorithm AM [2012] guided by GLSM-based estimate AM, P.Shukla [2010]) the requirement that

$$f^*J \approx 0,\ \Re e\left(f^*e^{i\theta}\Omega\right)\Big|_{\theta=\frac{\pi}{2}} \approx {\rm vol}(C_3),\ \Im m\left(f^*e^{i\theta}\Omega\right)\Big|_{\theta=\frac{\pi}{2}} \approx 0$$

where $f : C_3 \rightarrow CY_3$.

- $\mathcal{P}_{\Sigma_S}\Big|_{D3|_{\rm near\ C_3\hookrightarrow\Sigma_B}}, \mathcal{P}_{\Sigma_B}|_{\rm near\ C_3\hookrightarrow\Sigma_B} \sim z_1^{18} + z_2^{18}$

Single $D7$-Brane and a single $D3$-Brane

- The $\mathcal{N}=1$ coordinates Jockers and Louis, 2004:
  $S = \tau + \kappa_4^2\mu_7\mathcal{L}_{A\bar{B}}\zeta^A\bar{\zeta}^{\bar{B}}, \tau = c_0 + ie^{-\phi}; \mathcal{G}^a = c^a - \tau\mathcal{B}^a$
  $\mathcal{B} \equiv b^a - 2\pi\alpha' f^a$, where f^a are the components of elements of two-form fluxes valued in $i^*\left(H^2_-(CY_3)\right)$,
  $i : \Sigma^\Lambda = \Sigma_B \cup \sigma(\Sigma_B) \hookrightarrow CY_3$.
- $\mathcal{L}_{A\bar{B}} = \frac{\int_{\Sigma^\Lambda} \tilde{s}_A \wedge \tilde{s}_{\bar{B}}}{\int_{CY_3} \Omega\wedge\bar{\Omega}}$, $\tilde{s}_A$ forming a basis for $H^{(2,0)}_{\bar{\partial},-}(\Sigma^\Lambda)$.
- ζ represents the fluctuations of $D7$-brane in the CY_3 normal to Σ^Λ i.e. $\zeta \in H^0(\Sigma^\Lambda, N\Sigma^\Lambda)$.
- $T_\alpha = \frac{3i}{2}(\rho_\alpha - \frac{1}{2}\kappa_{\alpha bc}c^b\mathcal{B}^c) + \frac{3}{4}\mathrm{vol}(\Sigma_\alpha) + \frac{3i}{4(\tau-\bar{\tau})}\kappa_{\alpha bc}\mathcal{G}^b(\mathcal{G}^c - \bar{\mathcal{G}}^c)$
  $+3i\kappa_4^2\mu_7 l^2\delta^B_\alpha C^{I\bar{J}}_\alpha a_I\bar{a}_{\bar{J}} + \frac{3i}{4}\delta^B_\alpha\tau Q_{\tilde{f}} +$
  $\frac{3i}{2}\mu_3 l^2(\omega_\alpha)_{i\bar{j}}\left[z^i\bar{z}^{\bar{j}} - \frac{i}{2}\bar{z}^{\tilde{a}}(\bar{\mathcal{P}}_{\tilde{a}})^{\bar{j}}_l z^l\right]$.
- $C^{I\bar{J}}_\alpha = \int_{\Sigma^\Lambda} i^*\omega_\alpha \wedge A^I \wedge A^{\bar{J}}$, $\omega_\alpha \in H^{(1,1)}_{\bar{\partial},+}(CY_3)$ and $A^I \in H^{(0,1)}_{\bar{\partial},-}(\Sigma^\Lambda)$.

- Wilson line moduli $a_{I=1,\ldots,h^{0,1}_{-}(\Sigma^\Lambda)}$ are defined via:
  $A(x,y) = A_\mu(x)dx^\mu P_-(y) + a_I(x)A^I(y) + \bar{a}_{\bar{J}}(x)\bar{A}^{\bar{J}}(y)$, where $P_-(y) = 1$ if $y \in \Sigma^\Lambda$ and -1 if $y \in \sigma(\Sigma^\Lambda)$.
- $z^{\tilde{a}}$ are $D=4$ complex scalar fields due to c.s. deformations of the Calabi-Yau orientifold defined via:
  $\delta g_{\bar{i}\bar{j}}(z^{\tilde{a}}) = -\frac{i}{||\Omega||^2} z^{\tilde{a}} \left(\chi_{\tilde{a}}\right)_{\bar{i}jk} \left(\bar{\Omega}\right)^{jkl} g_{l\bar{j}}$, where $\left(\chi_{\tilde{a}}\right)_{\bar{i}jk}$ are components of elements of $H^{(2,1)}_{\bar{\partial},-}(CY_3)$
- $\left(\mathcal{P}_{\tilde{a}}\right)^i_{\bar{j}} \equiv \frac{1}{||\Omega||^2}\bar{\Omega}^{ikl}\left(\chi_{\tilde{a}}\right)_{kl\bar{j}}$, i.e.$\mathcal{P}: TCY_3^{(1,0)} \longrightarrow TCY_3^{(0,1)}$ via the transformation:$z^i \stackrel{\text{c.s. deform}}{\longrightarrow} z^i + \frac{i}{2} z^{\tilde{a}} \left(\mathcal{P}_{\tilde{a}}\right)^i_{\bar{j}} \bar{z}^{\bar{j}}$.
- z^i denotes the geometric fluctuations of $D3$-brane inside the Calabi-Yau: $z(x) = z^i(x)\partial_i + \bar{z}^{\bar{i}}(\bar{x})\bar{\partial}_{\bar{i}}$.
- $Q_{\tilde{f}} \equiv l^2 \int_{\Sigma^\Lambda} \tilde{f} \wedge \tilde{f}$; where $l = 2\pi\alpha'$
  $\tilde{f} \in \tilde{H}^2_-(\Sigma^\Lambda) \equiv \text{coker}\left(H^2_-(CY_3) \stackrel{i^*}{\rightarrow} H^2_-(\Sigma^\Lambda)\right)$.

- The most non-trivial example of *involutions which are meaningful only at large volumes is mirror symmetry implemented as three T-dualities a la S(trominger) Y(au) Z(aslow) to a Calabi-Yau* which *locally* can be thought of as a T^3-fibration over a (large) base; all Calabi-Yau's with mirrors (in the conventional sense) are expected to have such a local fibration.

- Four local appropriate harmonic distribution one-forms odd under a large-volume involution (analogous to the involutive SYZ mirror symmetry requiring a large base of T^3-fibration) that are in $coker\left(H^{(0,1)}_{\bar{\partial},-}(CY_3)\stackrel{i^*}{\rightarrow}H^{(0,1)}_{\bar{\partial},-}(\Sigma^\Lambda)\right)$ localized along C_3 corresponding to the location of the $D3$-brane can be written as: $A_I|_{C_3}\sim\delta\left(|z_1|-\mathcal{V}^{\frac{1}{36}}\right)\delta\left(|z_2|-\mathcal{V}^{\frac{1}{36}}\right)\mathbb{A}_I$, where :

$$\mathbb{A}_1\sim -z_1^{18}z_2^{19}dz_1+z_1^{19}z_2^{18}dz_2,$$
$$\mathbb{A}_2\sim -z_1^{18}z_2dz_1+z_2^{18}z_1dz_2,$$
$$\mathbb{A}_3\sim -z_1^{18}z_2^{37}dz_1-z_2^{18}z_1^{37}dz_1,$$
$$\mathbb{A}_4\sim -z_1^{36}z_2^{37}dz_1+z_2^{36}z_1^{37}dz_2 \text{ M.Dhuria, A.M. [2012]}.$$

- The intersection matrices $C^B_{I\bar{J}}$ were also estimated in M.Dhuria, AM [2012].

- For a single $D3$- and $D7$-brane, the following basis of fluctuations simultaneously diagonalizes $K_{\mathcal{I}\bar{\mathcal{J}}}$ and $Z_{\mathcal{I}\mathcal{J}}, \mathcal{I}, \mathcal{J}$ indexing $\delta a_I, \delta z_i$:

$$\delta\mathcal{A}_4 \sim \delta a_4 + \mathcal{V}^{-\frac{3}{5}}\delta a_3 + \mathcal{V}^{-\frac{6}{5}}\delta a_1 + \mathcal{V}^{-\frac{9}{5}}\delta a_2 + \mathcal{V}^{-2}\left(\delta z_1 + \delta z_2\right);$$
$$\delta\mathcal{A}_3 \sim -\delta a_3 + \mathcal{V}^{-\frac{3}{5}}\delta a_4 - \mathcal{V}^{-\frac{3}{5}}\delta a_1 - \mathcal{V}^{-\frac{7}{5}}\delta a_2 + \mathcal{V}^{-\frac{8}{5}}\left(\delta z_1 + \delta z_2\right);$$
$$\delta\mathcal{A}_1 \sim \delta a_1 - \mathcal{V}^{-\frac{3}{5}}\delta a_3 + \mathcal{V}^{-1}\delta a_2 - \mathcal{V}^{-\frac{6}{5}}\delta a_4 + \mathcal{V}^{-\frac{6}{5}}\left(\delta z_1 + \delta z_2\right);$$
$$\delta\mathcal{A}_2 \sim -\delta a_2 - \mathcal{V}^{-1}\delta a_1 + \mathcal{V}^{-\frac{7}{5}}\delta a_3 - \mathcal{V}^{-\frac{3}{5}}\left(\delta z_1 + \delta z_2\right);$$
$$\delta\mathcal{Z}_2 \sim -\frac{(\delta z_1 + \delta z_2)}{\sqrt{2}} - \mathcal{V}^{-\frac{6}{5}}\delta a_1 + \mathcal{V}^{-\frac{3}{5}}\delta a_2 + \mathcal{V}^{-\frac{8}{5}}\delta a_3 + \mathcal{V}^{-2}\delta a_4;$$
$$\delta\mathcal{Z}_1 \sim \frac{(\delta z_1 - \delta z_2)}{\sqrt{2}} - \mathcal{V}^{-\frac{6}{5}}\delta a_1 + \mathcal{V}^{-\frac{3}{5}}\delta a_2 + \mathcal{V}^{-\frac{8}{5}}\delta a_3 + \mathcal{V}^{-2}\delta a_4,$$

- With appropriate fluxes (DDF (2004); Ganor 1997,8): $W \sim W_{ED1-ED3} \sim \left(\mathcal{P}_{\Sigma_S}\Big|_{D3|_{\rm near\ C_3 \hookrightarrow \Sigma_B}} \sim z_1^{18}+z_2^{18}\right)^{n^s} e^{in^sT_s}\Theta_{n^s}(\mathcal{G}^a,\tau);$ $z_1=x_1/x_2,\ z_2=x_3/x_2,\ z_3=x_4/x_2^6$ in the non-singular $x_2=1$ coordinate patch (i.e. away from the $\mathbb{Z}_3$-singular $(0,0,0,x_4,x_5)$) , $n^s\equiv\mathcal{O}(1)$ $D3$-instanton number, $\Theta_{n^s}(\tau,\mathcal{G}^a)$ (which encodes the contribution of $D1$-instantons in an $SL(2,\mathbf{Z})$-covariant form)$\equiv$ the holomorphic Jacobi theta function of index n^s.

- The holomorphic pre-factor $\left(\mathcal{P}_{\Sigma_S}\Big|_{D3|_{\rm near\ C_3 \hookrightarrow \Sigma_B}} \sim z_1^{18}+z_2^{18}\right)^{n^s}$ represents a one-loop determinant of fluctuations around the $ED3$-instanton Baumann et al (2006); $e^{in^sT_s}=e^{-n^s{\rm vol}(\Sigma_S)+i...}$ being a section of the inverse divisor bundle $n^s[-\Sigma_S]$, the holomorphic prefactor has to be a section of $n^s[\Sigma_S]$ to compensate and the holomorphic prefactor, a section of $n^s[\Sigma_S]$ having no poles, must have zeros of order n^s on a manifold homotopic to Σ_S Ganor(1997,8).

- Also, coefficient of quadratic term $(\omega_\alpha)_{i\bar{j}}\, z^i \left(\bar{z}^{\bar{j}} - \frac{i}{2}\left(\mathcal{P}_{\tilde{a}}\right)^{\bar{j}}{}_l\, \bar{z}^{\tilde{a}} z^l\right)$ arising in T_B due to inclusion of position moduli z_i can be shown to be $\mathcal{O}(1)$ by calculating $(\omega_B)_{i\bar{j}} \sim (\omega_S)_{i\bar{j}} \sim \mathcal{O}(1)$ near $z_{1,2} \sim \frac{\mathcal{V}^{\frac{1}{36}}}{\sqrt{2}}$ M. Dhuria, AM [2012]. Therefore one can self-consistently show M. Dhuria, AM [2012]; AM, P. Shukla [2010] that near $\langle|z_{1,2}|\rangle \sim \mathcal{V}^{\frac{1}{36}} M_p, \langle|z_3|\rangle \sim \mathcal{V}^{\frac{1}{6}} M_p, \langle|a_1|\rangle \sim \mathcal{V}^{-\frac{2}{9}} M_p, \langle|a_2|\rangle \sim \mathcal{V}^{-\frac{1}{3}} M_p, \langle|a_3|\rangle \sim \mathcal{V}^{-\frac{13}{18}} M_p, \langle|a_4|\rangle \sim \mathcal{V}^{-\frac{11}{9}} M_p; \zeta^{A=1,\ldots,h^{0,2}_-(\Sigma_B|_{C_3})} \equiv 0$ (implying rigidity of the non-rigid Σ_B); $b^a/c^a \sim \frac{\pi}{\mathcal{O}(1)k^a(\sim\mathcal{O}(10))} M_p$, one obtains a local meta-stable dS-like minimum corresponding to the positive semi-definite potential $e^K G^{T_S\bar{T}_S}|D_{T_S}W|^2$

- Stabilized values: $\mathrm{vol}\,(\Sigma_B)=\Re e(\sigma_B)\sim\mathcal{V}^{\frac{2}{3}},\mathrm{vol}\,(\Sigma_S)=\Re e\sigma_S\sim\mathcal{V}^{\frac{1}{18}}$ such that $\Re eT_S\sim\mathcal{V}^{\frac{1}{18}}$ and in the dilute flux approximation, gauge couplings corresponding to the gauge theories living on stacks of $D7$ branes wrapping the “big" divisor Σ_B will given by: $g_{YM}^{-2}\sim\Re e(T_B)\sim\mathcal{V}^{\frac{1}{18}}\sim O(1)$ (justified by the partial cancelation between between σ_B and $C_{I\bar{J}}a_I\bar{a}_{\bar{J}}$ i.e $(Vol(\Sigma_B)+C_{I\bar{J}}a_I\bar{a}_{\bar{J}}+h.c.\sim\mathcal{V}^{\frac{1}{18}})$.

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$$1/g^2_{j=SU(3)\ \mathrm{or}\ SU(2)}=Re(T_B)+\mathcal{O}(F_j^2),$$

$F_j^2=F_j^\alpha F_j^\beta\kappa_{\alpha\beta}+\tilde{F}_j^a\tilde{F}_j^b\kappa_{ab}$, F_j^α are the components of the magnetic fluxes for the $j-$th stack expanded out in the basis of $i^*\omega_\alpha$, $\omega_\alpha\in H_-^{1,1}(CY_3)$, and $\tilde{F}_j^a$ are the components of the magnetic fluxes for the j-th stack expanded out in the basis $\tilde{\omega}_a\in coker\left(H_-^{(1,1)}(CY_3)\stackrel{i^*}{\rightarrow}H_-^{(1,1)}(\Sigma_B)\right)$;
$\kappa_{\alpha\beta}=\int_{\Sigma_B}i^*\omega_\alpha\wedge i^*\omega_\beta,\kappa_{ab}=\int_{\Sigma_B}\tilde{\omega}_a\wedge\tilde{\omega}_b$.

- The Dirac mass term in $\mathcal{N}=1$ SUGRA is given by $e^{\frac{K}{2}}\mathcal{D}_i D_j W \bar{\chi}_L \chi_R$ where

$$\mathcal{D}_i D_j W = \partial_i \partial_j W + (\partial_i \partial_j K) W + \partial_i K D_J W + \partial_J K D_i W - (\partial_i K \partial_j K) W - \Gamma^k_{ij} D_k W.$$

Considering fluctuations in $\mathcal{Z}_i : \mathcal{Z}_i \rightarrow \langle \mathcal{Z}_i \rangle + \delta \mathcal{Z}_i$, $\hat{Y}^{\rm eff}_{\delta\mathcal{Z}_i \delta\tilde{\mathcal{A}}_J \delta\tilde{\mathcal{A}}_K} \equiv \frac{\mathcal{O}(\delta\mathcal{Z}_i)-\text{term in } e^{\frac{K}{2}}\mathcal{D}_J D_K W}{\sqrt{K_{\delta\mathcal{Z}_i \delta\bar{\mathcal{Z}}_i} K_{\delta\mathcal{A}_J \delta\bar{\mathcal{A}}_J} K_{\delta\mathcal{A}_K \delta\bar{\mathcal{A}}_K}}}$; the corresponding Dirac mass will be given by $\langle \delta\mathcal{Z}_i \rangle \hat{Y}^{\rm eff}_{\delta\mathcal{Z}_i \delta\tilde{\mathcal{A}}_J \delta\tilde{\mathcal{A}}_K}$. One can show that under 1-loop RG flow, the Yukawas in our setup change by $\mathcal{O}(1)$ M.Dhuria, AM [2012] and that its possible that the Higgs vev flows down to 246 GeV AM, P.Shukla [2010].

- $$e^{\frac{K}{2}}\mathcal{D}_{\mathcal{A}_1}D_{\mathcal{A}_3}W\Big|_{\mathcal{Z}_i\sim\mathcal{O}(1)\mathcal{V}^{\frac{1}{36}}}(M_s)\sim\mathcal{V}^{-\frac{4}{9}}$$ implying that
$$246\ GeV\times\hat{Y}^{\rm eff}_{\delta\mathcal{Z}_i\delta\tilde{\mathcal{A}}_1\delta\tilde{\mathcal{A}}_3}\Big|_{\mathcal{V}\sim 10^5}\tilde{<}\ \mathcal{O}(1MeV)$$
- $$e^{\frac{K}{2}}\mathcal{D}_{\mathcal{A}_2}D_{\mathcal{A}_4}W\Big|_{\mathcal{Z}_i\sim\mathcal{O}(1)\mathcal{V}^{\frac{1}{36}}}(M_s)\sim\mathcal{V}^{-\frac{4}{9}}$$ implying that
$$246\ GeV\times\hat{Y}^{\rm eff}_{\delta\mathcal{Z}_i\delta\tilde{\mathcal{A}}_2\delta\tilde{\mathcal{A}}_4}\Big|_{\mathcal{V}\sim 10^5}\tilde{<}\ \mathcal{O}(1)\sim\mathcal{O}(10)MeV$$
- This suggests that possibly, the fermionic superpartners of $\mathcal{A}_1$ and $\mathcal{A}_3$ correspond respectively to e_L and e_R and the fermionic superpartners of $\mathcal{A}_2$ and $\mathcal{A}_4$ correspond respectively to the first generation u_L and u_R.

Multiple $D7$ Branes

- Like Intersecting Brane Models, using four stacks of wrapped $D7$ branes in groups of 3,2,1,1, after turning on of block-diagonal two-form fluxes on four two-cycles in Σ_B: $\mathcal{F} = \mathcal{F}_3 \oplus \mathcal{F}_2 \oplus \mathcal{F}_1 \oplus \mathcal{F}_1, \mathcal{F} = f \in i^*\left(H^2_-(CY_3)\right)/\tilde{f} \in coker\left(H^2_-(CY_3) \xrightarrow{i^*} H^2_-(\Sigma^\lambda)\right)$, guided by single $D7$-brane studies, bifundamental Wilson line super-moduli $\mathcal{A}_I$, will be represented as:

$$\mathcal{A}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{\nu}_e + \theta\nu_e & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{e} + \theta e & 0 \\ 0 & 0 & 0 & \bar{\tilde{\nu}}_e + \bar{\theta}\bar{\nu}_e & \bar{\tilde{e}} + \bar{\theta}\bar{e} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, etc.$$

- Assuming the single $D7$-brane diagonal basis to also be valid for multiple $D7$-branes but for matrix-valued a_I and $\tilde{z}_i$,

$$a_1 =$$

$$\left(\begin{array}{ccccccc}
0 & 0 & 0 & \frac{\xi_1^{14}\tilde{u}_L}{\mathcal{V}^{\frac{7}{5}}} & 0 & 0 & \frac{\xi_1^{17}\tilde{u}_R}{\mathcal{V}^{\frac{11}{5}}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\xi_1^{14}\bar{\tilde{u}}_L}{\mathcal{V}^{\frac{7}{5}}} & 0 & 0 & 0 & 0 & \xi_1^{46}\left(\frac{\tilde{e}_L}{2}+\frac{H_u}{\mathcal{V}^{\frac{8}{5}}}\right) & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\xi_1^{56}H_d}{\mathcal{V}^{\frac{8}{5}}} & 0 \\
0 & 0 & 0 & \bar{\xi}_1^{46}\left(\frac{\bar{\tilde{e}}_L}{2}+\frac{\bar{H}_u}{\mathcal{V}^{\frac{8}{5}}}\right) & \frac{\bar{H}_d\bar{\xi}_1^{56}}{\mathcal{V}^{\frac{8}{5}}} & 0 & -\frac{e_R\xi_1^{67}}{\mathcal{V}^{\frac{4}{5}}} \\
\frac{\bar{\tilde{u}}_R\bar{\xi}_1^{17}}{\mathcal{V}^{\frac{11}{5}}} & 0 & 0 & 0 & 0 & -\frac{\bar{\tilde{e}}_R\bar{\xi}_1^{67}}{\mathcal{V}^{\frac{4}{5}}} &
\end{array}\right)$$

[$\xi^{ab}_{I,i}, 1 \leq a \leq 6, 1 \leq b \leq 7$ are $\mathcal{O}(1)$ numbers]

$$\tilde{z}_1 =$$

$$\begin{pmatrix} 0 & 0 & 0 & \alpha_1^{14}\frac{\tilde{u}_L}{\mathcal{V}} & 0 & 0 & \frac{5\alpha_1^{17}\tilde{u}_R}{\mathcal{V}^{\frac{14}{5}}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\bar{\alpha}_1^{14}\bar{\tilde{u}}_L}{\mathcal{V}} & 0 & 0 & 0 & 0 & \alpha_1^{46}\left(\frac{\tilde{e}_L}{\mathcal{V}^{\frac{9}{5}}} - \frac{H_u}{\sqrt{2}}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\alpha_1^{56}H_d}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \bar{\alpha}_1^{46}\left(\frac{\bar{\tilde{e}}_L}{\mathcal{V}^{\frac{9}{5}}} - \frac{\bar{H}_u}{\sqrt{2}}\right) & \bar{\alpha}_1^{56}\frac{\bar{H}_d}{\sqrt{2}} & 0 & \frac{\alpha_1^{67}e_R}{\mathcal{V}^{\frac{11}{5}}} \\ \frac{5\bar{\alpha}_1^{17}\bar{\tilde{u}}_R}{\mathcal{V}^{\frac{14}{5}}} & 0 & 0 & 0 & 0 & \frac{\bar{\alpha}_1^{67}\bar{\tilde{e}}_R}{\mathcal{V}^{\frac{11}{5}}} & \end{pmatrix}$$

[$\alpha_{1,2}^{ab}, 1 \leq a \leq 6, 1 \leq b \leq 7$ are $\mathcal{O}(1)$ numbers]

- Assuming that the complex structure moduli $z^{\tilde{a}=1,...,h^{2,1}_{-}(CY_3)}$ are stabilized at very small values, which is in fact already assumed in writing T_α which has been written upon inclusion of terms up to linear in the complex structure moduli, let us define a modified intersection matrix in the $a_I - z_i$ moduli space:

$$\mathcal{C}^{\mathcal{I}\mathcal{J}} = \kappa_4^2\mu_7 C^{I\bar{J}},\ \mathcal{I}=I, \mathcal{J}=\bar{J};$$
$$\mathcal{C}^{\mathcal{I}\mathcal{J}} = \mu_3\left(2\pi\alpha'\right)^2(\omega_\alpha)^{i\bar{j}},\ \mathcal{I}=i, \mathcal{J}=\bar{j};$$
$$\mathcal{C}^{\mathcal{I}\mathcal{J}} = 0,\ \mathcal{I}=I, \mathcal{J}=\bar{j},\ \text{etc..}$$

- Now, one can show that for $|z_i| \sim 0.8\mathcal{V}^{\frac{1}{36}},\ \mathcal{V}\sim 10^5$:

$$\mathcal{C}^{\mathcal{I}\bar{\mathcal{J}}}Tr(\mathcal{M}_{\mathcal{I}}\mathcal{M}^\dagger_{\mathcal{J}}) \sim \kappa_4^2\mu_7\left(C^{a_1\bar{a}_1}|\tilde{e}_L|^2 + C^{a_2\bar{a}_2}|\tilde{u}_L|^2 + C^{a_3\bar{a}_3}|\tilde{e}_R|^2 + C^{a_4\bar{a}_4}|\tilde{u}_R|^2\right) + \mu_3\left(2\pi\alpha'\right)^2|H_u|^2,$$

where $\mathcal{M}_{\mathcal{I}} \equiv a_I, z_i$ which implies that in the large volume limit: $\mathcal{C}^{\mathcal{A}_I\bar{\mathcal{A}}_{\bar{J}}} \sim C^{a_I\bar{a}_{\bar{J}}}, \mathcal{C}^{\mathcal{Z}_i\bar{\mathcal{Z}}_{\bar{j}}} \sim \mathcal{C}^{z_i\bar{z}_j}$.

- Clubbing together the Wilson line moduli and the $D3$-brane position moduli into a single vector: $\mathcal{M}_\Lambda \equiv \mathcal{A}_I, \mathcal{Z}_i$, then one sees:

$$\mathcal{C}^{\Lambda\bar{\Sigma}} Tr\left(\mathcal{M}_\Lambda \mathcal{M}^\dagger_\Sigma\right) \sim \mathcal{C}^{\mathcal{I}\bar{\mathcal{J}}} Tr(\mathcal{M}_\mathcal{I} \mathcal{M}^\dagger_\mathcal{J}).$$

  In the large volume and rigid limit of $\Sigma_B(\zeta^A = 0$ which corresponds to a local minimum), perhaps $\mathcal{C}^{\Lambda\bar{\Sigma}} Tr\left(\mathcal{M}_\Lambda \mathcal{M}^\dagger_\Sigma\right)$ is invariant under moduli transformations in the $(\mathcal{A}_I, \mathcal{Z}_i)/(a_I, z_i)$-subspace of the open-string moduli space, which would imply that $\kappa_4^2 \mu_7 C^{I\bar{J}} a_I \bar{a}_{\bar{J}} + \mu_3 (\alpha')^2 (\omega_B)_{i\bar{j}} z^i \bar{z}^{\bar{j}}$ for multiple $D7$-branes, in a basis that diagonalizes $g_{\mathcal{M}_\mathcal{I} \bar{\mathcal{M}}_{\bar{\mathcal{J}}}}$ at stabilized values of the open string moduli, is replaced by $\mathcal{C}^{\mathcal{I}\bar{\mathcal{J}}} Tr(\mathcal{M}_\mathcal{I} \mathcal{M}^\dagger_\mathcal{J})$.

Soft SUSY Breaking Parameters

- The soft supersymmetry parameters are computed via the expansion of the Kähler potential and Superpotential for the open- and closed-string moduli as a power series in fluctuations of the open-string (the “matter fields") moduli about their vevs corresponding to a local minimum.

- Considering the fluctuations around the stabilized values $z_{1,2} = \mathcal{V}^{\frac{1}{36}} + \delta z_{1,2}$; $\mathcal{A}_I = \langle \mathcal{A}_I \rangle + \delta \mathcal{A}_I$ the superpotential and
- Kähler potential can be expanded out.

$$K(\{\sigma^B, \bar{\sigma}^B; \sigma^S, \bar{\sigma}^S; \mathcal{G}^a, \bar{\mathcal{G}}^a; \tau, \bar{\tau}\}; \{z_{1,2}, \bar{z}_{1,2}; \mathcal{A}_1, \bar{\mathcal{A}}_1\})$$

$$\sim -ln(-i(\tau - \bar{\tau})) - ln(i \int_{CY_3} \Omega \wedge \bar{\Omega}) - 2\, ln\mathcal{V} + (|\delta z_1|^2 + |\delta z_2|^2$$

$$+\delta z_1 \bar{\delta} z_2 + \delta z_2 \bar{\delta} z_1) K_{z_i \bar{z}_j} + [((\delta z_1)^2 + (\delta z_2)^2) Z_{z_i z_j} + c.c] + [|\delta \mathcal{A}_1|^2 K_{\mathcal{A}_1 \bar{\mathcal{A}}_1}$$

$$+(\delta \mathcal{A}_I)^2 Z_{\mathcal{A}_I \mathcal{A}_I} + c.c] + [\delta z_j \delta \bar{\mathcal{A}}_I K_{z_j \bar{\mathcal{A}}_I} + c.c] + [\delta z_i \delta \mathcal{A}_J Z_{z_i \mathcal{A}_J} + c.c.] + \ldots$$

Results summarized M.Dhuria, AM[2012]

Quark mass Lepton mass	$M_q \sim O(5) MeV$ $M_l \sim \mathcal{O}(1) MeV$
Gravitino mass Gaugino mass Neutralino mass	$m_{\frac{3}{2}} \sim \mathcal{V}^{-\frac{n^s}{2}-1} m_{pl}; n_s = 2$ $M_{\tilde{g}} \sim V^{\frac{2}{3}} m_{\frac{3}{2}}$ $M_{\chi_3^0} \sim V^{\frac{2}{3}} m_{\frac{3}{2}}$
$D3$-brane position moduli (Higgs) mass Wilson line moduli mass	$m_{\mathcal{Z}_i} \sim \mathcal{V}^{\frac{59}{72}} m_{\frac{3}{2}}$ $m_{\tilde{\mathcal{A}}_I} \sim \mathcal{V}^{\frac{1}{2}} m_{\frac{3}{2}}$ $I = 1, 2, 3, 4$
A-terms	$A_{pqr} \sim n^s \mathcal{V}^{\frac{37}{36}} m_{\frac{3}{2}}$ $\{p, q, r\} \in \{\tilde{\mathcal{A}}_I, \mathcal{Z}_i\}$
Physical μ-terms (Higgsino mass)	$\hat{\mu}_{\mathcal{Z}_i \mathcal{Z}_j}$ $\sim \mathcal{V}^{\frac{37}{36}} m_{\frac{3}{2}}$
Physical $\hat{\mu}B$-terms	$(\hat{\mu}B)_{\mathcal{Z}_1 \mathcal{Z}_2} \sim \mathcal{V}^{\frac{37}{18}} m_{\frac{3}{2}}^2$

$m_\nu \leq 1eV$

- The non-zero neutrino masses are generated through the Weinberg(-type) dimension-five operators written out schematically as: $\int d^4x \int d^2\theta e^{\hat{K}/2} \times \left(\mathcal{Z}^2 \mathcal{A}_1^2 \in \frac{\partial^2 W}{\partial \mathcal{A}_1^2}\Big|_{\theta=0} \mathcal{A}_1^2\right)$, where one picks out the $\mathcal{O}(Z_i^2)$ term in $\frac{\partial^2 W}{\partial \mathcal{A}_1^2}\Big|_{\theta=0}$ and is given as: $m_\nu = v^2 sin^2\beta \hat{\mathcal{O}}_{\mathcal{Z}_1 \mathcal{Z}_1 \mathcal{A}_1 \mathcal{A}_l}/2M_p$ where $\hat{\mathcal{O}}_{\mathcal{Z}_1 \mathcal{Z}_1 \mathcal{A}_1 \mathcal{A}_l} = \frac{e^{\frac{\hat{K}}{2}} \hat{\mathcal{O}}_{\mathcal{Z}_1 \mathcal{Z}_1 \mathcal{A}_1 \mathcal{A}_l}}{\sqrt{\hat{K}^2_{\mathcal{Z}_1 \bar{\mathcal{Z}}_{\bar{1}}} \hat{K}^2_{\mathcal{A}_1 \bar{\mathcal{A}}_{\bar{1}}}}}$, $vsin\beta \equiv \langle H_u \rangle$ and $sin\beta$ is defined via $tan\beta = \langle H_u \rangle / \langle H_d \rangle$.

$m_\nu \leq 1eV$ (continued)

- Using RG-flow arguments of AM, P.Shukla [2010], one can show that one produces $m_\nu \stackrel{<}{\sim} 1eV$ M.Dhuria, AM [2012].
- The squark/slepton masses do not vary significantly, e.g., under an MSSM RG-flow AM, P.Shukla [2010].

μ-Split SUSY Scenario

- In case of “split supersymmetry scenario" N.A-Hamed, S.Dimopoulos [2004], SUSY breaking scale is high and fine tuning is done in order to get one light Higgs at EW scale and super heavy squarks/sleptons (of the order of high supersymmetry breaking scale) along with light fermions and a small $\hat{\mu}_{\mathcal{Z}_1\mathcal{Z}_2}$ (Higgsino mass parameter).
- In an alternate approach to split SUSY scenario called “μ-split SUSY scenario" Cheng and Cheng [2005], one can assume $\hat{\mu} \sim m_{\mathcal{A}_I} \sim B$, i.e., large $\hat{\mu}$ parameter to get $\hat{\mu}B \sim m^2_{\mathcal{A}_I}$. This is in conformity with the requirement of EW symmetry breaking at the EW scale: $\frac{M_Z^2}{2} = \frac{m^2_{H_1} - m^2_{H_2} tan^2\beta}{tan^2\beta - 1} - \hat{\mu}^2$. “This choice appears more natural and also helps to alleviate the ‘μ problem’".

- There is lack of universality in moduli masses but universality in trilinear A_{ijk} couplings - to get an estimate, using solution of RG flow equation for moduli masses $m^2_{\mathcal{Z}_{1,2}}$ and Higgsino mass $\hat{\mu}_{\mathcal{Z}_1\mathcal{Z}_2}$ as given in Nath, Arnowitt [1998], $A_{\mathcal{Z}_i\mathcal{Z}_i\mathcal{Z}_i} \sim n^s \hat{\mu}_{\mathcal{Z}_1\mathcal{Z}_2}$ AM, P. Shukla [2009]; M.Dhuria, AM [2012], one-loop ln-running for the $U(1)$ gauge couplings in 2HDM/(MS)SM, assuming $\hat{\mu}B \sim \xi\hat{\mu}^2$ AM,Pramod Shukla [2009]; M.Dhuria, AM [2012] (verified at M_s for $\mathcal{O}(1)\xi$) as per EW symmetry breaking, the Higgs mass matrix at the EW-scale can thus be expressed as:

$$\left(\begin{array}{cc} m^2_{H_1} & \hat{\mu}B \\ \hat{\mu}B & m^2_{H_2} \end{array}\right) \sim \left(\begin{array}{cc} m^2_{H_1} & \xi\hat{\mu}^2 \\ \xi\hat{\mu}^2 & m^2_{H_2} \end{array}\right).$$

- The eigenvalues are given by:

$$\frac{1}{2}\left(m^2_{H_1} + m^2_{H_2} \pm \sqrt{\left(m^2_{H_1} - m^2_{H_1}\right)^2 + 4\xi^2\hat{\mu}^4}\right)$$

$$m_0^2 - 0.06S_0 + ... \pm \sqrt{\left(m_0^2 + 0.06S_0 + ...\right)^2 + 4\xi^2\left(m_0^2 - 0.03S_0\right)}$$

where $S_0 = Tr(Ym^2) = m^2_{\mathcal{Z}_2} - m^2_{\mathcal{Z}_1} + \sum_{i=1}^{n_g}(m^2_{\tilde{q}_L} - 2m^2_{\tilde{u}_R} + m^2_{\tilde{d}_R} - m^2_{\tilde{l}_L} + m^2_{\tilde{e}_R})$ and all the masses are at the string scale and n_g is the number of generations.

- Hence, assuming non-universality w.r.t. to the $D3$-brane position moduli masses ($m_{Z_{1,2}}$) and the squark/slepton masses, if

  $S_0 \sim (-4.23) m_0^2$ and $\xi^2 \sim \frac{1}{5} + \frac{1}{16}\frac{m_{EW}^2}{m_0^2}$

  one sees that one obtains one light Higgs (corresponding to the negative sign of the square root) with mass $125 GeV$ and one heavy Higgs (corresponding to the positive sign of the square root).
- Note, however, the Higgsino mass parameter $\hat{\mu}_{\mathcal{Z}_1 \mathcal{Z}_2}$ then turns out to be heavy with a value, at the EW scale of around $\mathcal{V}^{\frac{59}{72}} m_{3/2}$, which is indicative of μ-split SUSY scenario.

String Particle Cosmology - DM Studies

- To calculate the decay widths of all important 2- and 3-body (N)LSP decay channels, we will be utilizing/generalizing results of H. Jockers [2005] in the $\mathcal{N}=1$ gauged supergravity action of Wess and Bagger with the understanding that $m_{\rm moduli/modulini} << m_{\rm KK}\left(\sim \frac{M_s}{\mathcal{V}^{\frac{1}{6}}}(\mathcal{V}\sim 10^{5/6})\sim 10^{14}GeV\right), M_s = \frac{M_p}{\sqrt{\mathcal{V}}}(\mathcal{V}\sim 10^5)\sim 10^{15}GeV$, and that for multiple $D7$-branes, the non-abelian gauged isometry group [corresponding to gauging of a Pecci-Quinn/shift symmetry along the RR two-form axions c^a and the zero-form axion ρ_B due to the dualization of the Green-Schwarz term $Tr\left(Q_B\int_{\mathbf{R}^{1,3}}dD^{(2)}_B\wedge A\right)$ - $D^{(2)}_B$ being an RR two-form axion modifies the covariant derivative of T_B by an additive shift given by $6i\kappa_4^2\mu_7\left(2\pi\alpha'\right)Tr(Q_BA_\mu)$] can be identified with the SM group (i.e. A_μ is the SM-like adjoint-valued gauge field Wess+Bagger); $Q_B = 2\pi\alpha'\int_{\Sigma_B}i^*\omega_\alpha\wedge P_-\tilde{f}$.

$$\mathcal{L}^{\mathcal{N}=1}_{\text{Wess Bagger};Jockers;\text{Dhuria,AM}} =$$

$$g_{YM} g_{T_B \bar{\mathcal{J}}} Tr\left(X^{T_B} \bar{\chi}_L^{\bar{\mathcal{J}}} \lambda_{\tilde{g},\, R}\right) + i g_{\mathcal{I}\bar{\mathcal{J}}} Tr\left(\bar{\chi}_L^{\bar{\mathcal{I}}}\left[\not{\partial}\chi_L^{\mathcal{I}} + \Gamma^i_{Mj}\not{\partial}a^M \chi_L^{\mathcal{J}}\right.\right.$$

$$\left.\left. + \frac{1}{4}\left(\partial_{a_M} K \not{\partial} a_M - \text{c.c.}\right)\chi_L^{\mathcal{I}}\right]\right) + \frac{e^{\frac{K}{2}}}{2}\left(\mathcal{D}_{\bar{\mathcal{I}}} D_{\mathcal{J}} \bar{W}\right) Tr\left(\chi_L^{\mathcal{I}} \chi_R^{\mathcal{J}}\right) - \frac{f_{ab}}{4} F^a_{\mu\nu} F^{b\mu\nu}$$

$$+ \frac{1}{8} f_{ab} \epsilon^{\mu\nu\rho\lambda} F^a_{\mu\nu} F^b_{\rho\lambda} + g_{T_B \bar{T}_B} Tr\left[\left(\partial_\mu T_B - A_\mu X^{T_B}\right)\left(\partial^\mu T_B - A^\mu X^{T_B}\right)^\dagger\right]$$

$$+ g_{T_B \mathcal{J}} Tr\left(X^{T_B} A_\mu \bar{\chi}_L^{\mathcal{J}} \gamma^\nu \gamma^\mu \psi_{\nu,\, R}\right) + \bar{\psi}_{L,\, \mu} \sigma^{\rho\lambda} \gamma^\mu \lambda_{\tilde{g},\, L} F_{\rho\lambda} +$$

$$+ Tr\left[\bar{\lambda}_{\tilde{g},\, L} \not{A}\left(6\kappa_4^2 \mu_7 (2\pi\alpha') Q_B K + \frac{12\kappa_4^2 \mu_7 (2\pi\alpha') Q_B v^B}{\mathcal{V}}\right)\lambda_{\tilde{g},\, L}\right]$$

$$+ \frac{e^K G^{T_B \bar{T}_B}}{\kappa_4^2} 6i\kappa_4^2 (2\pi\alpha') Tr\left[Q_B A^\mu \partial_\mu\left(\kappa_4^2 \mu_7 (2\pi\alpha')^2 C^{I\bar{J}} a_I \bar{a}_{\bar{J}}\right)\right] + \text{h.c.}$$

$$- \frac{i\sqrt{2}}{4} g \partial_{i/I} f_{ab} Tr\left(\frac{12\kappa_4^2 \mu_7 (2\pi\alpha') Q_B^a v^B}{\mathcal{V}} \bar{\lambda}^b_{\tilde{g},L} \chi_R^{i/I}\right) + \text{h.c.}$$

$$- \frac{\sqrt{2}}{4} \partial_{i/I} f_{ab} Tr\left(\bar{\lambda}^a_{\tilde{g},R} \sigma^{\mu\nu} \chi_L^{i/I}\right) F^b_{\mu\nu} + \bar{\psi}_{L,\, \mu} \sigma^{\rho\lambda} \gamma^\mu \lambda_{\tilde{g},\, L} W^+_\rho W^-_\lambda + \text{h.c.}.$$

Gluino Lifetime

- Since the squarks which mediate gluino decay are ultra-heavy implies gluinos are long-lived in split SUSY scenarios.
- From the neutralino mass matrix, one obtains the lightest neutralino: $\chi_3^0 \sim -\lambda_g + \tilde{f}\mathcal{V}^{\frac{5}{6}} \frac{v}{M_p}(\tilde{H}_1 + \tilde{H}_2)$ with a mass $\sim V^{\frac{2}{3}} m_{\frac{3}{2}}$.
- Using RG analysis of coefficients of the effective dimension-six gluino decay operators as given in Guidice et al [2005], it can be shown in M.Dhuria+AM [2011] that these coefficients at the EW scale are of the same order as that at the squark mass scale.
- $\tilde{f}^2 < 10^{-8}[$ Using $\langle V_F \rangle \sim \langle e^K G^{T_S \bar{T}_S} D_{T_S} W \bar{D}_{\bar{T}_S} \bar{W} \rangle \sim \mathcal{V}^{19/18} m_{3/2}^2 \sim \mathcal{V}^{-3} m_{pl}^2 > \langle V_D \rangle = \langle \frac{108\kappa_4^2 \mu_7}{\mathcal{V}^2 Re T_B} (Q_B v^B)^2 \rangle$ M.Dhuria+AM[2011] in dilute flux approximation.

Life time of various Gluino decay channels

Particle decay	Decay Modes	Life Time	Remarks
Gluino decays	$\tilde{g} \longrightarrow \chi^{o}_{\rm n} q_{_I} \bar{q}_{_J}$	$10s$	(Large lifetime
	$\tilde{g} \longrightarrow \tilde{\chi}^0_3 g$	$10^{10}s$	from
	$\tilde{g} \longrightarrow \psi_\mu q_{_I} \bar{q}_{_J}$	$10^3 s$	collider point
	$\tilde{g} \longrightarrow \psi_\mu g$	$10^{-1}s$	of view)

N(LSP) Decay Channels

- A very important constraint: the hadronic/electromagnetic energy released from decay products of next-to-lightest supersymmetric particle (NLSP) must not alter the observed abundance of light elements in the universe essentially fixed by average lifetime around $\tau \sim 10^2 sec$ referred to as the B(ig) B(ang) N(ucleosynthesis)) constraint; the same is satisfied by NLSP candidates if decay of same occurs before BBN era Kawasaki et al [2004].
- In addition to this, taking R-parity violating couplings into account, the (lightest) neutralino might decay into leptons/quarks rather than gravitino and hence elude the relic abundance of gravitino coming from decay of neutralino (Co-NLSP) if life time for the former decay is less than the latter; via explicit calculations, *we ensure that this does not happen*. For the same one needs to calculate the decay widths of all important 2- and 3-body decay channels.

Gravitino(LSP) Decays

- The viable dark matter particle should have life time of the order or greater than the age of the universe. Unlike assuming R-parity to be conserved and hence stability of LSP, we first calculate the contribution of possible trilinear R-parity violating couplings λ_{ijk}, λ'_{ijk} and λ''_{ijk}:

$$W_{\not R_p} = \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \lambda''_{ijk} U^c_i D^c_j D^c_k + \mu_i H L_i.$$

  in the effective $\mathcal{N} = 1$ gauged supergravity action.

Life time of various N(LSP) decay channels

Particle decay	Decay Modes	Life Time	Remarks
Neutralino/Gaugino	$\tilde{B} \rightarrow \psi_\mu Z/\gamma$	$10^{-30}s$	BBN
decays	$\tilde{W} \xrightarrow{\tilde{f}} \psi_\mu u\bar{u}$	$10^{-25}s$	constraint
	$\tilde{B} \xrightarrow{Z} \psi_\mu u\bar{u}$	$10^{-13}s$	
	$\tilde{l} \rightarrow l'\psi_\mu V$	$10^{-28}s$	"
Slepton decays	$\tilde{l}/\tilde{q} \rightarrow l/q\psi_\mu$	$10^{-25.5}s$	
RPV Neutralino decay	$\chi^0_3 \rightarrow u\bar{d}e^-$	$10s$	doesn't affect
			ψ_μ abundance
Gravitino decays	$\psi_\mu \rightarrow \nu\gamma, \nu Z$	$10^{21}s$	Life time
	$\psi_\mu \rightarrow h\nu_e$	$10^{17}s$	greater
	$\psi_\mu \xrightarrow{\lambda} l_i l_j e^c_k$	$10^{22}s$	than age
	$\psi_\mu \xrightarrow{\lambda'} l_i q_j d^c_k$	$10^{20}s$	of
	$\psi_\mu \xrightarrow{\lambda''} u^c_i d^c_j d^c_k$	$10^{18}s$	Universe

Relic Abundance of Gravitino

- If the gravitino LSP produced by decay of Co-NLSP's is to account for all the gravitinos, the relic abundance of gravitino is given as

$$\Omega_{\tilde{G}} h^2 = \Omega_{\chi_3^0} h^2 \times \frac{m_{\frac{3}{2}}}{m_{\chi_3^0}}$$

if Co-NLSP's freeze out with appropriate thermal relic density ($\Omega_{\chi_3^0}$) before decaying into the gravitino Wang et al [2005].

- The freeze out condition depends on thermal cross-section $\sigma v_{\rm Ml}$ of such particles which in partial wave expansion approach, is given as:

$$\langle \sigma v_{\rm Ml} \rangle \equiv a + bx + \mathcal{O}(x^2)$$

where analytical expression of a and b are given for each annihilation channel in Nihei et al [2002].

- Evaluation of Relic density depends sensitively on the annihilation cross section ($\sigma v_{\rm Ml}$) of such particles. To get the idea of same, we have calculated annihilation cross-section of all important channels: $\chi_3^0\chi_3^0 \xrightarrow{h_s,\chi_i^0\ t} hh$, $\chi_3^0\chi_3^0 \xrightarrow{h_s,\tilde{f}_t} ff$ in case of neutralino annihilation and ($\widetilde{\ell_a}\widetilde{\ell_b^*} \xrightarrow{h_s,\tilde{l}_c\ t} ZZ$, $\widetilde{\ell_a}\widetilde{\ell_b^*} \xrightarrow{h_s,\tilde{l}_c\ t} Zh$,$\widetilde{\ell_a}\widetilde{\ell_b^*} \xrightarrow{h_s,\tilde{l}_c\ t} hh$, $\widetilde{\ell_a}\widetilde{\ell_b^*} \xrightarrow{{\rm cont},\tilde{l}_c\ t} \gamma\gamma$, $\widetilde{\ell_a}\widetilde{\ell_b^*} \xrightarrow{\tilde{l}_c\ t} \gamma h$, $\widetilde{\ell_a}\widetilde{\ell_b^*} \xrightarrow{\tilde{l}_c\ t} ll$) in case of slepton annihilation.
- Relic abundance of gravitino $\Omega_{\tilde{G}}h^2$ comes out to be 0.16 by considering neutralino to be NLSP and 10^{-22} by considering sleptons to be NLSP.

EDM of the electron/neutron

- The $e/n \equiv f$ EDM is defined via : $\mathcal{L}_I = -\frac{i}{2} d_f \bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu}$. In supersymmetric theories, non-zero phases are given by complex soft SUSY breaking parameters (off-diagonal L-R sfermion mass-matrix mixing, $\mathcal{A}_{IJK}, \mu B, \mu$).

One-Loop EDM

- At one loop level, for ψ_f interacting with other heavy ψ_i's and heavy ϕ_k's with masses m_i, m_k and charges Q_i, Q_k, the interaction that contains CP violation in general is given by Ibrahim, Nath [1997]:

  $-\mathcal{L}_{int} = \sum_{ik} \bar{\psi}_f (K_{ik} \frac{1-\gamma^5}{2} + L_{ik} \frac{1+\gamma^5}{2}) \psi_i \phi_k + h.c.$; $\mathcal{L}$ violates CP invariance iff $\mathrm{Im}(K_{ik} L^*_{ik}) \neq 0$ and one-loop EDM of the fermion in this case is given by

  $\sum_{ik} \frac{m_i}{(4\pi)^2 m_k^2} \mathrm{Im}(K_{ik} L^*_{ik})(Q_i A(\frac{m_i^2}{m_k^2}) + Q_k B(\frac{m_i^2}{m_k^2}))$,

  $A(r) = \frac{1}{2(1-r)^2}(3 - r + \frac{2lnr}{1-r})$, $B(r) = \frac{1}{2(r-1)^2}(1 + r + \frac{2rlnr}{1-r})$,

  $Q_k = Q_f - Q_i$.

- The scalar (sfermion) mass matrix:

$$M_{\tilde{f}}^2 = \begin{pmatrix} M_{\tilde{f}_L}^2 & m_u(A_f^* - \mu \cot\beta) \\ m_u(A_f - \mu^* \cot\beta) & M_{\tilde{f}_R}^2 \end{pmatrix}.$$

where $A_f^* = \mathcal{A}_{IJK}/\mathcal{Y}^{\rm eff}_{IJK}$.

- Diagonalized $\tilde{f}_L$ and $\tilde{f}_R$ Ibrahim, Nath[1997]:

$$\tilde{f}_L = D_{f_{11}}\tilde{f}_1 + D_{f_{12}}\tilde{f}_2$$
$$\tilde{f}_R = D_{f_{21}}\tilde{f}_1 + D_{f_{22}}\tilde{f}_2.$$

where f corresponds to first generation leptons and quarks.

-

$$D_f = \begin{pmatrix} \cos\frac{\theta_f}{2} & -\sin\frac{\theta_f}{2}e^{-i\phi_f} \\ \sin\frac{\theta_f}{2}e^{i\phi_f} & \cos\frac{\theta_f}{2} \end{pmatrix}$$

- $$D_f^\dagger M_{\tilde{f}}^2 d_f = \mathrm{diag}(M_{\tilde{f}1}^2, M_{\tilde{f}2}^2)$$

  where $\tan\theta_f = \frac{2|M_{\tilde{f}21}^2|}{M_{\tilde{f}11}^2 - M_{\tilde{f}22}^2} \Rightarrow \theta_f \approx \frac{\pi}{2}$ (assume $\phi_f \in \left[0, \frac{\pi}{2}\right]$), the eigenvalues $M_{\tilde{f}1}^2$ and $M_{\tilde{f}2}^2$ are as follows:

$$M_{\tilde{f}(1)(2)}^2 = \frac{1}{2}(M_{\tilde{f}11}^2 + M_{\tilde{f}22}^2)(+)(-)\frac{1}{2}[(M_{\tilde{f}11}^2 - M_{\tilde{f}22}^2)^2 + 4|M_{\tilde{f}21}^2|^2]^{\frac{1}{2}} \sim \mathcal{V} m_{3/2}^2.$$

- Similar considerations for Higgs mass matrix.

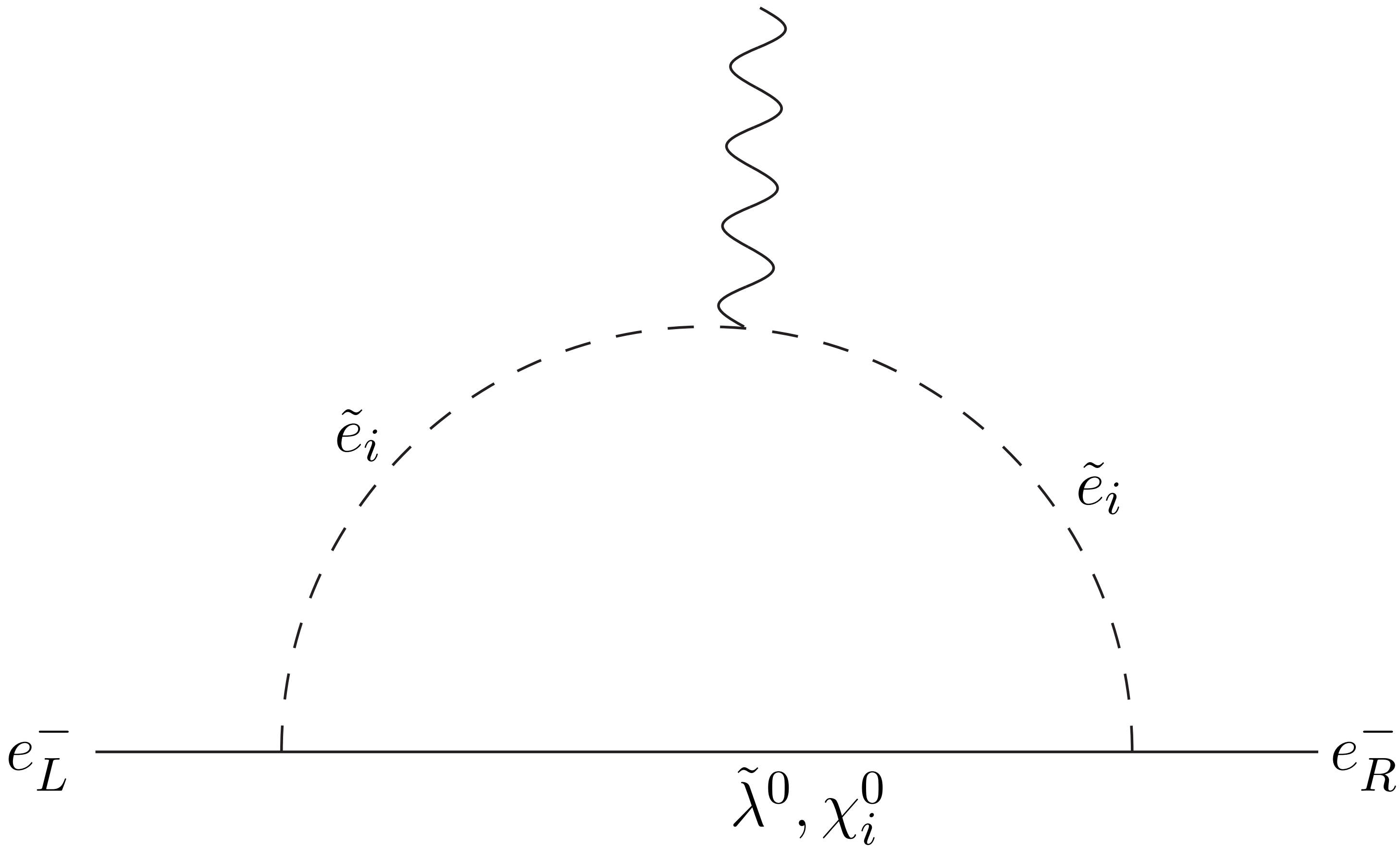
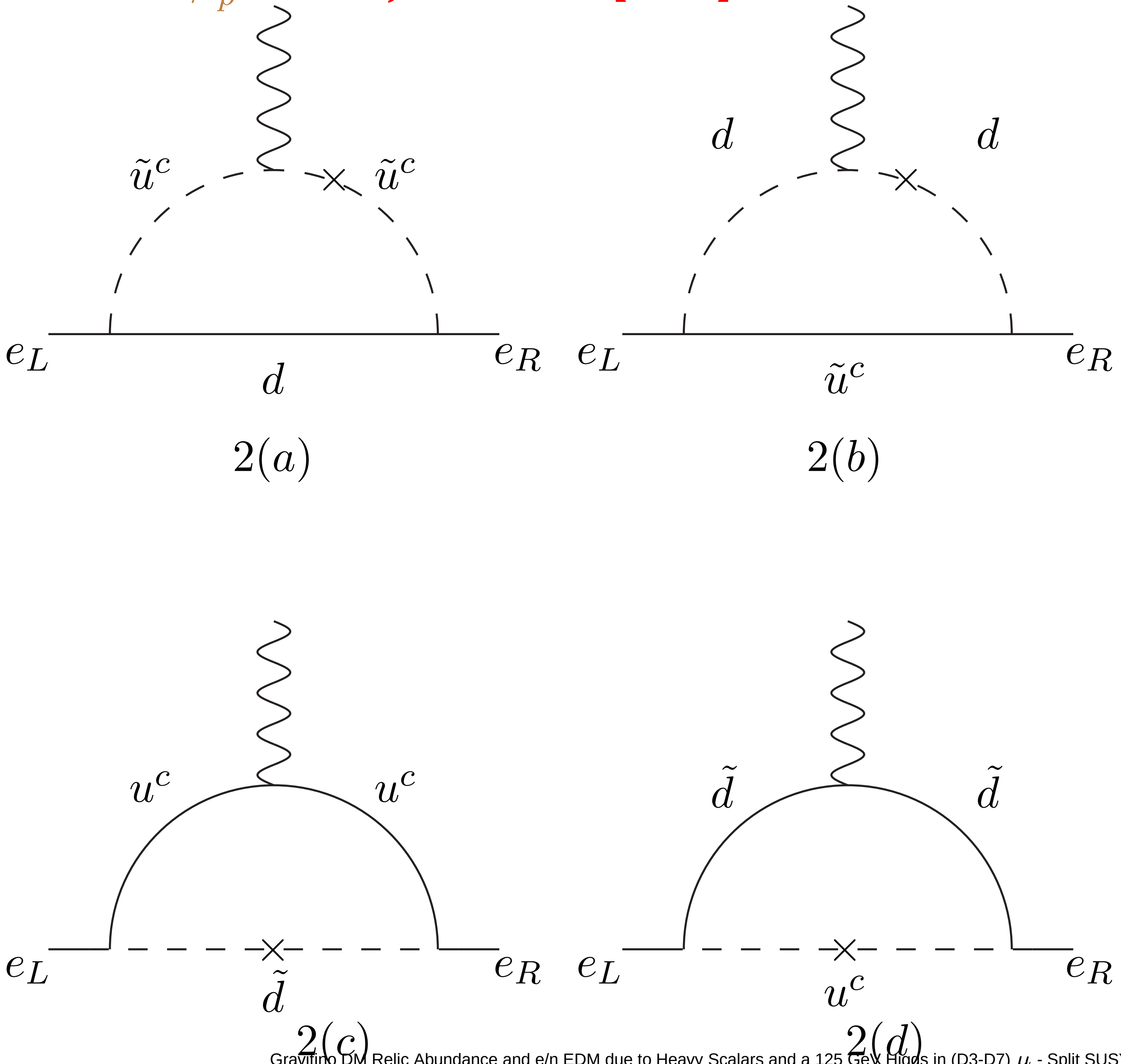


Fig. 1

$\not{R}_p$ Franck, Hamidian [1997]



$$\tilde{\chi}_1^+ = -\tilde{H}_u^+ + \left(\frac{v}{M_p}\tilde{f}\mathcal{V}^{\frac{5}{6}}\right)\tilde{\lambda}_i^+, \tilde{\chi}_1^- = -\tilde{H}_d^- + \left(\frac{v}{M_p}\tilde{f}\mathcal{V}^{\frac{5}{6}}\right)\tilde{\lambda}_i^-,$$

$$m_{\tilde{\chi}_1^\pm} \sim \mathcal{V}^{\frac{59}{72}} m_{\frac{3}{2}};$$

$$\tilde{\chi}_2^+ = \tilde{\lambda}_i^+ + \left(\frac{v}{M_p}\tilde{f}\mathcal{V}^{\frac{5}{6}}\right)\tilde{H}_u^+, \tilde{\chi}_2^- = \tilde{\lambda}_i^- + \left(\frac{v}{M_p}\tilde{f}\mathcal{V}^{\frac{5}{6}}\right)\tilde{H}_d^-,$$

$$m_{\tilde{\chi}_2^\pm} \sim \mathcal{V}^{\frac{2}{3}} m_{\frac{3}{2}}.$$

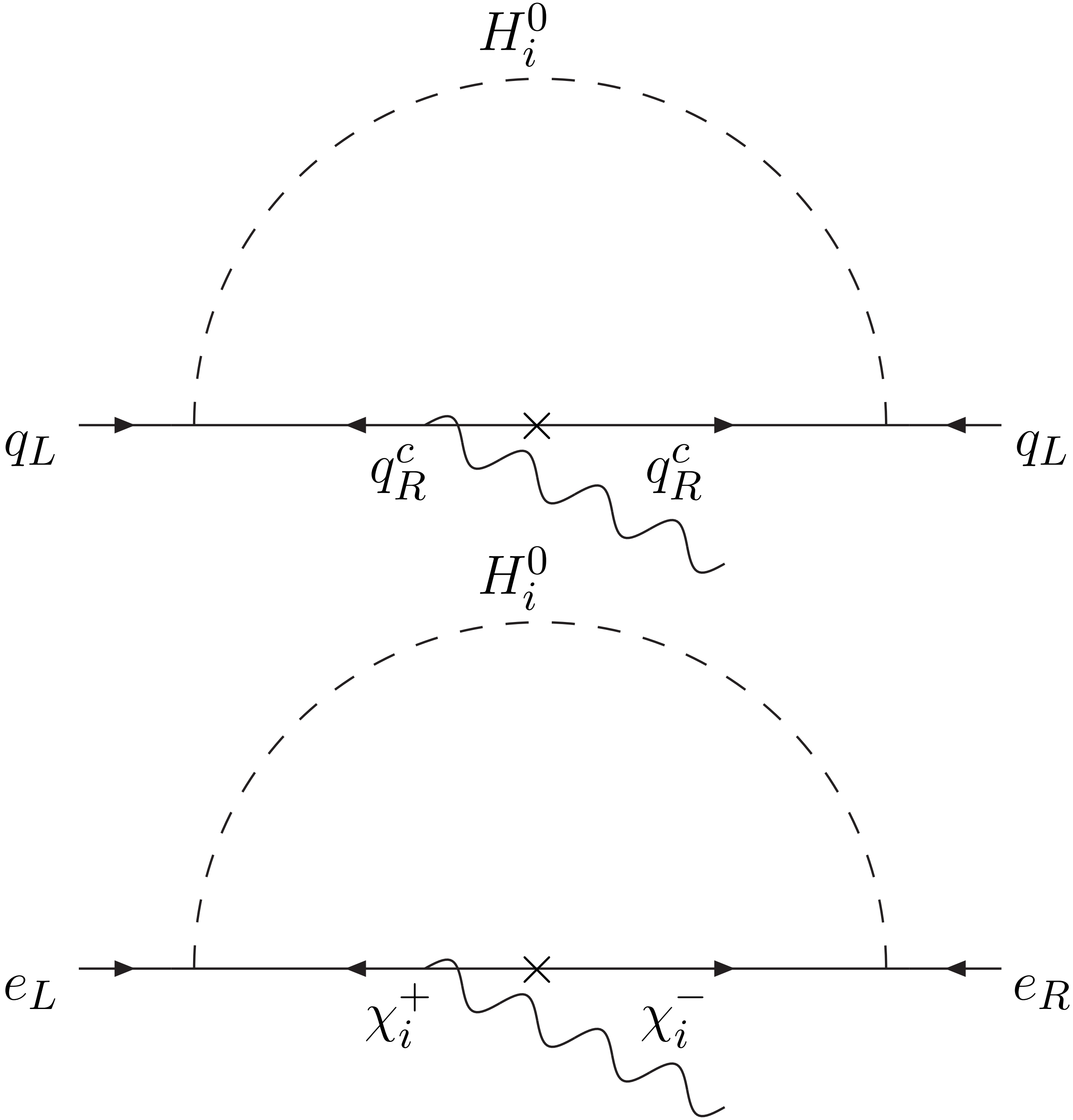
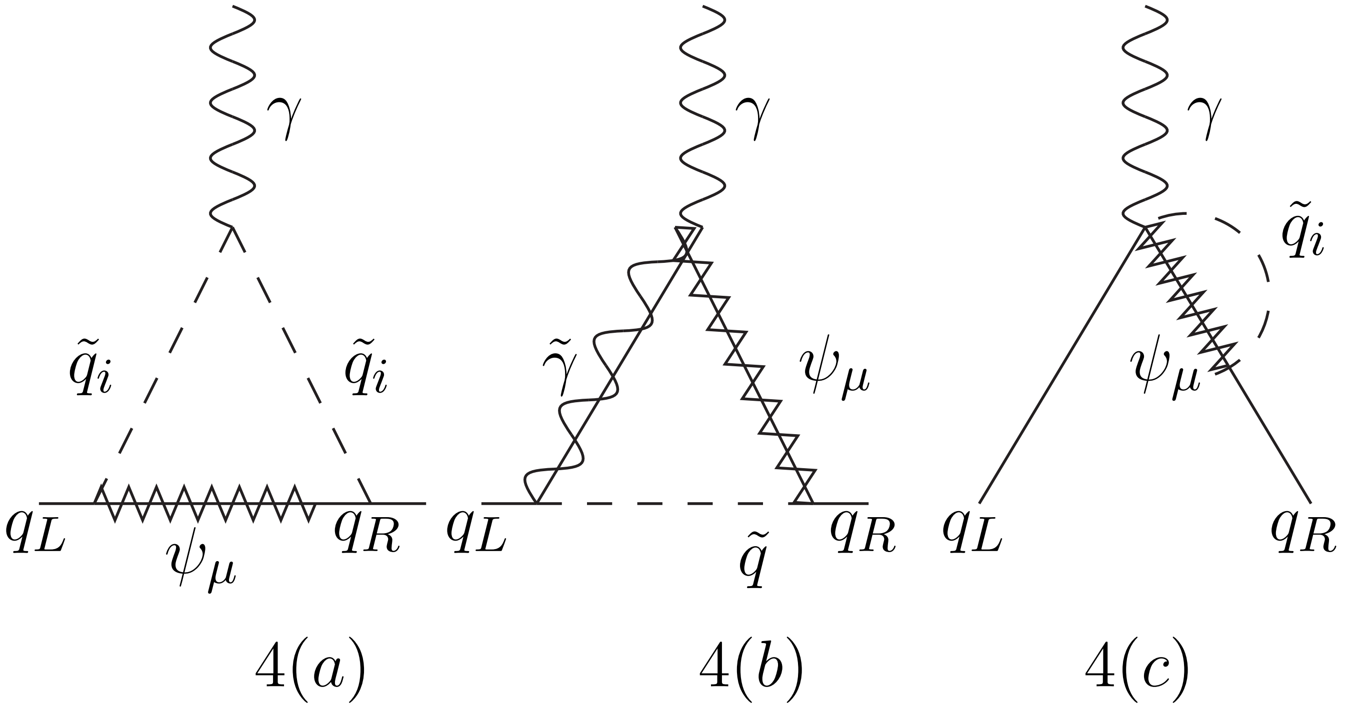
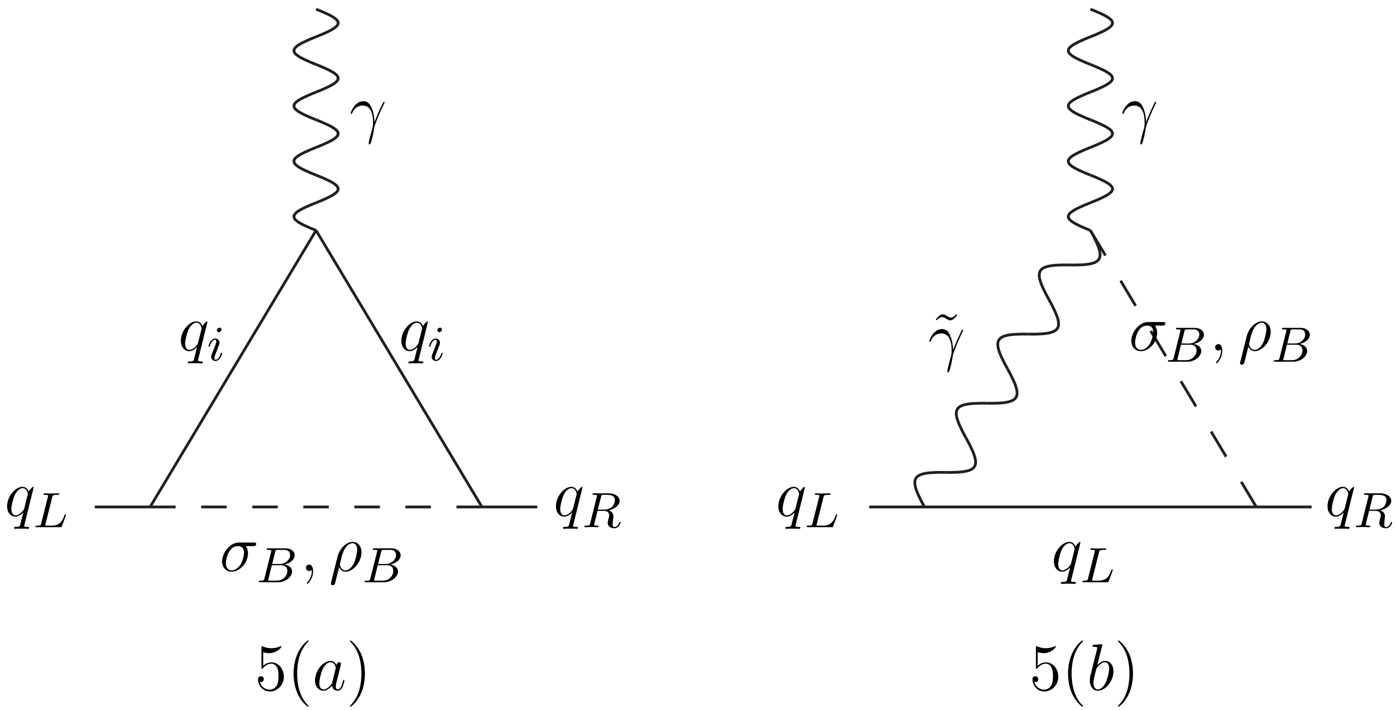


Fig 3



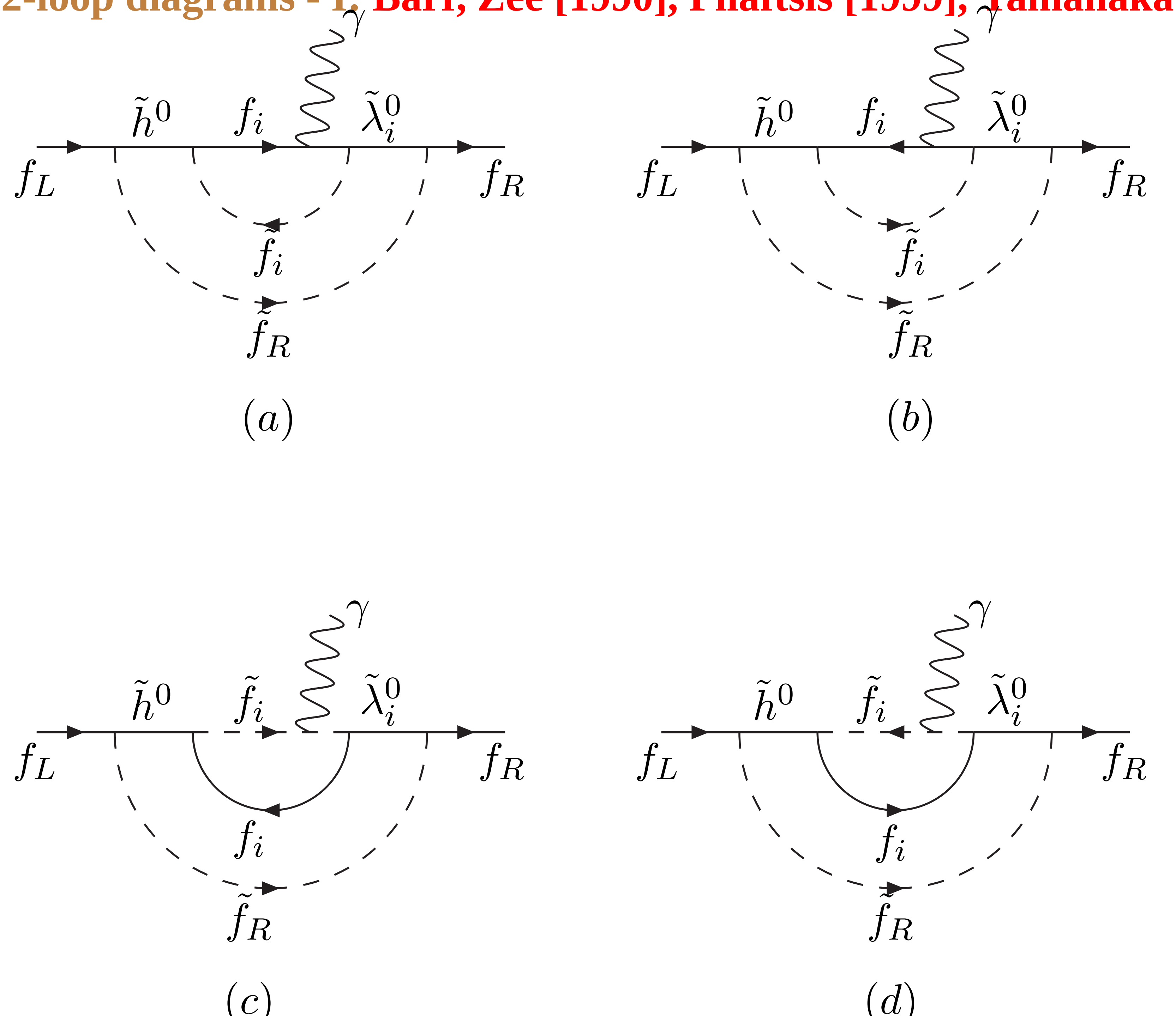
One loop diagrams involving gravitino Mèndez, Orte [1985]



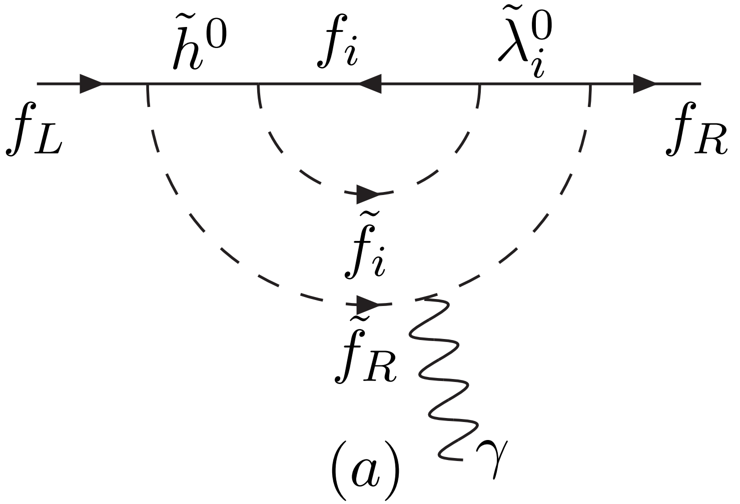
One loop diagrams involving sGoldstino $\tau_B = \sigma_B + i\rho_B$

Brignole, Perazzi, Zwirner[1999]

R_p^+ 2-loop diagrams - I: Barr, Zee [1990], Pilaftsis [1999], Yamanaka [2012]



R_p^+ 2-loop diagrams-II Yamanaka [2012]



Diagrams involving W-boson in the internal loop Leigh, Paban, Xu [198

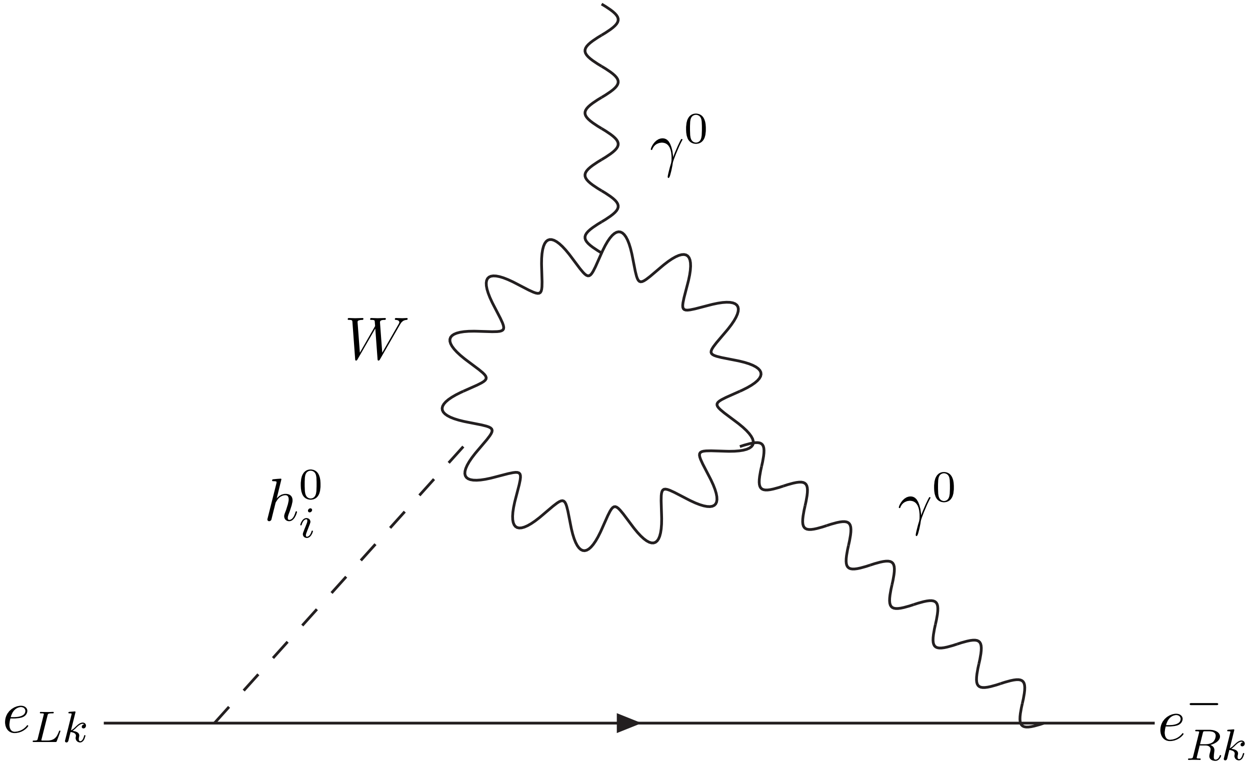


Fig.8

One-loop particle exchange	Origin of $\mathbb{C}$ phase	$\frac{d_e}{e}$(cm)	$\frac{d_n}{e}$(cm)
$\lambda^0 \tilde{f}$	$\tilde{f}$ mass eigenstates	10^{-39}	10^{-38}
$\chi_i^0 \tilde{f}$	"	10^{-37}	10^{-34}
$f \tilde{f}$	"	10^{-40}	10^{-40}
$f h_i^0$	Higgs mass eigenstates	10^{-34}	10^{-33}
$\chi^{\pm} h_i^0$	"	10^{-32}	$-$
$\mathrm{gravitino}\ \tilde{f}$	$\tilde{f}$ mass eigenstates	10^{-57}	10^{-57}
$\mathrm{sgoldstino}\ \tilde{f}$	"	10^{-72}	10^{-68}

Two-loop particle exchange	Origin of $\mathbb{C}$ phase	$\frac{d_e}{e}$(cm)	$\frac{d_n}{e}$(cm)
$h_i^0 \gamma f$	$\hat{Y}_{\rm eff} \in \mathbb{C}$	10^{-36}	10^{-36}
$h_i^0 \gamma \chi_i^{\pm}$	”	10^{-47}	10^{-47}
$\tilde{f} f \gamma$	”	10^{-70}	10^{-70}
$\tilde{f} h_i^0 \gamma$	”	10^{-29}	10^{-29}
$\gamma W^{\pm} h_i^0$	Higgs exchange	10^{-27}	10^{-27}
$\tilde{h}^0 \tilde{f} \lambda_i^0$ (R_p Rainbow type)	Diagonalized $\tilde{f}$ mass eigenstates and $\hat{Y}_{\rm eff}$	10^{-55}	10^{-54}
$\nu^0 \tilde{f} \lambda_i^0$ ($\not{R}_p$ Rainbow type)	”	10^{-52}	10^{-52}

Summary

- Possibility of realizing big-divisor $D3/D7$ μ-Split Supersymmetry (light fermions (in the process obtained the first generation leptonic (including ν) and quark masses), heavy sleptons/squarks, one light ($125GeV$) and one heavy Higgs, heavy Higgsino, relatively long-lived gluinos), sleptons/squarks, neutralino/gauginos (with $\mathcal{O}(1)$ mass difference for $\mathcal{V} \sim 10^5$) as the co-NLSPs.
- Gravitino (LSP) is a viable DM candidate.
- Obtain a healthy EDM up to two loops.

Extra Slides

Squark/Slepton Masses

- $[M_a/{g_a}^2]$ is a one-loop RG invariant.
- RG equations of first family of squark and slepton masses result in the following set of equations which represent the difference in their mass-squared values between Q_{EW} and Q_0 at one-loop level Martin (1997): $M^2_{\tilde{d}_L,\tilde{u}_L}|_{Q_{EW}} - M^2_{\tilde{d}_L,\tilde{u}_L}|_{Q_0} = \mathcal{K}_3 + \mathcal{K}_2 + \frac{1}{36}\mathcal{K}_1 + \tilde{\Delta}_{\tilde{d}_L}$, etc. where $\mathcal{K}_a \sim \mathcal{O}(1/10)\int_{lnQ_0}^{lnQ_{\rm EW}} dt g_a^2(t) M_a^2(t) \equiv$ $\mathcal{O}(1/10)(M_a/g_a^2)^2|_{Q_0}[g_a^4|_{Q_{EW}} - g_a^4|_{Q_0}]_{1-{\rm loop}}$, $\tilde{\Delta}_{\tilde{x}}(\tilde{x} \in$ the first family of squarks and sleptons)$\equiv [T_{3\tilde{x}} - Q_{\tilde{x}}{\rm Sin}^2(\theta_W)]{\rm Cos}(2\beta)m_Z^2$ Martin (1997).

- For $m_{3/2} \sim 10TeV$ (which can be realized in our setup, one obtains $K_a \sim 3.5(TeV)^2$ AM, P.Shukla [2010] to be compared with $0.5(TeV)^2$ as obtained in Conlon et al [2007]; an mSUGRA point on the “SPS1a slope" has a value of around $(TeV)^2$.

Proton Decay

- The possibility of proton decay in Grand unified theories is caused by higher dimensional B-number-violating operators.
- The B-number-violating dimension-five operators in SUSY GUT-type models relevant to proton decay are of the type: $(\mathrm{squark})^2(\mathrm{quark})(\mathrm{lepton})$ or $(\mathrm{squark})^2(\mathrm{quark})^2$ Ellis et al [1982], Nath and Perez [2006]. This would correspond to $\partial^2 W/\partial\mathcal{A}_I^2|_{\theta=0}(\chi^I)^2$, in our setup. There is no $\mathcal{A}_I$-dependence of W implying the stability of the proton up to dimension-five operators.

- Using two local involutively-odd harmonic one-forms on the big divisor Σ^Λ that lie in $coker\left(H^{(0,1)}_{\bar\partial,-}(CY_3)\stackrel{i^*}{\longrightarrow}H^{(0,1)}_{\bar\partial,-}(\Sigma^\Lambda)\right)$ localized around the mobile $D3$-brane AM, Pramod Shukla [2009, 2010] - one estimates

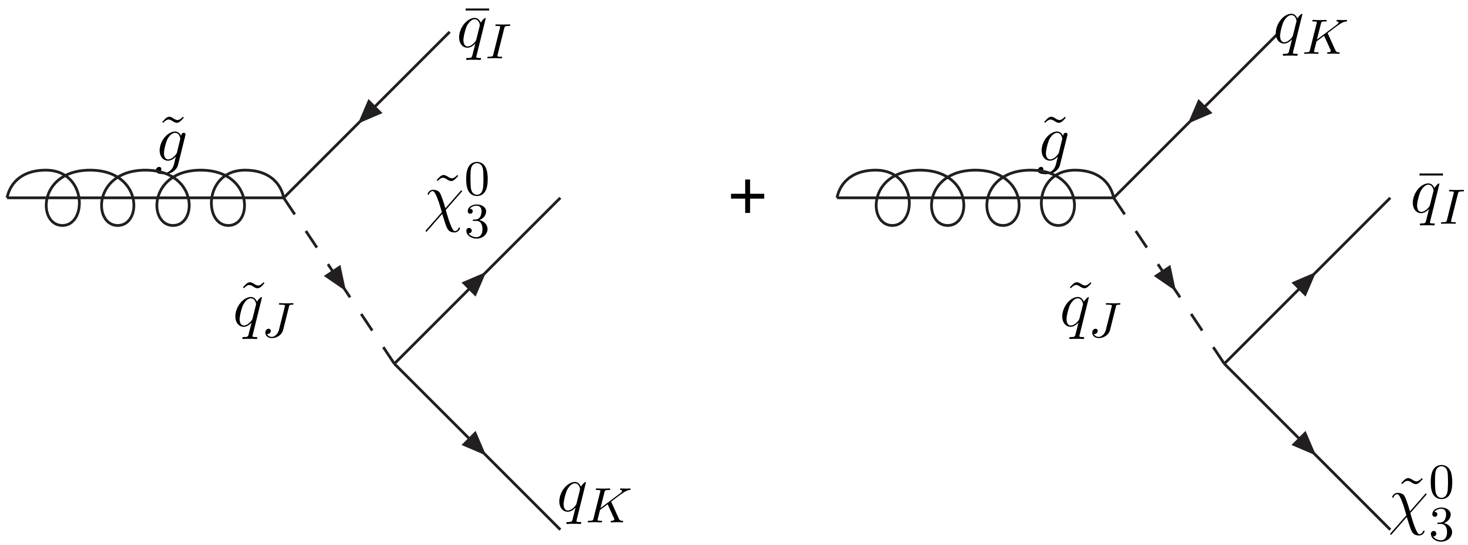
$$i\kappa_4^2\mu_7C_{I\bar J}a_I\bar a_{\bar J}\sim\mathcal{V}^{7/6}|a_1|^2+\mathcal{V}^{2/3}(a_1\bar a_{\bar 2}+c.c.)+\mathcal{V}^{1/6}|a_2|^2,$$

a_2 being another Wilson line modulus.

- The Wilson line moduli a_I can be stabilized at around $\mathcal{V}^{-1/4}$ and hence a partial cancelation between $vol(\Sigma_B)$ and $i\kappa_4^2\mu_7C_{1\bar 1}|a_1|^2$ in T_B is possible.

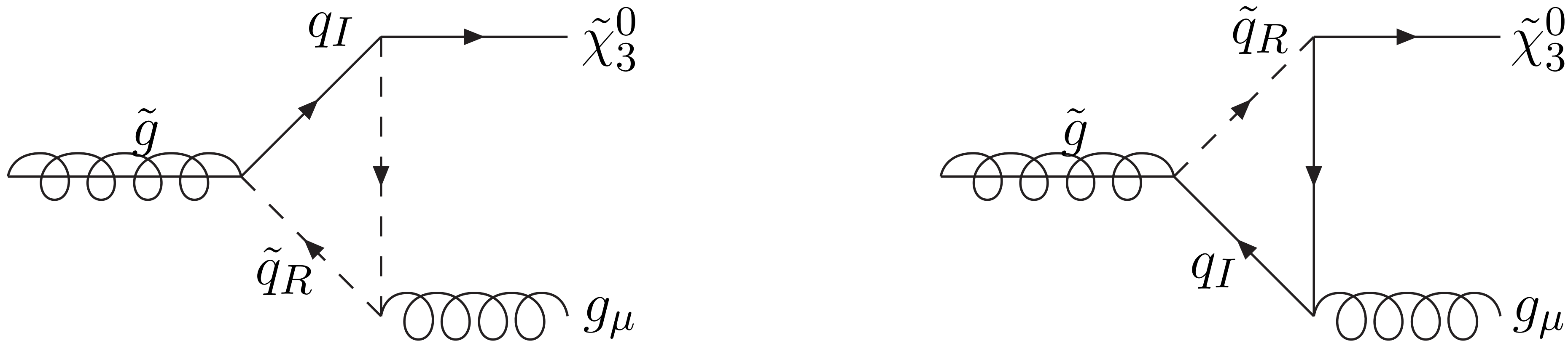
- With the idea of considering fluctuations in a_2 about $\mathcal{V}^{-1/4}$ - $a_2 \to \mathcal{V}^{-\frac{1}{4}} + a_2$ - keeping a_1 fixed with a_2 promoted to the Wilson line modulus superfield $\mathcal{A}_2$ in the Kähler potential, when expanded in powers of the canonically normalized $\hat{\mathcal{A}}_2$, the SUSY GUT-type four-fermion dimension-six proton decay operator obtained from $\int d^2\theta d^2\bar{\theta}(\mathcal{A}_2)^2(\mathcal{A}_2^\dagger)^2/M_p^2(\in K(\hat{\mathcal{A}}_I, \hat{\mathcal{A}}_I^\dagger, ...))$, for $\mathcal{V} \sim 10^6$ would correspond to a proton lifetime P.Nath, P.F.Peres [2006], Klebanov, Witten[2003]; Friedmann, Witten[2002]:
$$\frac{\mathcal{O}(1)\times L_{\Sigma_B}^{-4/3}(10^{9/2}M_p)^4}{(\alpha^2(M_s)m_p^5)},$$
$L_{\Sigma_B} \equiv$ Ray-Singer torsion of Σ_B.
- Assuming $L_{\Sigma_B} \sim \mathcal{O}(1)$ and obtain an upper bound on the proton lifetime to be around 10^{61} years, in conformity with the very large sparticle masses in our setup.

- Consider three-body decay (as an example): $\tilde{g} \to q\bar{q}\chi_n$; $\tilde{g}$ being a gaugino, $q/\bar{q}$ being quark/anti-quark and χ_n being a neutralino.



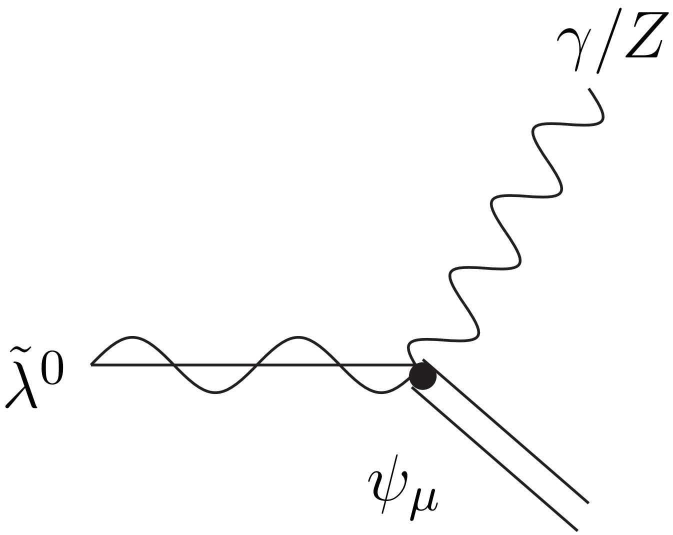
Three-body gluino decay diagrams

$\tilde{g} \longrightarrow \tilde{\chi}_3^0 + g$

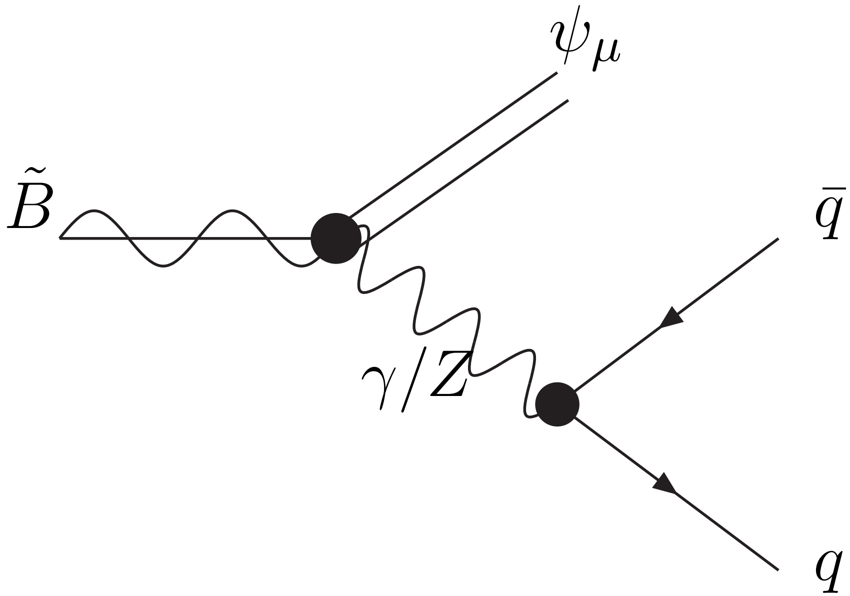


Diagrams contributing to one-loop two-body gluino decay

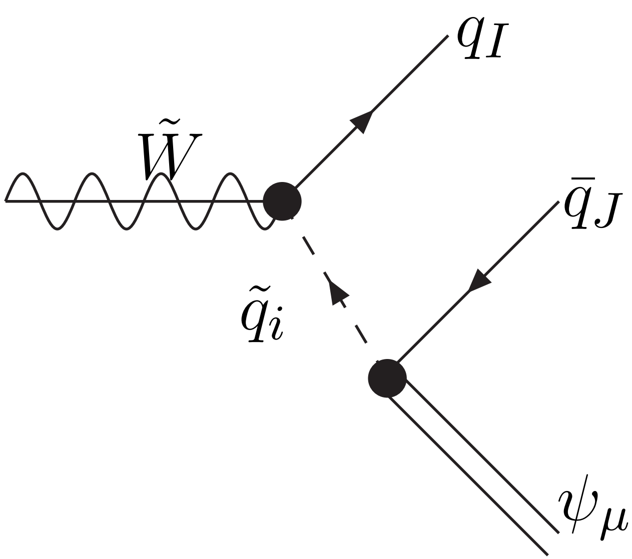
Gaugino Decays



Two-body gaugino decay

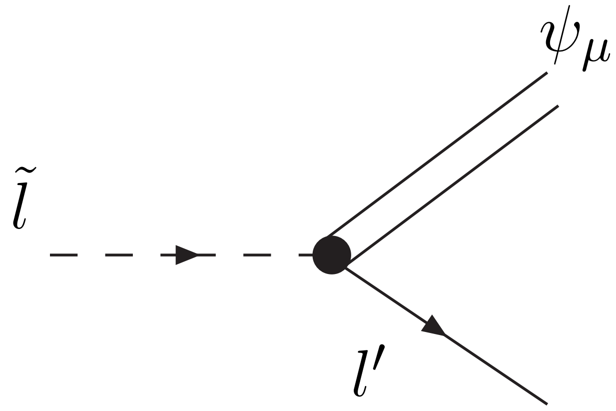


Three-body gaugino-decay diagrams

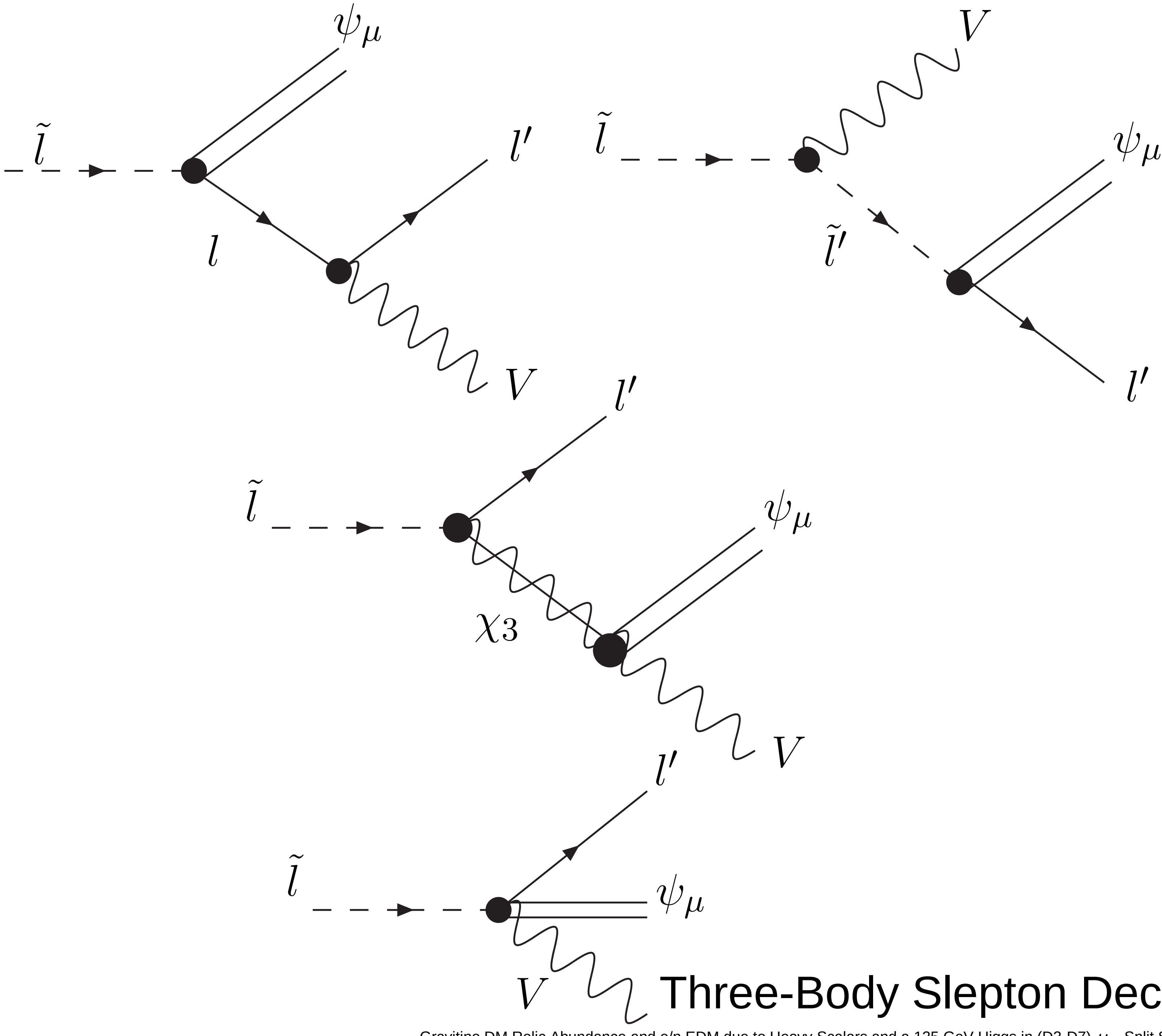


Three-body gaugino decays into the gravitino

Slepton/Squark Decays

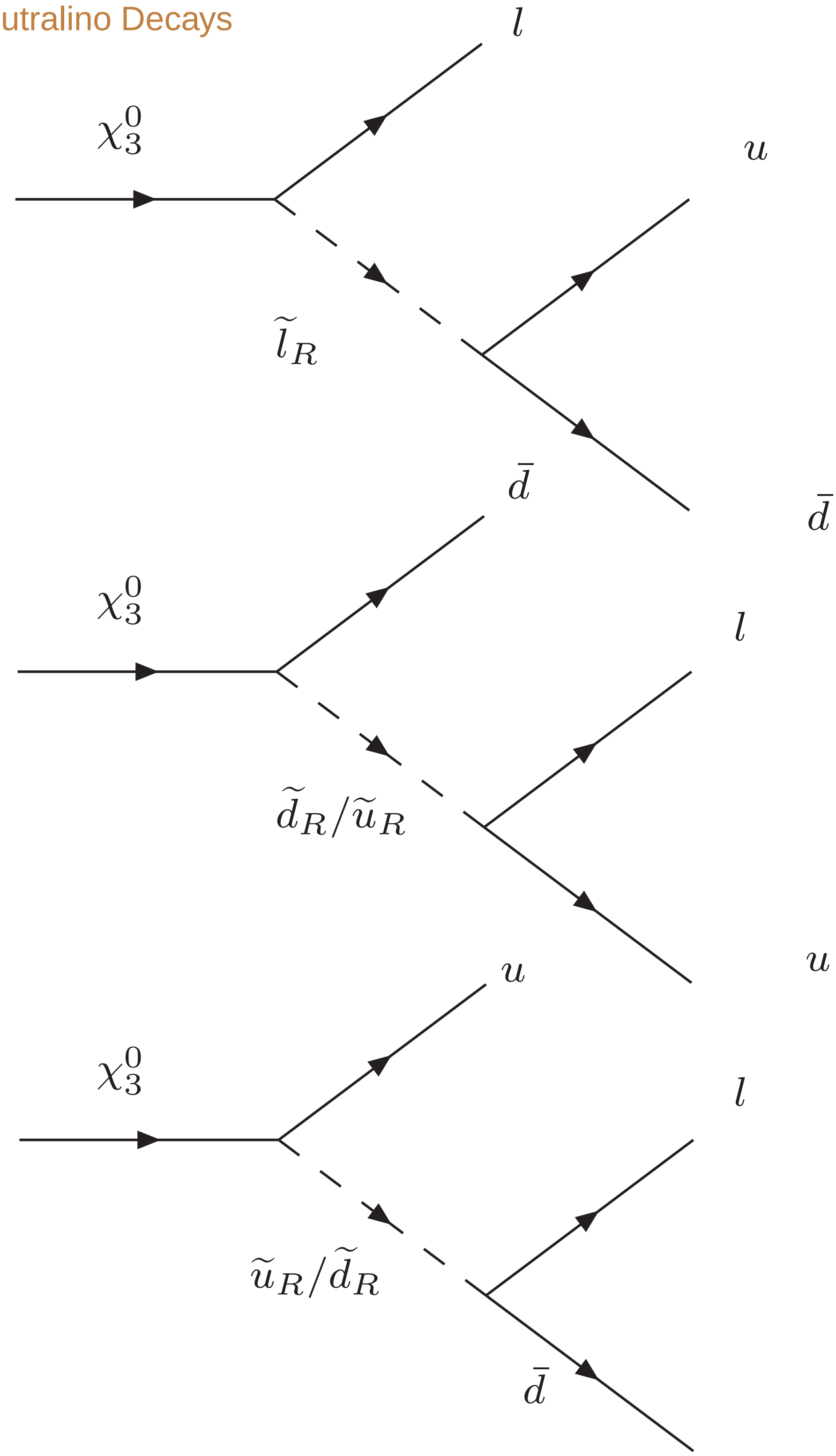


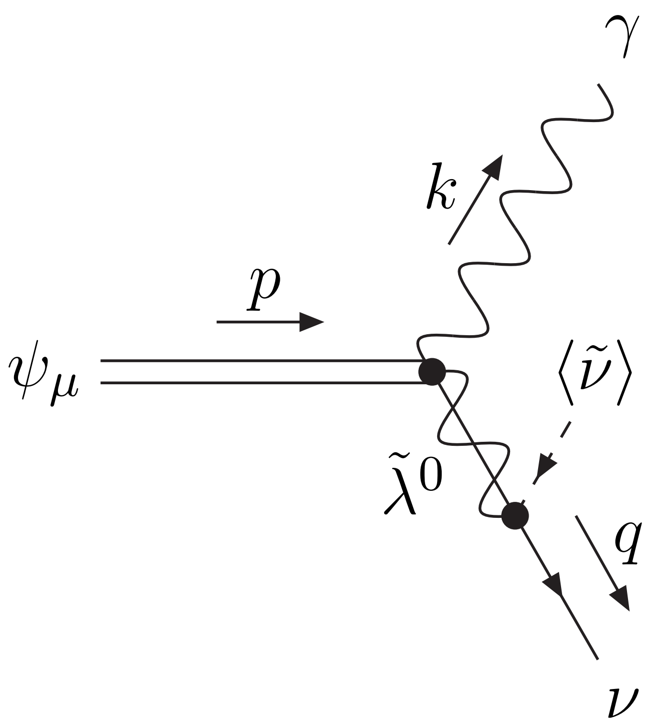
Two-Body Slepton/Squark Decay



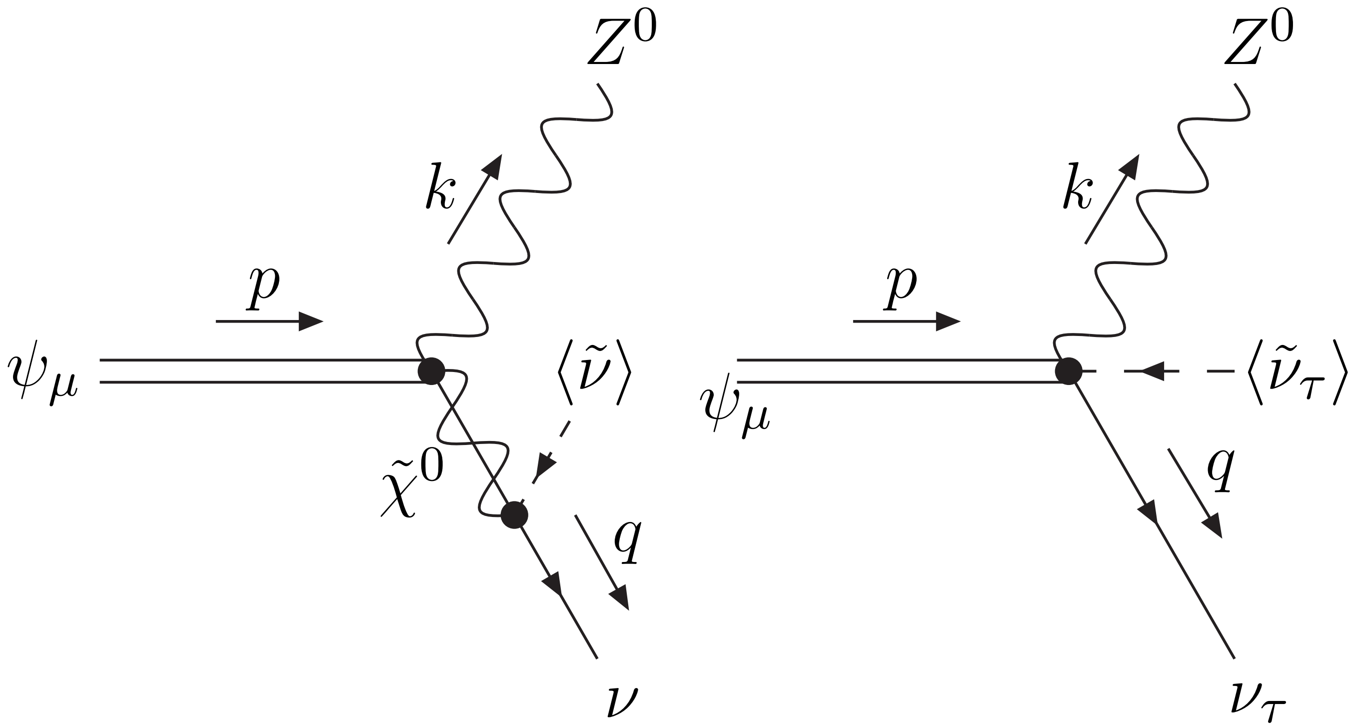
Three-Body Slepton Decays

Neutralino Decays

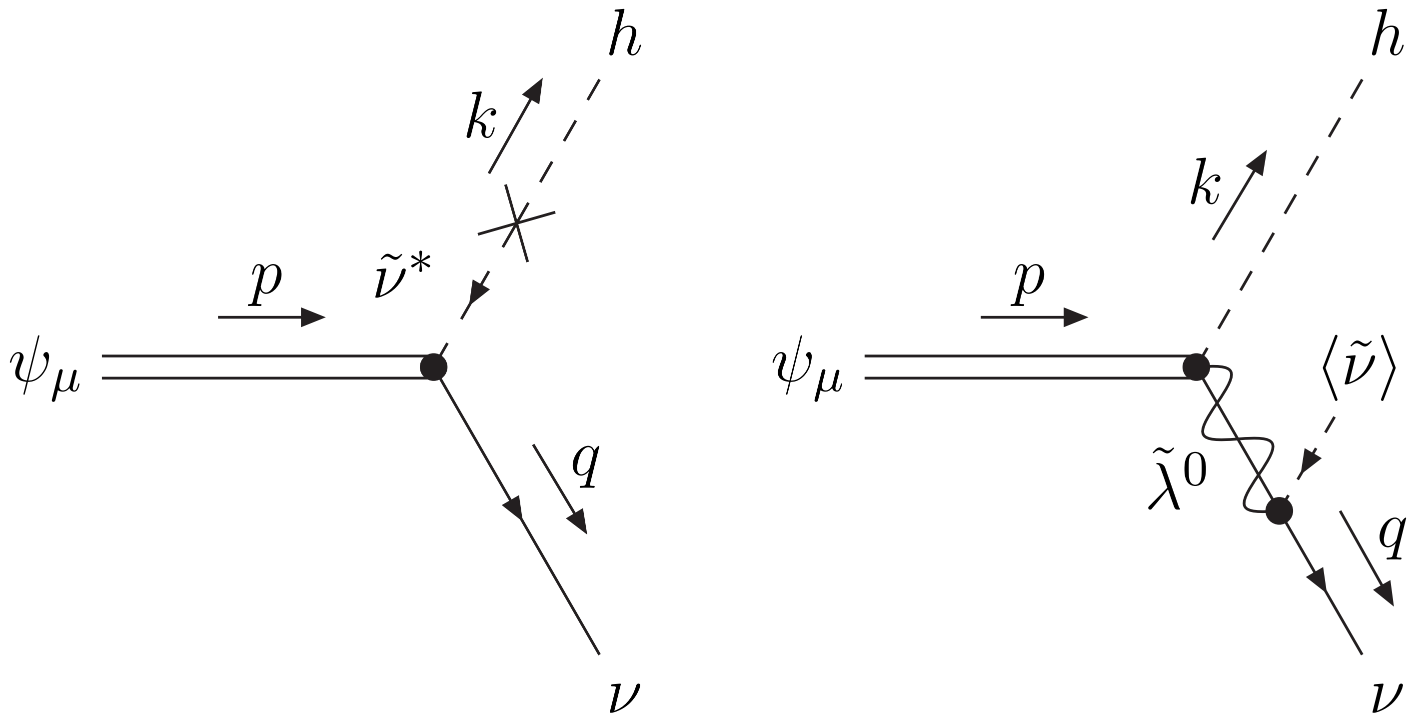




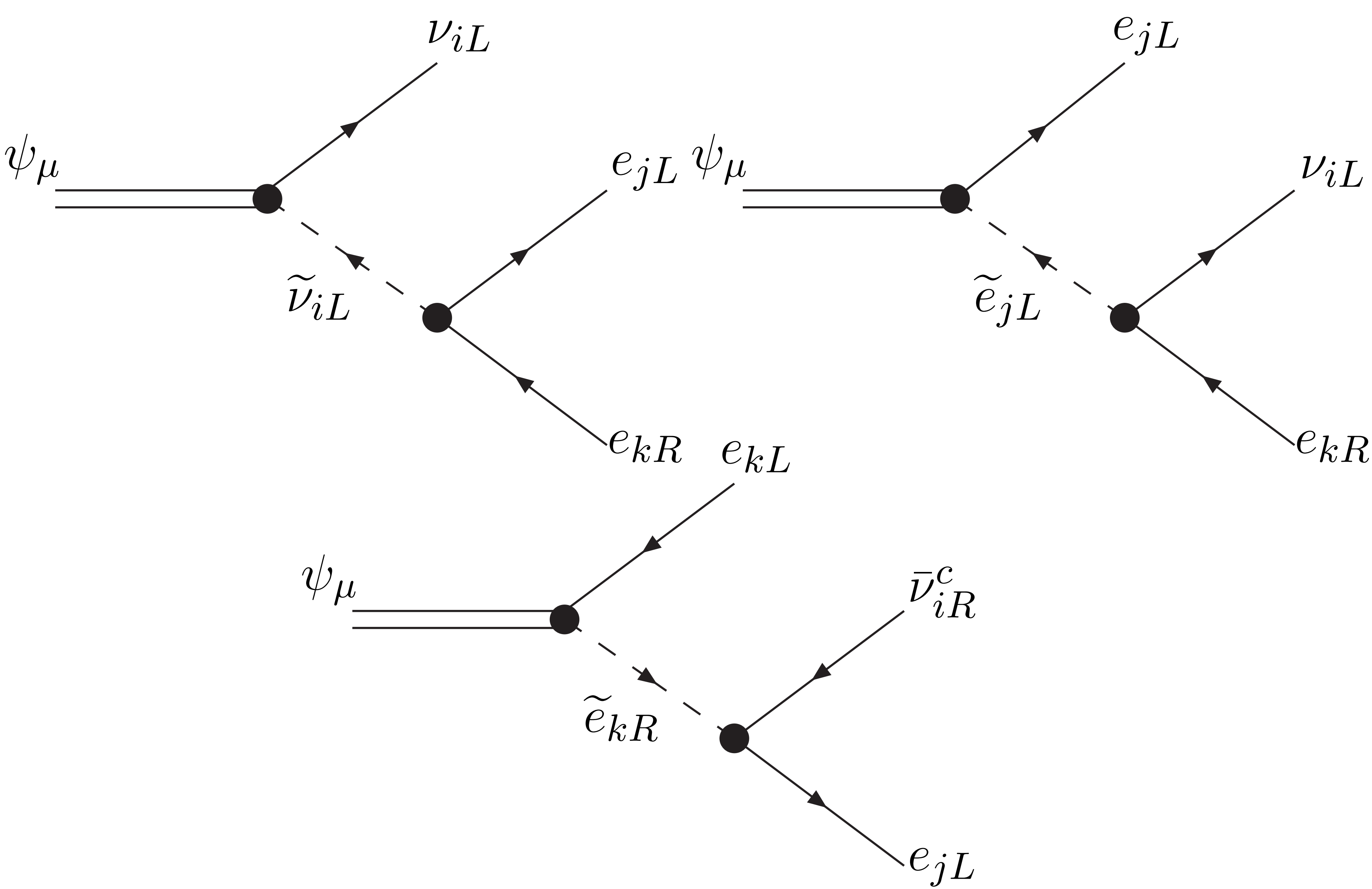
Two-body gravitino decay: $\psi_\mu \to \nu + \gamma$



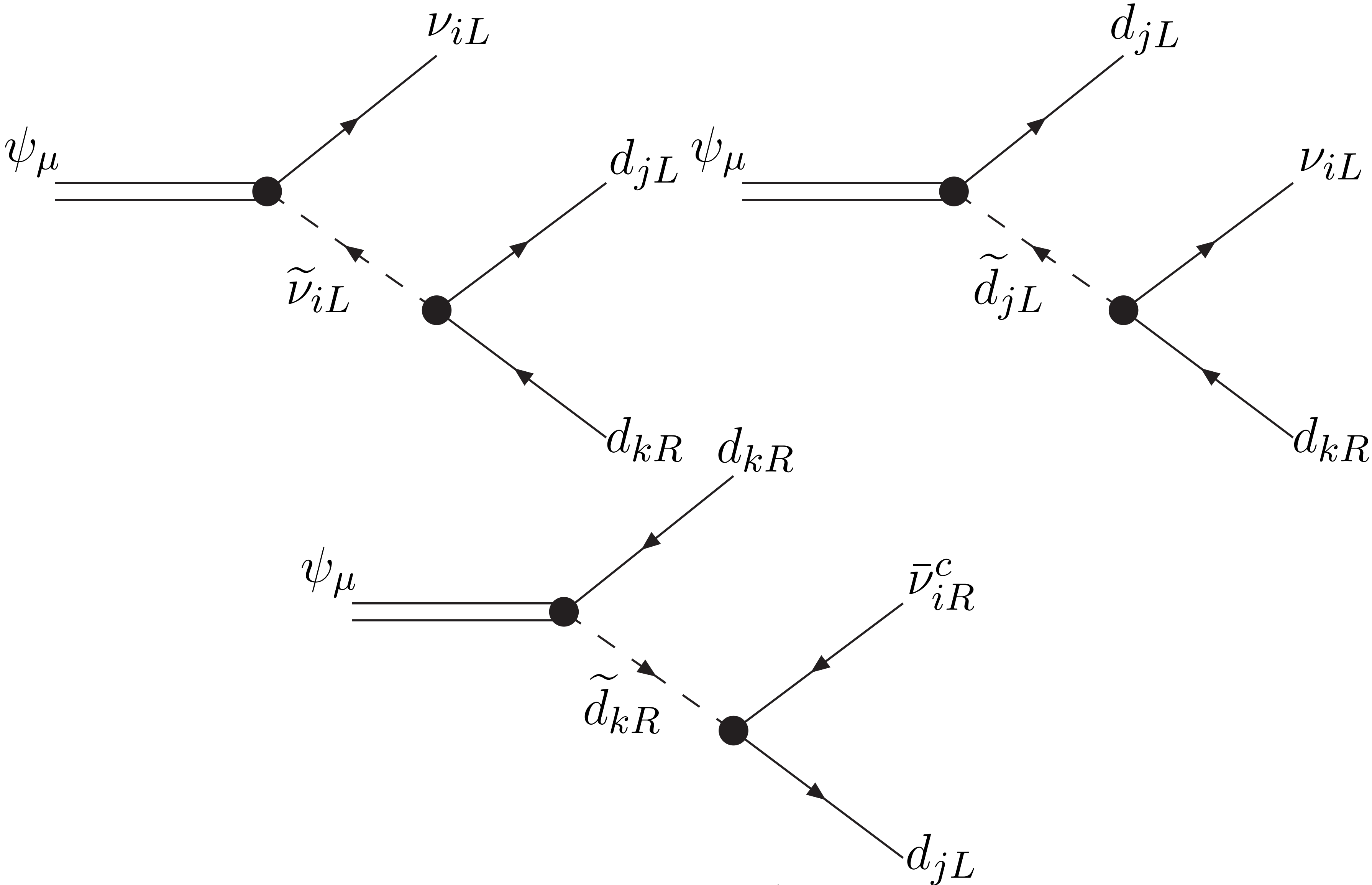
Two-body gravitino decay: $\psi_\mu \rightarrow Z^0 + \nu$



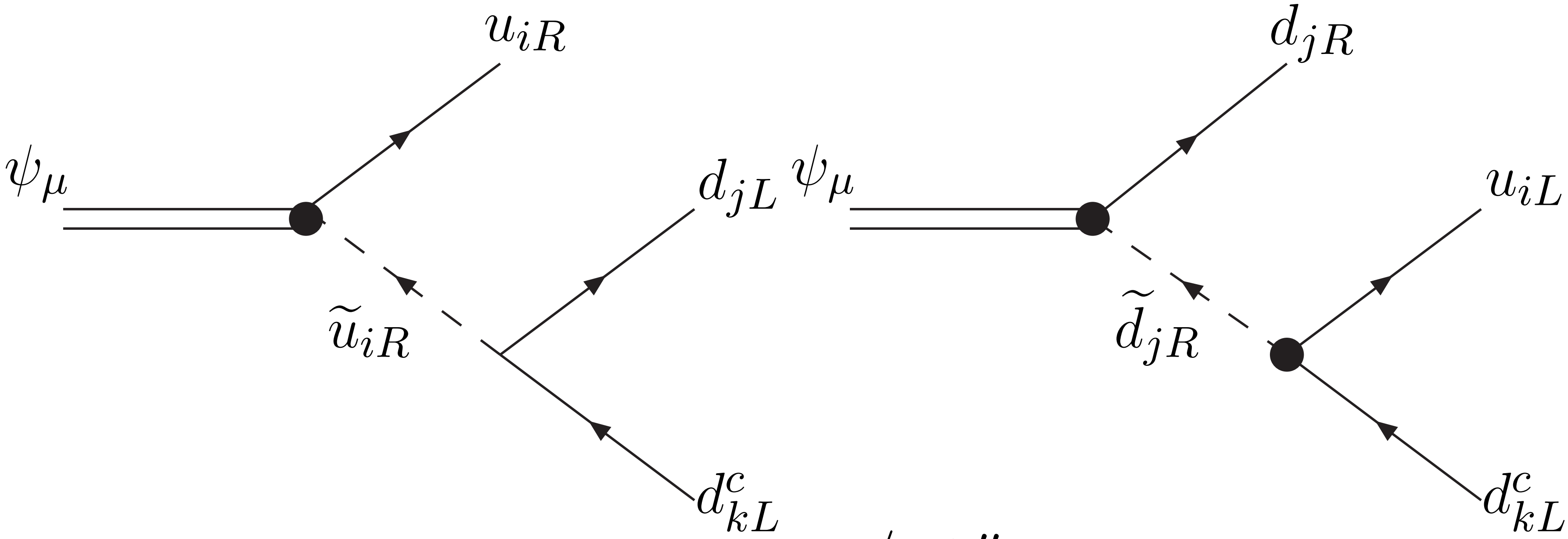
Two-body gravitino decay: $\psi_\mu \rightarrow h + \nu$



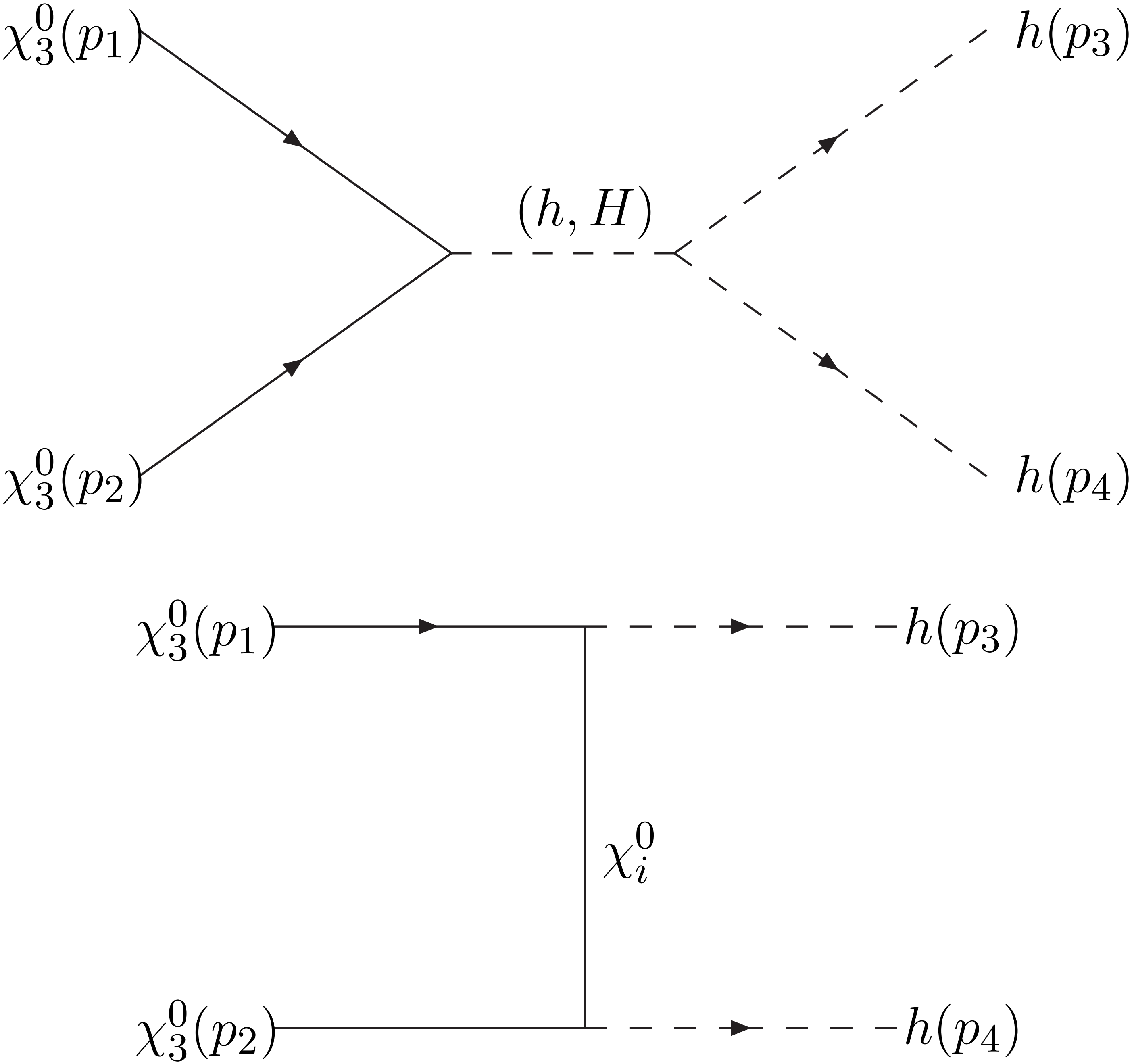
Three-body gravitino decays involving $\not{R}_p$ λ_{ijk} coupling

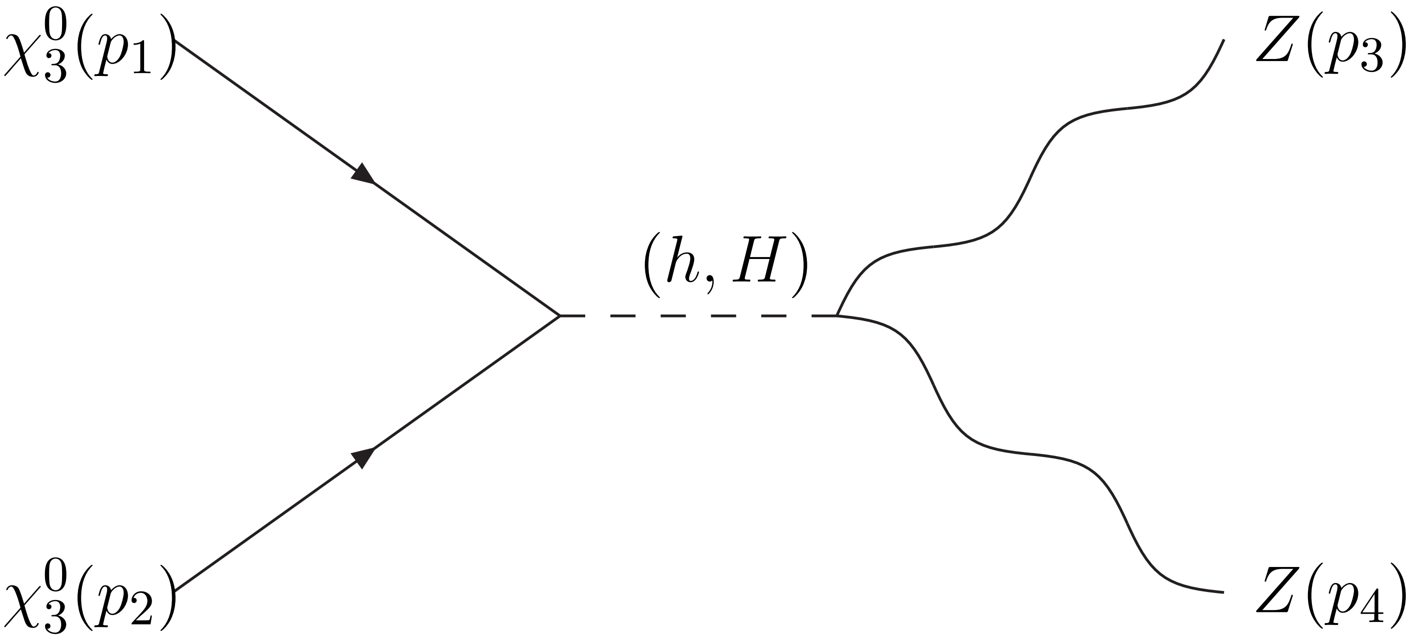


Three-body gravitino decays involving $\not R_p$ λ'_{ijk} *coupling*

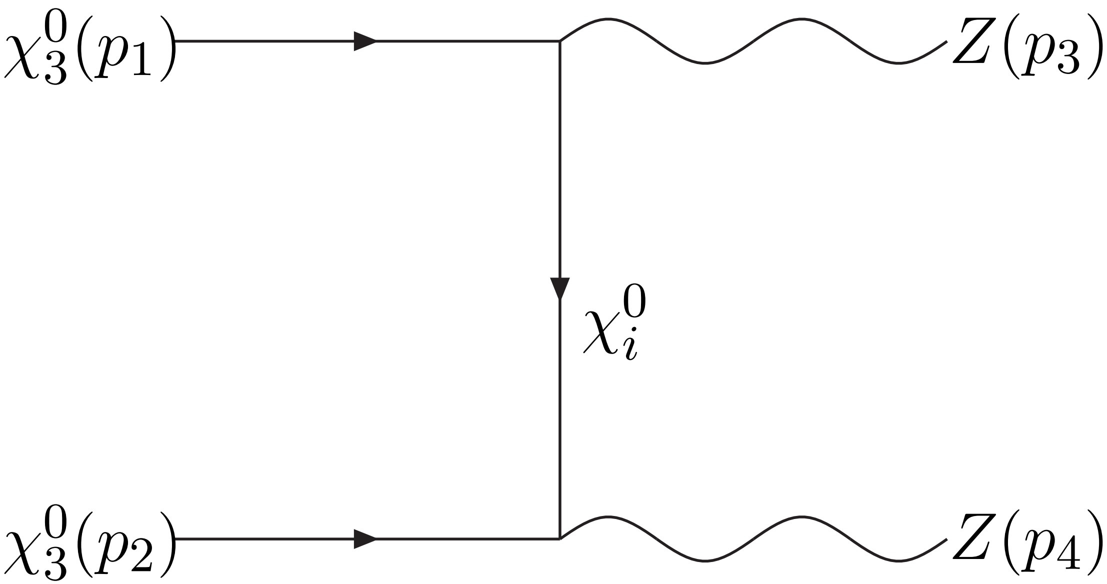


Three-body gravitino decays involving $\not{R}_p\ \lambda''_{ijk}$ *coupling*

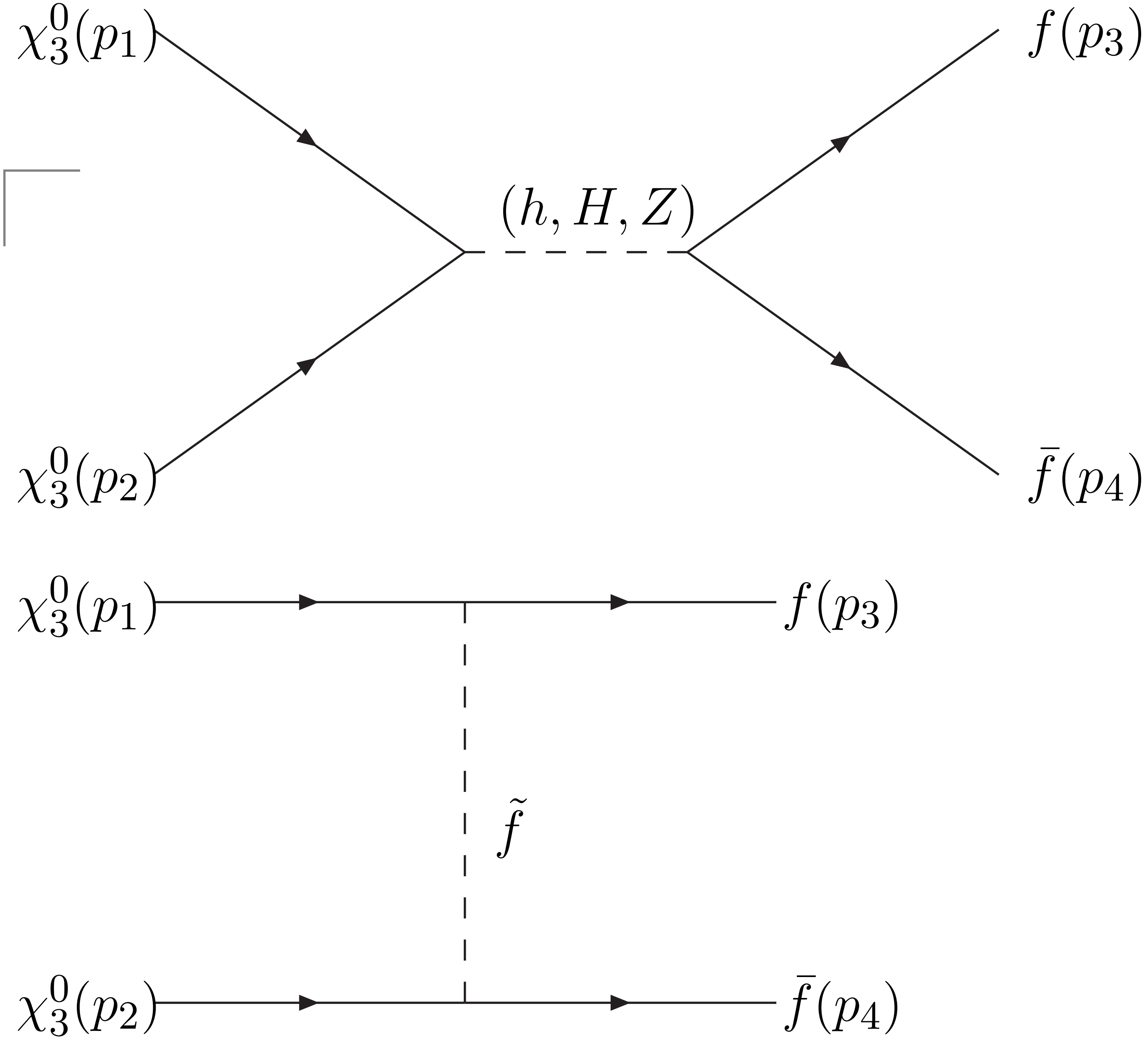

$\chi_3^0(p_1)$
$h(p_3)$
(h, H)
$\chi_3^0(p_2)$
$h(p_4)$
$\chi_3^0(p_1)$
$h(p_3)$
χ_i^0
$\chi_3^0(p_2)$
$h(p_4)$



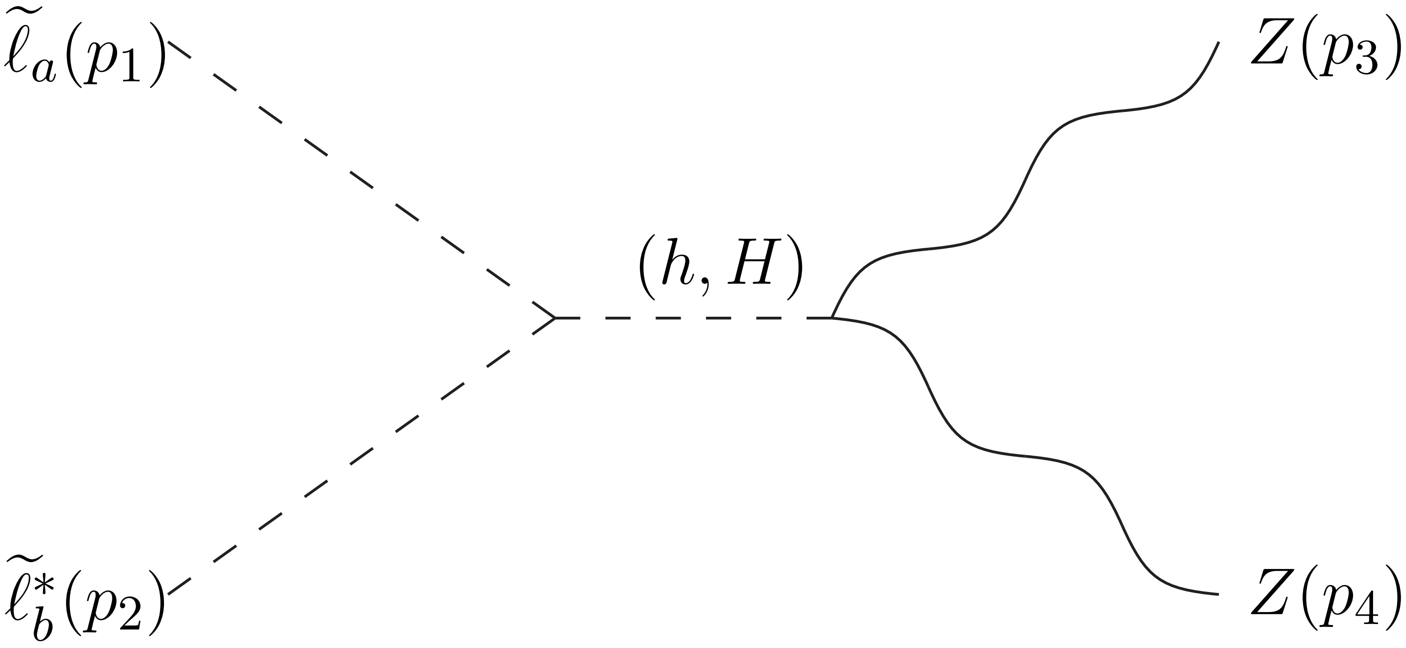
$\chi_3^0(p_1)$
$Z(p_3)$
(h, H)
$\chi_3^0(p_2)$
$Z(p_4)$



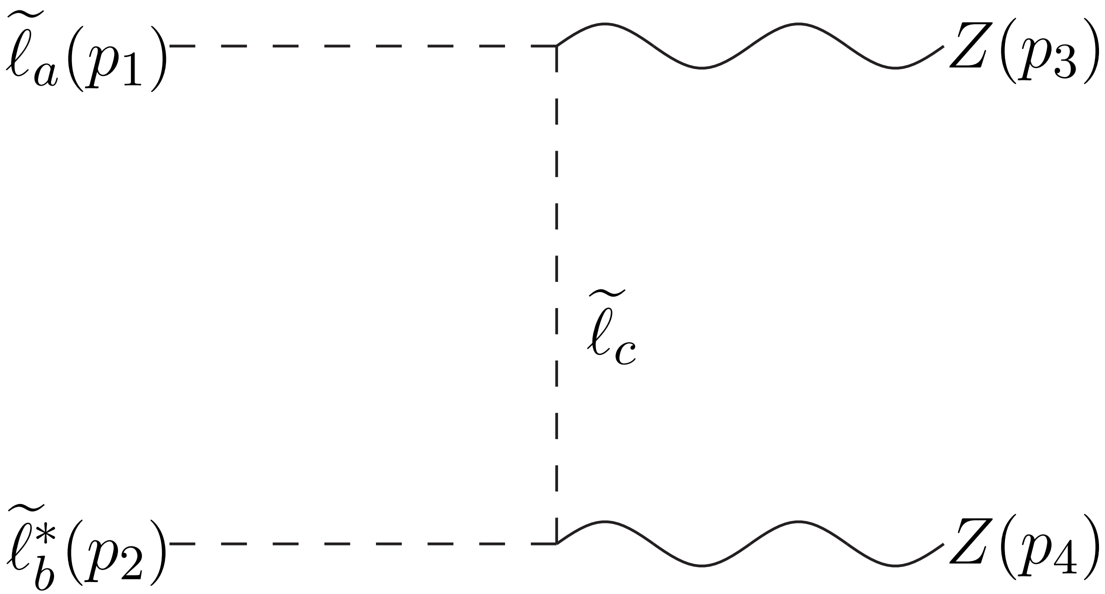
$\chi_3^0(p_1)$
$Z(p_3)$
χ_i^0
$\chi_3^0(p_2)$
$Z(p_4)$



Feynman diagrams for $\chi_3^0\chi_3^0 \rightarrow f\bar{f}$ *via*



$\widetilde{\ell}_a(p_1)$
$Z(p_3)$
(h, H)
$\widetilde{\ell}_b^*(p_2)$
$Z(p_4)$



$\widetilde{\ell}_a(p_1)$
$Z(p_3)$
$\widetilde{\ell}_c$
$\widetilde{\ell}_b^*(p_2)$
$Z(p_4)$

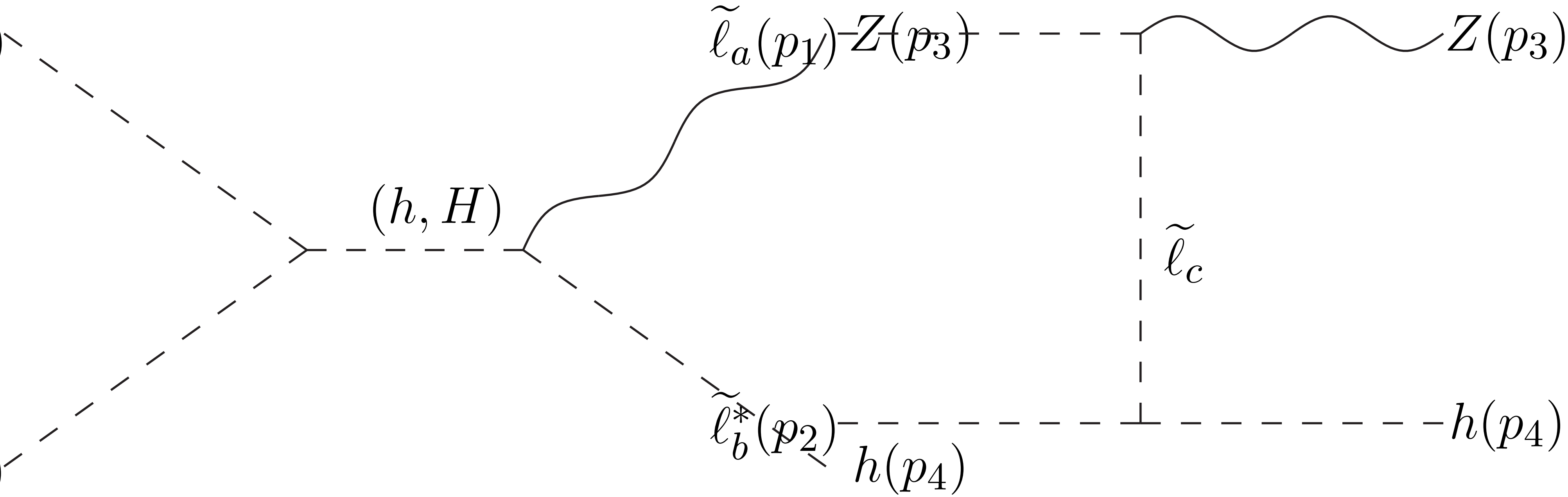
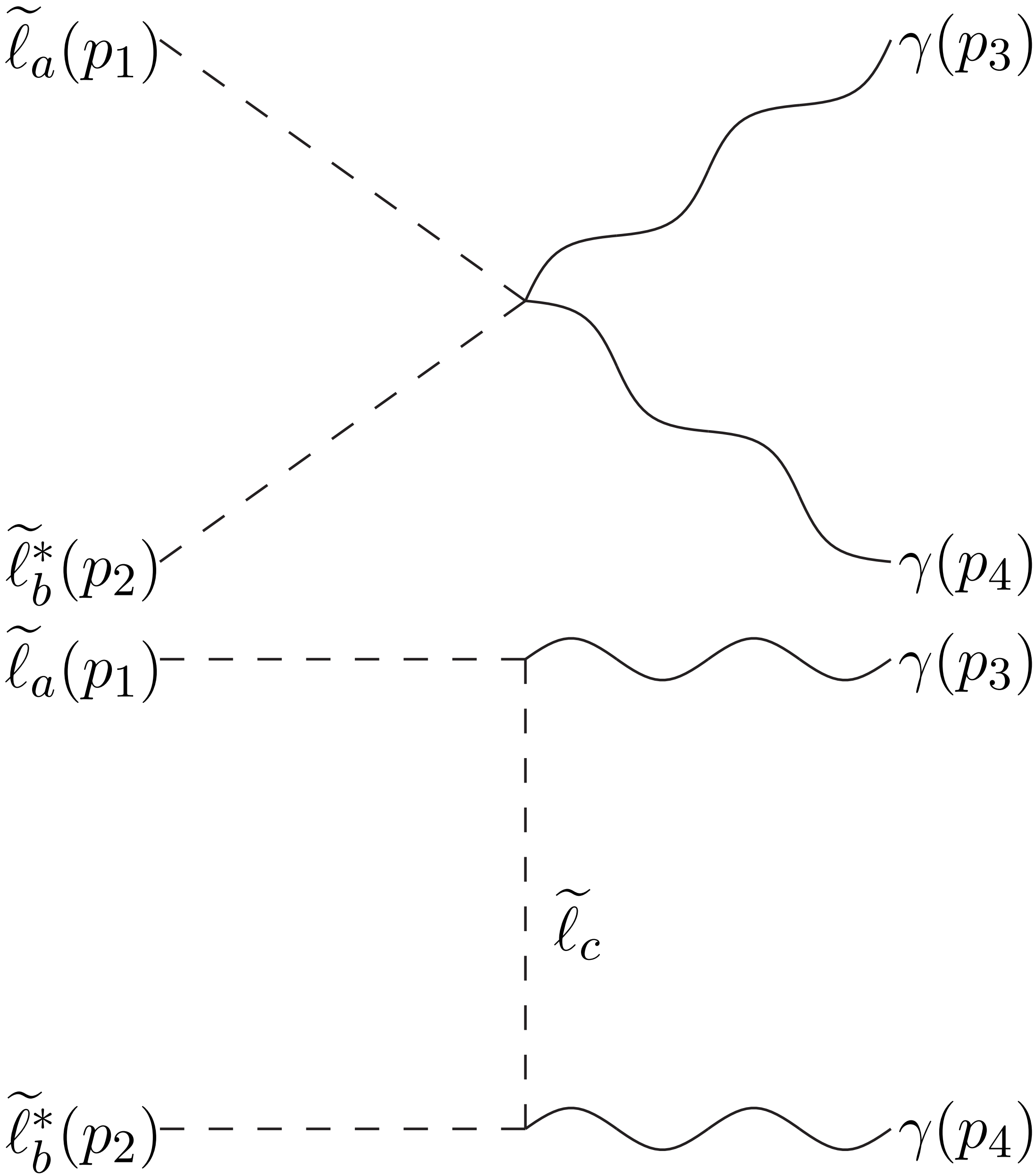


Fig. 18 Feynman diagrams for $\widetilde{\ell}_a\widetilde{\ell}_b^* \rightarrow Zh$ via s–channel Higgs exchange and t-channel $\widetilde{\ell}_c$ exchange.


$\widetilde{\ell}_a(p_1)$
$\gamma(p_3)$
$\widetilde{\ell}^*_b(p_2)$
$\gamma(p_4)$
$\widetilde{\ell}_a(p_1)$
$\gamma(p_3)$
$\widetilde{\ell}_c$
$\widetilde{\ell}^*_b(p_2)$
$\gamma(p_4)$