Cabibbo's dream

Belén Gavela

(Alonso, Gavela, D.Hernandez, Merlo, Rigolin)

(Alonso, Gavela, Isidori, Maiani)

invisibles
neutrinos, dark matter & dark energy physics

Neutrino masses and mixings from a minimal principle

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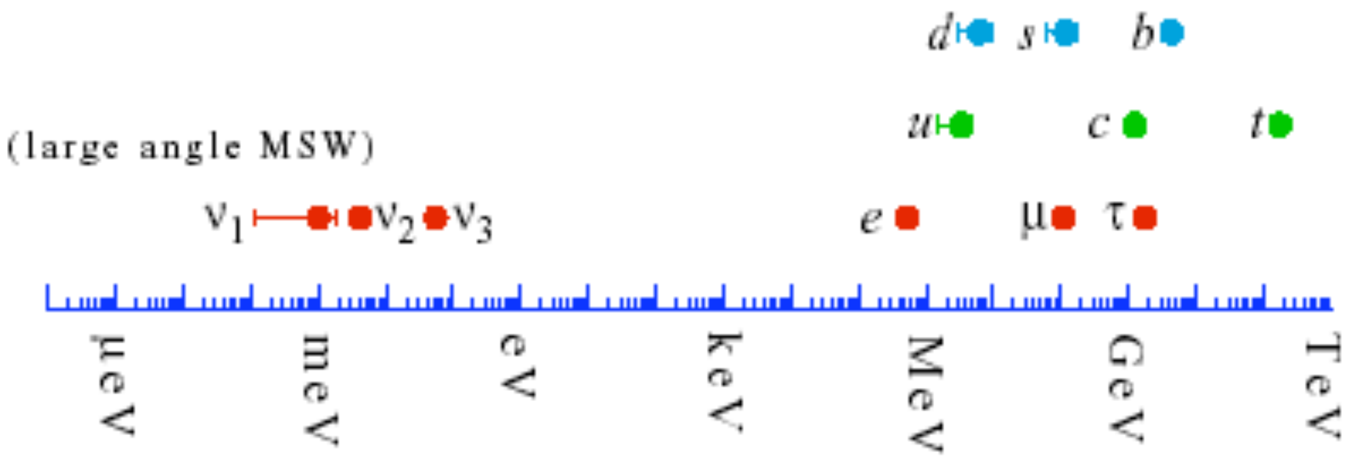
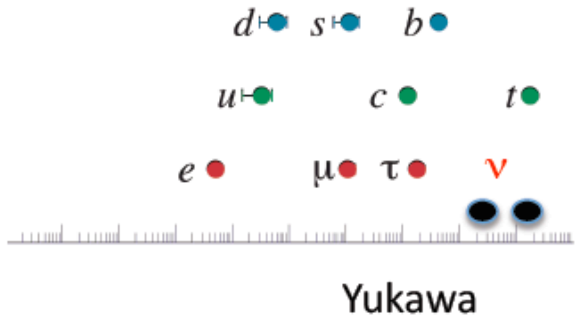
Neutrino light on flavour ?



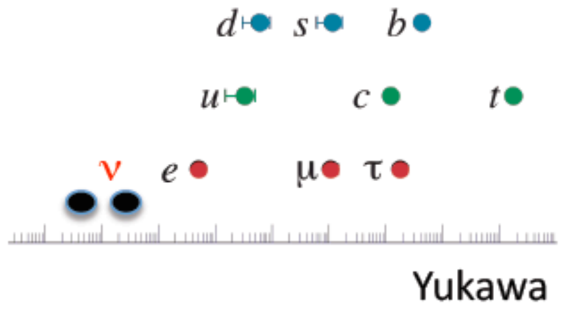
Neutrinos lighter because Majorana?

Within seesaw, the size of ν Yukawa couplings is alike to that for other fermions:

$\Lambda \leq$ GUT



Λ = TeV



Pilar Hernandez drawings

Minkowski; Gell-Mann, Ramond Slansky; Yanagida, Glashow...

Leptons

$$V_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & \sim 9^o \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & +0.7 \end{pmatrix}$$

Quarks

$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

Why so different?

Leptons

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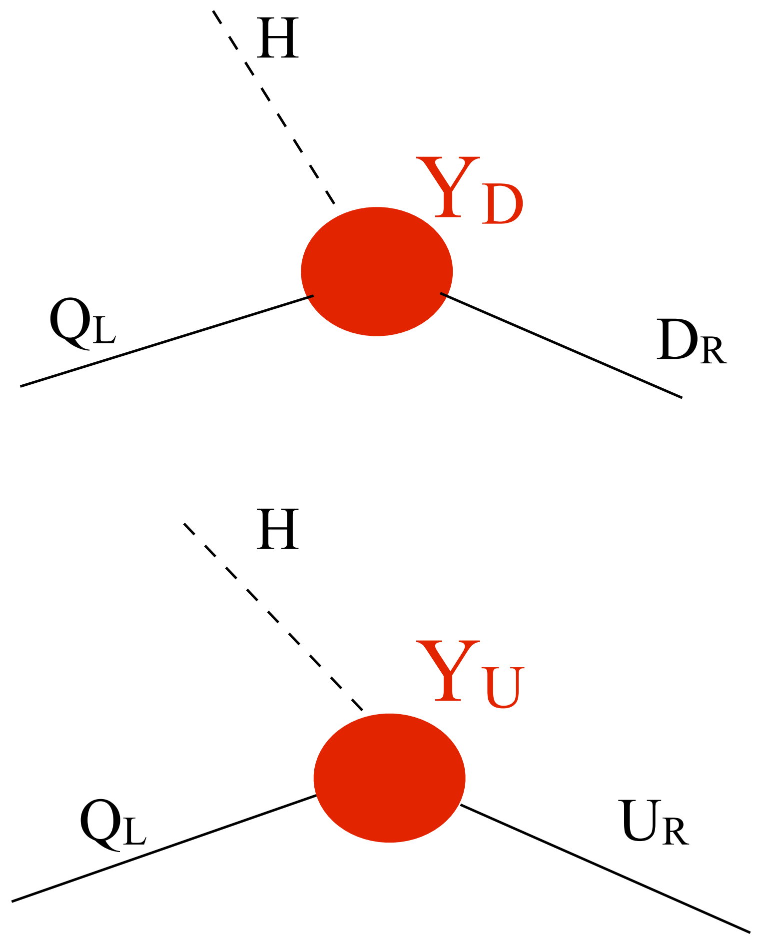
Perhaps also because ν_s may be Majorana?

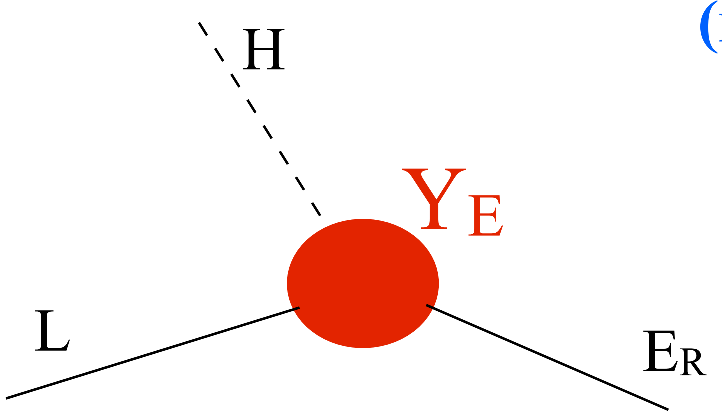
- Dynamical Yukawas

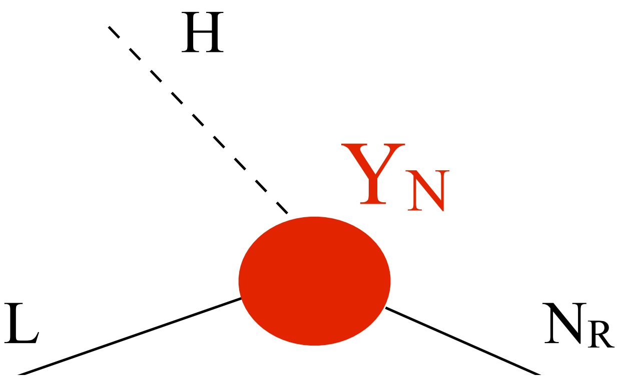
Yukawa couplings are the source of flavour in the SM



Yukawa couplings are a source of flavour in the ν-SM

(i.e. Seesaw type-I)





$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\partial\!\!\!/ N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R} M N_R{}^c + h.c.\right]$$

May they correspond to dynamical fields (e.g. vev of fields that carry flavor) ?

Instead of inventing an ad-hoc symmetry group,

why not use the continuous flavour group

suggested by the SM itself?

We have realized that the different pattern for

quarks versus leptons

may be a simple consequence of the

continuous flavour group of the SM (+ seesaw)

(Alonso, Gavela, D.Hernandez, Merlo, Rigolin)

(Alonso, Gavela, Isidori, Maiani) 2013

We have realized that the different pattern for

quarks versus leptons

may be a simple consequence of the

continuous flavour group of the SM (+ seesaw)

Our guideline is to use:

- maximal symmetry
- minimal field content

(Alonso, Gavela, D.Hernandez, Merlo, Rigolin)

2013

(Alonso, Gavela, Isidori, Maiani)

Global flavour symmetry of the SM

* QCD has a global -chiral- symmetry in the limit of massless quarks. For n generations:

$$\mathcal{L}_{QCD}^{\text{fermions}} = \bar{\Psi}(i\not{D} - m)\Psi \rightarrow \bar{\Psi} i\not{D}\Psi = \overline{\Psi_L} i\not{D}\Psi_L + \overline{\Psi_R} i\not{D}\Psi_R$$

$$SU(n)_L \times SU(n)_R \times U(1)'s$$

* In the SM, fermion masses and mixings result from Yukawa couplings. For massless quarks, the SM has a global flavour symmetry:

Quarks

$$\mathscr{L}_{SM}^{\text{fermions}} = i \sum_{\psi=Q_L}^{D_R} \overline{\psi} \not{D} \psi \qquad G_{\text{flavour}} = U(n)_{Q_L} \times U(n)_{U_R} \times U(n)_{D_R}$$

[Georgi, Chivukula, 1987]

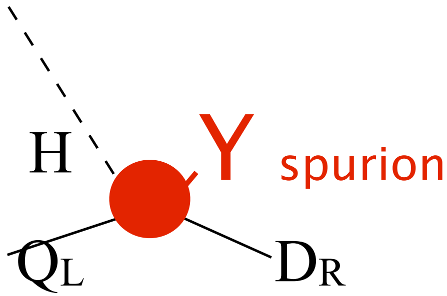
This continuous symmetry of the SM

$$G_{\text{flavour}} = U(n)_{Q_L} \times U(n)_{U_R} \times U(n)_{D_R}$$

is phenomenologically very successful and

at the basis of Minimal Flavour Violation

in which the Yukawa couplings are only spurions



D'Ambrosio+Giudice+Isidori+Strumia;
Cirigliano+Isidori+Grinstein+Wise

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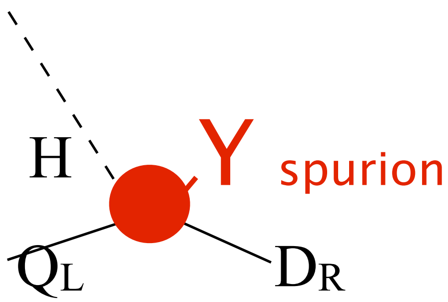


$$\frac{Y_{\alpha\beta}^{+}\, Y_{\delta\gamma}}{\Lambda_f^2}\; \bar{Q}_\alpha \gamma_\mu Q_\beta\, \bar{Q}_\gamma \gamma^\mu Q_\delta$$

D'Ambrosio+Giudice+Isidori+Strumia;
Cirigliano+Isidori+Grinstein+Wise

One step further :

dynamical Ys

(Alonso, Gavela, D.Hernandez, Merlo, Rigolin, 2012 -2013)

(Alonso, Gavela, Isidori, Maiani, 2013)

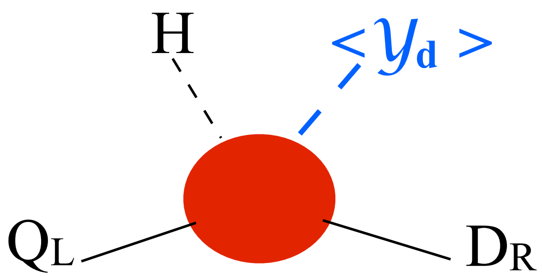
Quarks

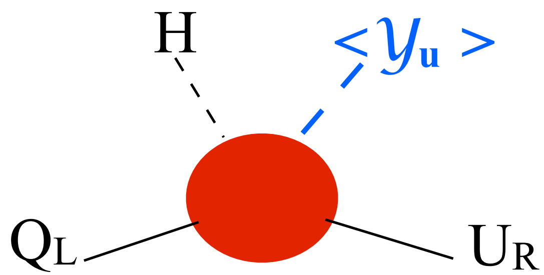
For this talk:

each Y_{SM} -- >one single field $\mathcal{Y}$

$$Y_{SM} \sim \frac{<\mathcal{Y}>}{\Lambda_f}$$

quarks:





Anselm+Berezhiani 96; Berezhiani+Rossi 01... Alonso+Gavela+Merlo+Rigolin 11...

$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$ ....

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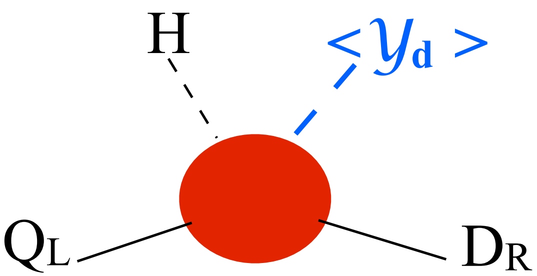
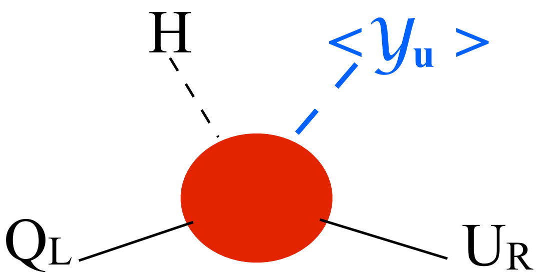
quarks:





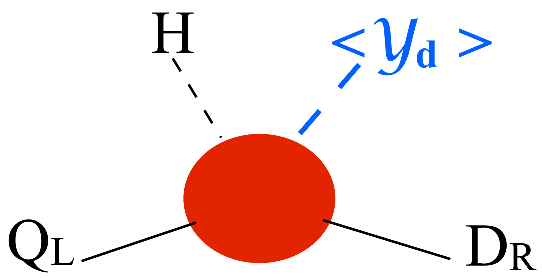
$\mathcal{Y}_d \sim (3,1,\bar{3})$ "bifundamentals" $\mathcal{Y}_u \sim (3,\bar{3},1)$

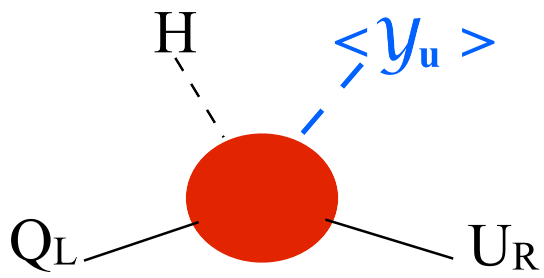
$$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \;\ldots.$$

$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$ ....

$\mathcal{Y}_d \sim (3,1,\bar{3})$ $\mathcal{Y}_u \sim (3,\bar{3},1)$

That is, two dynamical scalars





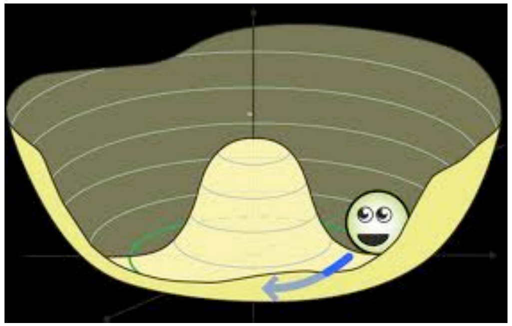
$$\mathcal{Y}_d \sim (3, \bar{3}, 1) \qquad\qquad \mathcal{Y}_u \sim (3, 1, \bar{3})$$

$$\boxed{\frac{<\mathcal{Y}_d>}{\Lambda_f} = Y_D = V_{CKM} \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}}, \quad \boxed{\frac{<\mathcal{Y}_u>}{\Lambda_f} = Y_U = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}}$$

Flavour Symmetry Breaking

Spontaneous breaking of flavour symmetry dangerous

To prevent Goldstone Bosons the symmetry can be Gauged



[Grinstein, Redi, Villadoro
Guadagnoli, Mohapatra, Sung
Feldman]

$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$ ....

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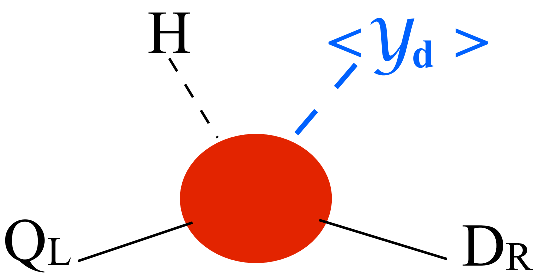
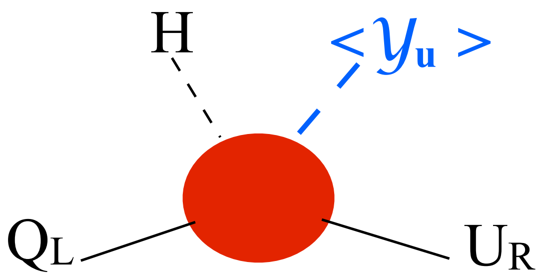


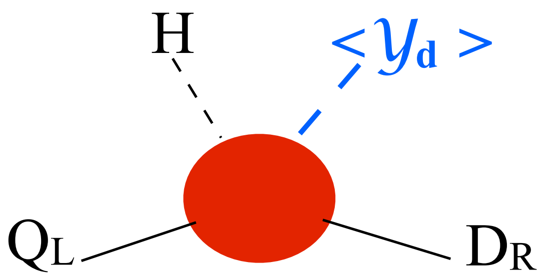


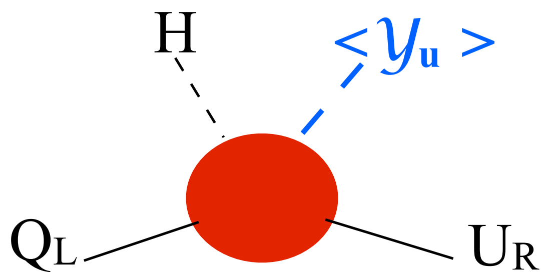
¿$V(\mathcal{Y}_d, \mathcal{Y}_u)$?

$$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \;\; ....$$

$\mathcal{Y}_d \sim (3,1,\bar{3})$ $\mathcal{Y}_u \sim (3,\bar{3},1)$

That is, two dynamical scalars





*** Does the minimum of the scalar potential justify the observed masses and mixings?**

$$\mathbf{V}(\mathcal{Y}_d, \mathcal{Y}_u)$$

* Invariant under the SM gauge symmetry

* Invariant under its global flavour symmetry $G_{flavour}$

$$G_{flavour} = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$$

The basis of the game is to find the minima of the invariants that you can construct out of Yukawa couplings

L. Michel+Radicati 70, Cabibbo+Maiani71 for the spectrum of masses

List of possible invariants for quarks: Hanani, Jenkins, Manohar 2010

The basis of the game is to find the minima of the invariants that you can construct out of Yukawa couplings

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Cabibbo's dream

$$\mathbf{V}(\mathcal{Y}_d, \mathcal{Y}_u)$$

* Invariant under the SM gauge symmetry

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$$G_{flavour} = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$$

There are as many independent invariants I as physical variables

$$\mathbf{V}(\mathcal{Y}_d, \mathcal{Y}_u) = \mathbf{V}(\mathbf{I}(\mathcal{Y}_d, \mathcal{Y}_u))$$

Minimization

a variational principle fixes the vevs of the Fields

$$\delta V = 0$$

$$\sum_j \frac{\partial I_j}{\partial y_i}\frac{\partial V}{\partial I_j} \equiv J_{ij}\frac{\partial V}{\partial I_j} = 0\,,$$

masses, mixing angles etc.

This is an homogenous linear equation;
if the rank of the Jacobian $J_{ij} = \partial I_j/\partial y_i$, is:

Maximum:
then the only solution is: $\frac{\partial V}{\partial I_j} = 0\,,$

Less than Maximum:
then the number of equations reduces to a number equal to the rank

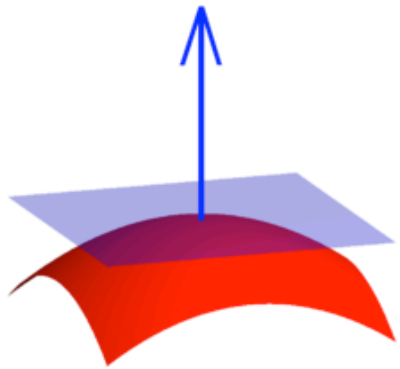
Boundaries

for a reduced rank of the Jacobian,

$\det(J) = 0$

there exists (at least) a direction δy_i for which
a variation of the field variables does
not vary the invariants

$$\delta I_j = \sum_i \frac{\partial I_j}{\partial y_i} \delta y_i = 0$$



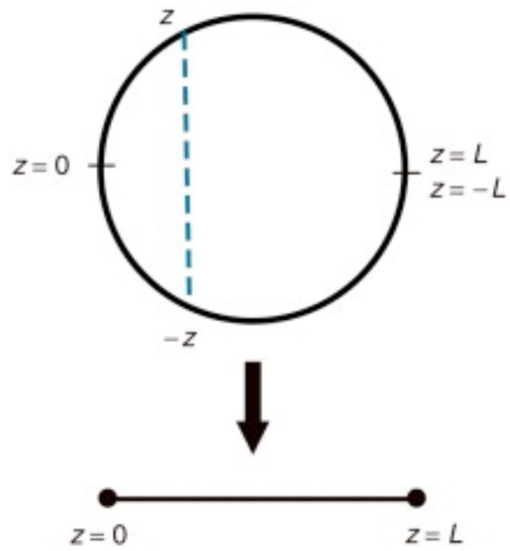
that is a Boundary of the *I-manifold*

[Cabibbo, Maiani, 1969]

Boundaries Exhibit Unbroken Symmetry [Michel, Radicati, 1969]
(maximal subgroups)

Boundaries Exhibit Unbroken Symmetry

Extra-Dimensions Example



<u>The smallest boundaries are extremal points of any function</u>

[Michel, Radicati, 1969]

Bi-fundamental Flavour Fields

For quarks: 10 independent invariants (because 6 masses+ 3 angles + 1 phase) that we may choose as

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger\right], \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D \mathcal{Y}_D^\dagger\right],$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^2\right], \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right],$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^3\right], \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^3\right],$$

$$I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right], \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right],$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right], \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right].$$

[Feldmann, Jung, Mannel; Jenkins, Manohar] quark case

Bi-fundamental Flavour Fields

$$\mathrm{Tr}[\mathbf{y}_U \mathbf{y}_U] = \sum y_\alpha^2$$

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger\right], \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D \mathcal{Y}_D^\dagger\right],$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^2\right], \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right],$$

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only masses

$$I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right], \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right],$$

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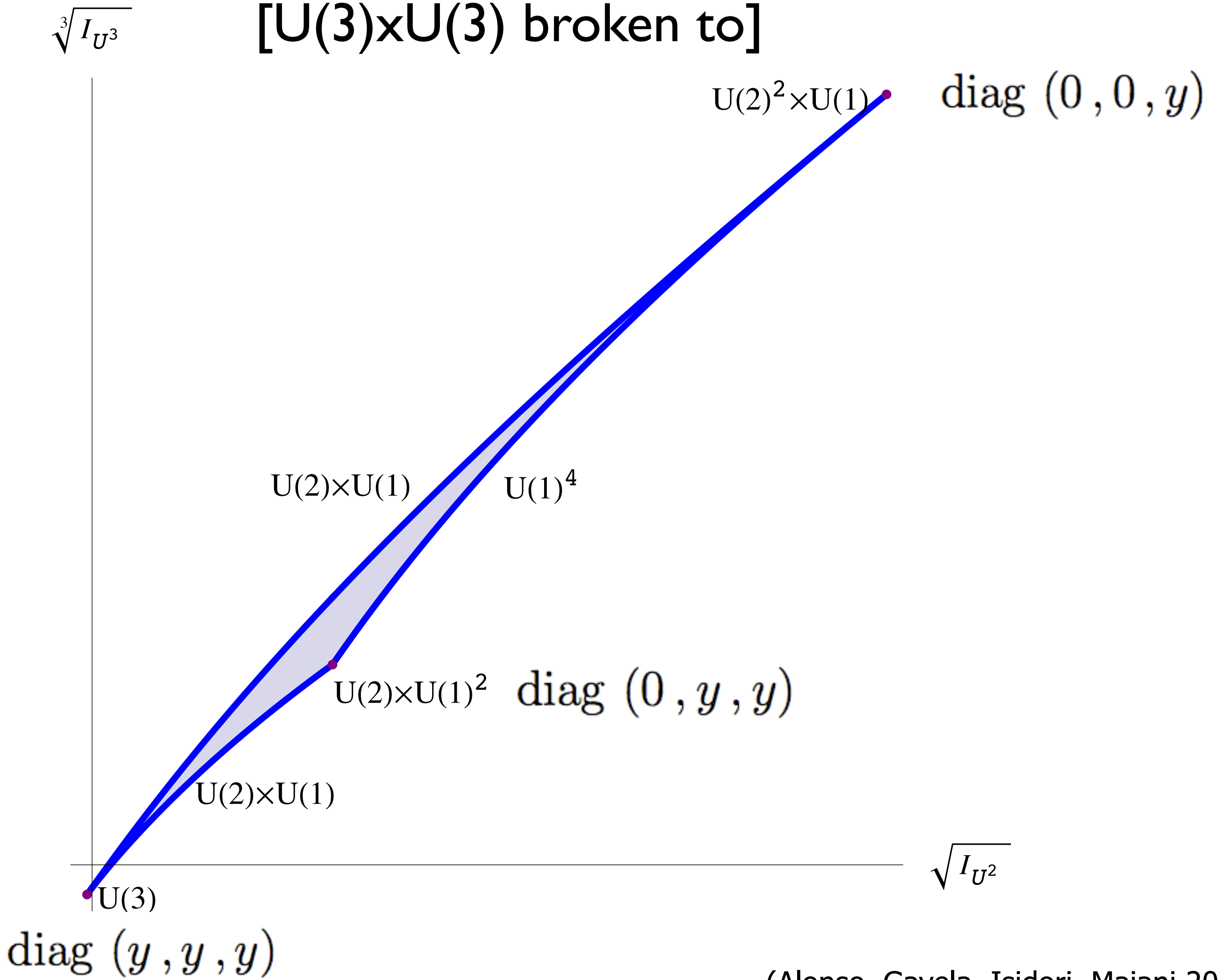
masses and mixing

[Feldmann, Jung, Mannel; Jenkins, Manohar
Alonso. Gavela. Isidori. Maiani 20131

Jacobian Analysis: Masses

[U(3)xU(3) broken to]



(Alonso, Gavela, Isidori, Maiani 2013)

Well before the electroweak SM: masses

Jacobian Analysis: [40 years ago...]

Breaking of $SU(3) \times SU(3)$ [Cabibbo, Maiani]

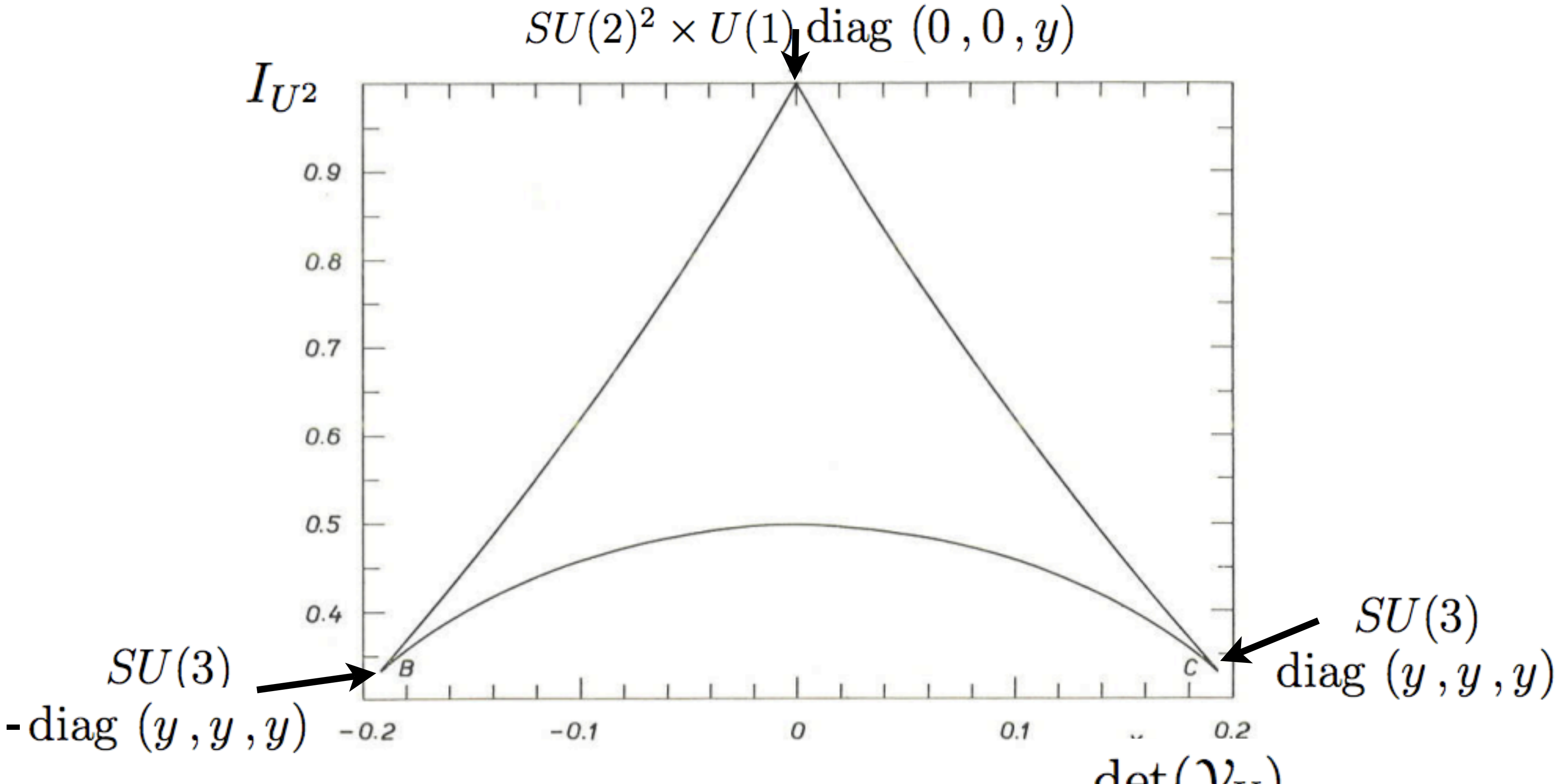


Fig. 4.

ancestors of dynamical Yukawas decades ago (only to explain the mass spectrum) in

Cabibbo

Michel,+Radicati, Cabibbo+Maiani ...

C. D. Froggat, H. B. Nielsen

Jacobian Analysis: Mixing

$$\det(J_{UD}) = (y_u^2 - y_t^2)(y_t^2 - y_c^2)(y_c^2 - y_u^2)$$
$$(y_d^2 - y_b^2)(y_b^2 - y_s^2)(y_s^2 - y_d^2)$$
$$\times |V_{ud}||V_{us}||V_{cd}||V_{cs}|$$

the rank is reduced the most for:

V_{CKM}= PERMUTATION

no mixing: reordering of states

(Alonso, Gavela, Isidori, Maiani 2013)

Quark Natural Flavour Pattern

Summarizing, a possible and natural breaking pattern arises:

G$_{\text{flavour (quarks)}}$: $U(3)^3 \to U(2)^3 \times U(1)$

giving a hierarchical mass spectrum **without mixing**

$$< \mathcal{Y}_{\mathrm{D}} > = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix} , \quad < \mathcal{Y}_{\mathrm{U}} > = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix} ,$$

a good approximation to the observed Yukawas to order $(\lambda_C)^2$

And what happens for leptons ?

Any difference with Majorana neutrinos?

Leptons

Global flavour symmetry of the SM + seesaw

* In the SM, for quarks the maximal global symmetry in the limit of massless quarks was:

$$\mathscr{L}^{\text{quarks}}_{\text{SM}} = i \sum_{\psi=Q_L}^{D_R} \overline{\psi} \not{D} \psi \qquad G_{\text{flavour}} = U(n)_{Q_L} \times U(n)_{U_R} \times U(n)_{D_R}$$

* In SM +type I seesaw, for leptons

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\not{\partial}N_R - \left[\overline{N_R}Y_N\tilde{\phi}^{\dagger}\ell_L + \frac{1}{2}\overline{N_R}MN_R{}^c + h.c.\right]$$

the maximal leptonic global symmetry in the limit of massless light leptons is $U(n)_L \times U(n)_{E_R} \times O(n)_{N_R}$

-> degenerate heavy neutrinos

Ilustration: 2 families (Casas-Ibarra parametrization)

for 2 generations, the mixing terms in $\mathbf{V}(\mathcal{Y}_E, \mathcal{Y}_\nu)$ is :

Leptons

$$\mathrm{Tr}(\mathcal{Y}_E \mathcal{Y}_E^{+} \mathcal{Y}_\nu \mathcal{Y}_\nu^{+}) \propto$$

$$(m_\mu^2 - m_e^2)\left[\cos 2\omega(m_{\nu_2} - m_{\nu_1})\cos 2\theta + 2\sin 2\omega\sqrt{m_{\nu_2} m_{\nu_1}}\, sin2\alpha \sin 2\theta\right]$$

Casas-Ibarra variable in R

where $U_{PMNS} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$

Quarks

$$\mathrm{Tr}(\mathcal{Y}_u \mathcal{Y}_u^{+} \mathcal{Y}_d \mathcal{Y}_d^{+}) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

* For instance for two generations: $O(2)_{N_R}$

e.g. two families

$$m_\nu \sim Y_\nu \frac{v^2}{M} Y_\nu^T = y_1\, y_2\, \frac{v^2}{M} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$U_{PMNS} = \boxed{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} e^{i\pi/4} & 0 \\ 0 & e^{-i\pi/4} \end{pmatrix}}$$

Degenerate neutrino masses

Generically, O(2) allows :

- **one mixing angle maximal**
- **one relative Majorana phase of $\pi/2$**
- **two degenerate light neutrinos**

Now for three generations and considering all possible independent invariants

easier using the bi-unitary parametrization as we did for quarks

Bi-fundamental Flavour Fields

Physical parameters
=Independent Invariants

Very direct results using the bi-unitary parametrization:

$$\mathbf{Y}_\nu = \frac{<\mathcal{Y}_\nu>}{\Lambda_f} = \mathcal{U}_L \mathbf{y}_\nu \mathcal{U}_R\,, \qquad \mathbf{Y}_E = \frac{<\mathcal{Y}_E>}{\Lambda_f} = \mathbf{y}_E$$

$$\mathcal{U}_L \mathcal{U}_L^\dagger = 1\,, \quad \mathcal{U}_R \mathcal{U}_R^\dagger = 1\,,$$

→ * $m_{e,\mu,\tau} = v\, \mathbf{y}_E$

*But the relation of $\mathcal{Y}_\nu$ with light neutrino masses is through:

$$m_\nu = \mathbf{Y} \frac{v^2}{M} \mathbf{Y}^T$$

Bi-fundamental Flavour Fields

Physical parameters
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$$\mathbf{Y_\nu} = \frac{<\mathcal{Y}_\nu>}{\Lambda_f} = \mathcal{U}_L \mathbf{y}_\nu \mathcal{U}_R \,, \qquad \mathbf{Y_E} = \frac{<\mathcal{Y}_E>}{\Lambda_f} = \mathbf{y_E}$$

$$\mathcal{U}_L \mathcal{U}_L^\dagger = 1 \,, \quad \mathcal{U}_R \mathcal{U}_R^\dagger = 1 \,,$$

→ * $m_{e,\mu,\tau} = v\, \mathbf{y_E}$

*But the relation of $\mathcal{Y}_\nu$ with light neutrino masses is through:

→ $$U_{PMNS}\, \mathbf{m}_\nu\, U_{PMNS}^T = \frac{v^2}{2M} \mathcal{U}_L\, \mathbf{y}_\nu \mathcal{U}_R \mathcal{U}_R^T\, \mathbf{y}_\nu \mathcal{U}_L^T \,,$$

Bi-fundamental Flavour Fields

Physical parameters
=Independent Invariants

Very direct results using the bi-unitary parametrization:

$$\mathbf{Y}_\nu = \frac{< \mathcal{Y}_\nu >}{\Lambda_f} = \mathcal{U}_L \mathbf{y}_\nu \mathcal{U}_R \,, \qquad \mathbf{Y}_E = \frac{< \mathcal{Y}_E >}{\Lambda_f} = \mathbf{y}_E$$

$$\mathcal{U}_L \mathcal{U}_L^\dagger = 1 \,, \quad \mathcal{U}_R \mathcal{U}_R^\dagger = 1 \,,$$

* $m_{e,\mu,\tau} = v \, \mathbf{y}_E$

*But the relation of $\mathcal{Y}_\nu$ with light neutrino masses is through:

$\mathcal{U}_R$ is relevant for leptons

$$U_{PMNS} \, \mathbf{m}_\nu \, U_{PMNS}^T = \frac{v^2}{2M} \mathcal{U}_L \, \mathbf{y}_\nu \, \mathcal{U}_R \mathcal{U}_R^T \, \mathbf{y}_\nu \, \mathcal{U}_L^T \,,$$

Number of Physical parameters = number of Independent Invariants

15 invariants for $G_{\text{flavour (leptons)}} = U(3)_L \times U(3)_{E_R} \times O(3)_{N_R}$

Leptons

$$I_E = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger\right], \qquad I_\nu = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right],$$

$$I_{E^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right], \qquad I_{\nu^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right],$$

$$I_{E^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^3\right], \qquad I_{\nu^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^3\right],$$

$$I_L = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

$$I_{L^2} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

$$I_{L^3} = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger \left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right],$$

$$I_{L^4} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

U_L and eigenvalues

$$I_R = \mathrm{Tr}\left[\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right],$$

$$I_{R^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)^2 \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right],$$

$$I_{R^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right)^2\right],$$

U_R and eigenvalues

$$I_{LR} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right], \quad I_{RL} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_E^* \mathcal{Y}_E^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

New Invariants wrt Quarks

(Alonso, Gavela, Isidori, Maiani 2013)

Number of Physical parameters = number of Independent Invariants

15 invariants for $G_{\text{flavour (leptons)}} = U(3)_L \times U(3)_{E_R} \times O(3)_{N_R}$

Leptons

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$$I_{L^3} = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger \left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right],$$

$$I_{L^4} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

U_L and eigenvalues

$$I_R = \mathrm{Tr}\left[\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)^{\mathrm{T}}\right]$$

$$I_{R^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)^2 \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right],$$

$$I_{R^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right)^2\right],$$

U_R and eigenvalues

New Invariants wrt Quarks

$$I_{LR} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right], \qquad I_{RL} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_E^* \mathcal{Y}_E^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

(Alonso, Gavela, Isidori, Maiani 2013)

Number of Physical parameters = number of Independent Invariants

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$$I_{L^2} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

$$I_{L^3} = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger \left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right],$$

$$I_{L^4} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right],$$

U_L and eigenvalues

$$\mathrm{Tr}(\mathbf{y}_\nu^2 \mathcal{U}_R \mathcal{U}_R^T \mathbf{y}_\nu^2 \mathcal{U}_R^* \mathcal{U}_R^\dagger)$$

$$I_{R^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)^2 \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right],$$

$$I_{R^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right)^2\right],$$

U_R and eigenvalues

New Invariants wrt Quarks

$$I_{LR} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right], \qquad I_{RL} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_E^* \mathcal{Y}_E^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right],$$

(Alonso, Gavela, Isidori, Maiani 2013)

Jacobian

$$J = \begin{pmatrix} \partial_{\mathbf{y}_E} I_{E^n} & 0 & 0 & \partial_{\mathbf{y}_E} I_{L^n} & \partial_{\mathbf{y}_E} I_{LR} \\ 0 & \partial_{\mathbf{y}_\nu} I_{\nu^n} & \partial_{\mathbf{y}_\nu} I_{R^n} & \partial_{\mathbf{y}_\nu} I_{L^n} & \partial_{\mathbf{y}_\nu} I_{LR} \\ 0 & 0 & \partial_{\mathcal{U}_R} I_{R^n} & 0 & \partial_{\mathcal{U}_R} I_{LR} \\ 0 & 0 & 0 & \partial_{\mathcal{U}_L} I_{L^n} & \partial_{\mathcal{U}_L} I_{LR} \\ 0 & 0 & 0 & 0 & \partial_{\mathcal{U}_L \mathcal{U}_R} I_{LR} \end{pmatrix},$$

$$\mathrm{Diag}(J) \equiv (J_E\,,\, J_\nu\,,\, J_{\mathcal{U}_R}\,,\, J_{\mathcal{U}_L}\,,\, J_{LR})$$

Jacobian Analysis: Mixing

$$\det\left(J_{\mathcal{U}_L}\right) = \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)\left(y_{\nu_3}^2 - y_{\nu_1}^2\right)$$
$$\left(y_e^2 - y_\mu^2\right)\left(y_\mu^2 - y_\tau^2\right)\left(y_\tau^2 - y_e^2\right)|\mathcal{U}_L^{e1}||\mathcal{U}_L^{e2}||\mathcal{U}_L^{\mu 1}||\mathcal{U}_L^{\mu 2}|\,.$$

same as for V_{CKM}

O(3) vs U(3)

$$\det J_{\mathcal{U}_R} = \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)^3\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)^3\left(y_{\nu_3}^2 - y_{\nu_1}^2\right)^3$$
$$\times\,|\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{11}||\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{22}||\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{12}|$$

the rank is reduced the most for $\mathcal{U}_R\mathcal{U}_R^T$ *being a permutation*

Jacobian Analysis: Mixing

...which is now **not** trivial mixing...

$$\frac{v^2}{M}\begin{pmatrix} y_{\nu_1}^2 & 0 & 0 \\ 0 & 0 & y_{\nu_2}y_{\nu_3} \\ 0 & y_{\nu_2}y_{\nu_3} & 0 \end{pmatrix} = U_{PMNS}\begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_2} \end{pmatrix} U_{PMNS}^T\,,$$

...in fact it allows maximal mixing:

Jacobian Analysis: Mixing

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$$\frac{v^2}{M}\begin{pmatrix} y_{\nu_1}^2 & 0 & 0 \\ 0 & 0 & y_{\nu_2}y_{\nu_3} \\ 0 & y_{\nu_2}y_{\nu_3} & 0 \end{pmatrix} = U_{PMNS}\begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_2} \end{pmatrix} U_{PMNS}^T \,,$$

...in fact it leads to one maximal mixing angle:

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}, \quad m_{\nu 2} = m_{\nu 3} = \frac{v^2}{M} y_{\nu_2} y_{\nu_3} \,, \quad m_{\nu_1} = \frac{v^2}{M} y_{\nu_1}^2 \,.$$

and maximal Majorana phase

Jacobian Analysis: Mixing

...which is now **not** trivial mixing...

$$\frac{v^2}{M}\begin{pmatrix} y_{\nu_1}^2 & 0 & 0 \\ 0 & 0 & y_{\nu_2}y_{\nu_3} \\ 0 & y_{\nu_2}y_{\nu_3} & 0 \end{pmatrix} = U_{PMNS}\begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_2} \end{pmatrix} U_{PMNS}^T\,,$$

...in fact it leads to one maximal mixing angle:

$\theta_{23} = 45°$;

Majorana Phase Pattern (1,1,i)

& at this level mass degeneracy: $m_{\nu 2} = m_{\nu 3}$

related to the O(2) substructure [Alonso, Gavela, D. Hernández, L. Merlo;
[Alonso, Gavela, D. Hernández, L. Merlo, S. Rigolin]

if the three neutrinos are quasidegenerate,

$$U_{PMNS}\begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U_{PMNS}^T = \frac{y_\nu v^2}{M}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

This very simple structure is signaled by the extrema of the potential and

has eigenvalues (1,1,-1) → 3 degenerate light neutrinos + a maximal Majorana phase

and is diagonalized by a maximal θ = 45°

What is the symmetry in this boundary?

a very intriguing

$$U(1)_{diag}$$

Generalization to any seesaw model

the effective Weinberg Operator

$$\bar{\ell}_L \tilde{H} \frac{C^{d=5}}{M} \tilde{H}^T \ell_L^c$$

shall have a flavour structure that breaks $U(3)_L$ to $O(3)$

$$\frac{v^2 \; C^{d=5}}{M} = m_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

then the results apply to any seesaw model

First conclusion:

* at the same order in which the minimum of the potential

does NOT allow quark mixing,

it allows:

- hierarchical charged leptons
- quasi-degenerate neutrino masses
- one angle of ~45 degrees
- one maximal Majorana phase and the other one trivial

Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$U_{PMNS}\begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U^T_{PMNS} = \frac{y_\nu v^2}{M}\begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

$$\theta_{23} \simeq \pi/4 \ , \ \theta_{12} \text{ large} \ , \ \theta_{13} \simeq \epsilon$$

Fixed Majorana phases: $(1, 1, i)$

degenerate spectrum

Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$U_{PMNS}\begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U^T_{PMNS} = \frac{y_\nu v^2}{M}\begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

$$\theta_{23} \simeq \pi/4 \ , \ \theta_{12} \text{ large} \ , \ \theta_{13} \simeq \epsilon$$

only this vanishes with the perturbations

Fixed Majorana phases: $(1, 1, i)$

this angle does not vanish with vanishing perturbations

degenerate spectrum

Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$U_{PMNS} \begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U_{PMNS}^T = \frac{y_\nu v^2}{M} \begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

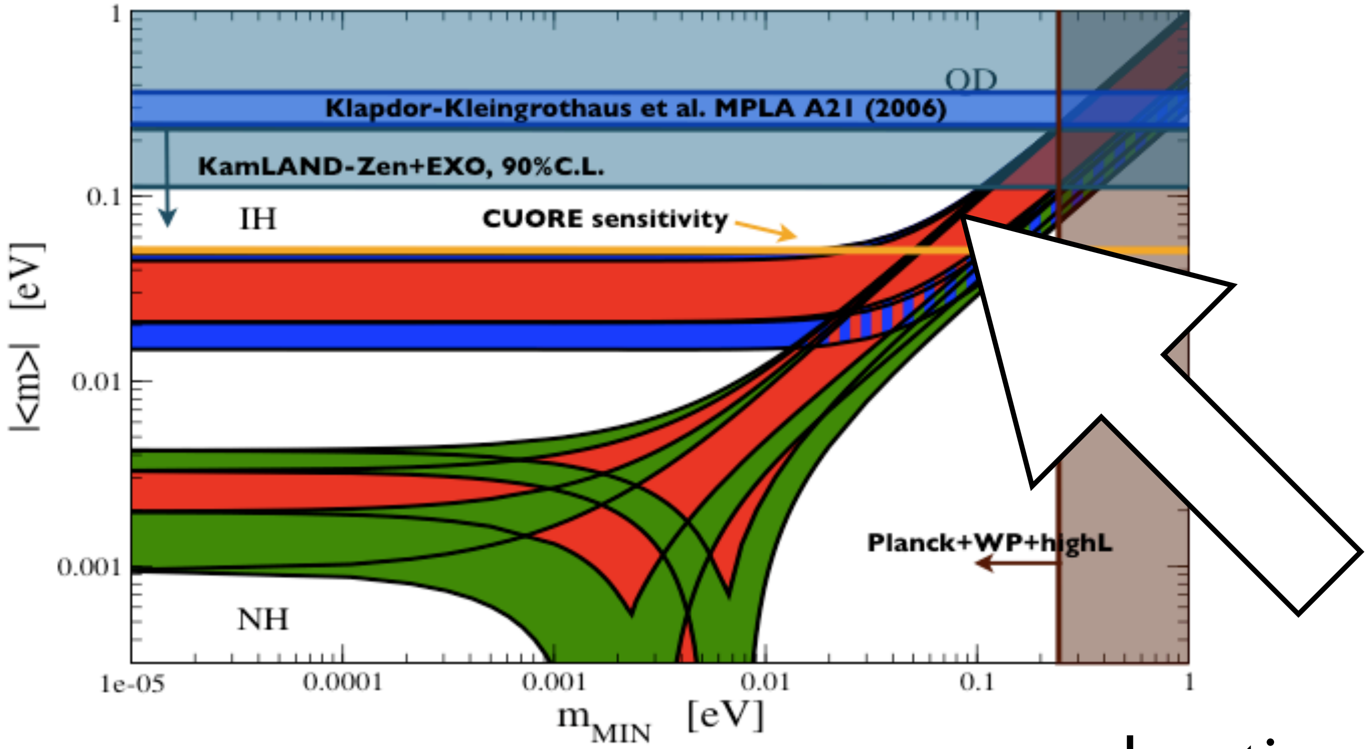
$$\theta_{23} \simeq \pi/4 \ , \ \theta_{12} \text{ large} \ , \ \theta_{13} \simeq \epsilon$$

Fixed Majorana phases: $(1, 1, i)$

degenerate spectrum

accommodation of angles requires degenerate spectrum

at reach in future neutrinoless double β exps.!



rough estimate

m~0.1eV

Slide from Laura Baudis talk presenting the new Gerda data at Invisibles13 workshop this summer

The physics

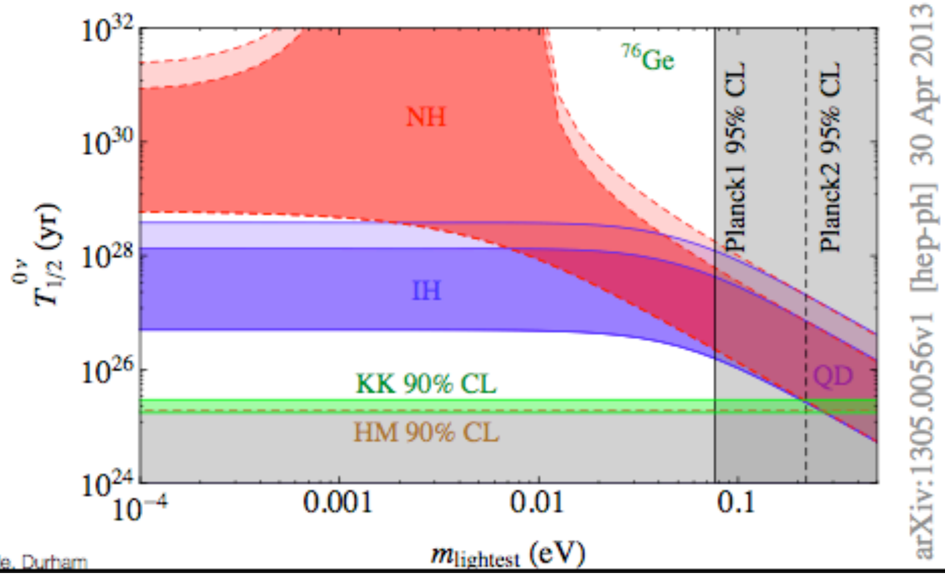
- Detect the neutrinoless double beta decay in ^{76}Ge:
  - ➡ lepton number violation
  - ➡ information on the nature of neutrinos and on the effective Majorana neutrino mass

$$\Gamma^{0\nu} = \frac{1}{T^{0\nu}_{1/2}} = G^{0\nu}(Q,Z)|M^{0\nu}|^2 \frac{|m_{\beta\beta}|^2}{m_e^2}$$

Alonso, Gavela, Isidori, Maiani
(4×10^{25} - 8×10^{26} yr)
arXiv:1306.5927 [hep-ph]

~ 2 10^{26} yr

current sensitivities



latest from Planck....

$$\sum m_\nu = 0.22 \pm 0.09 \text{ eV}$$

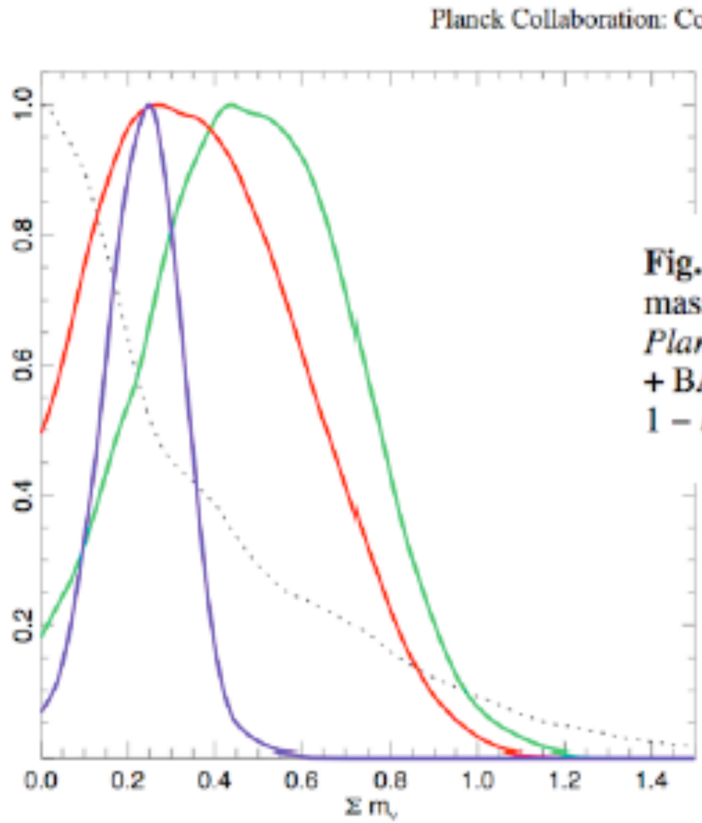


Fig. 12. Cosmological constraints when including neutrino masses $\sum m_\nu$ from: *Planck* CMB data alone (black dotted line); *Planck* CMB + SZ with $1-b$ in [0.7, 1] (red); *Planck* CMB + SZ + BAO with $1-b$ in [0.7, 1] (blue); and *Planck* CMB + SZ with $1-b=0.8$ (green).

Where do the differences in Mixing originated?

in the symmetries of the

Quark and Lepton sectors

$\mathcal{G}_{\mathcal{F}}^{q} \sim U(3)^3$ $\mathcal{G}_{\mathcal{F}}^{l} \sim U(3)^2 \times O(3)$

for the type I seesaw employed here;

in general $U(n_g)$ vs $O(n_g)$

Where do the differences in Mixing originate?

From the

MAJORANA vs DIRAC nature of fermions

Conclusions

- Spontaneous Flavour Symmetry Breaking is a predictive dynamical scenario
- Simple solutions arise that resemble nature in first approximation
- The differences in the mixing pattern of Quarks and Leptons arise from their Dirac vs Majorana nature (U vs. O symmetries). O(2) singled out -> O(3).
- A correlation between large angles and degenerate spectrum emerges. Explicitly, for neutrinos we find: fixed Majorana phases (1,1,i) , θ_{23} =45°, θ_{12} large, θ_{13} small and deg. ν's
- This scenario will be tested in the near future by $0\nu2\beta$ experiments (~.1eV).... or cosmology!!!

The prediction:

large mixing angles $\Leftrightarrow$ Majorana degenerate neutrinos

leads to neutrinoless double beta decay and CMB signals that could be observed in a not too distant future !!

Back-up slides

We set the perturbations by hand.
Can we predict them also dynamically?

Fundamental Fields

May provide dynamically the perturbations

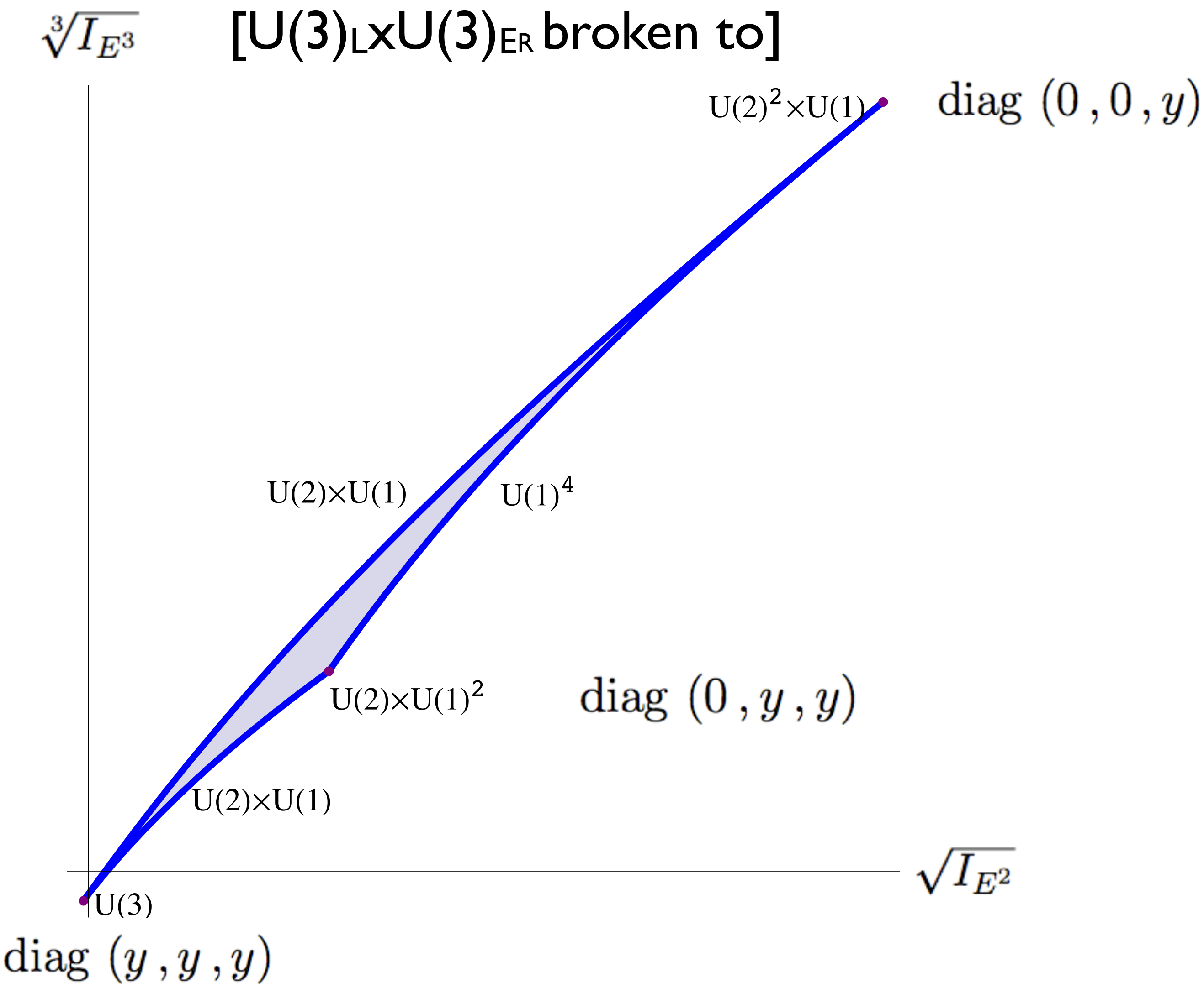
In the case of quarks they can give
the right corrections:

$$\frac{\mathcal{Y}_U}{\Lambda_f} + \frac{\chi_U^L \chi_U^{R\dagger}}{\Lambda_f^2} \sim \begin{pmatrix} 0 & \sin\theta_c\, y_c & 0 \\ 0 & \cos\theta_c\, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

[Alonso, Gavela, Merlo, Rigolin]

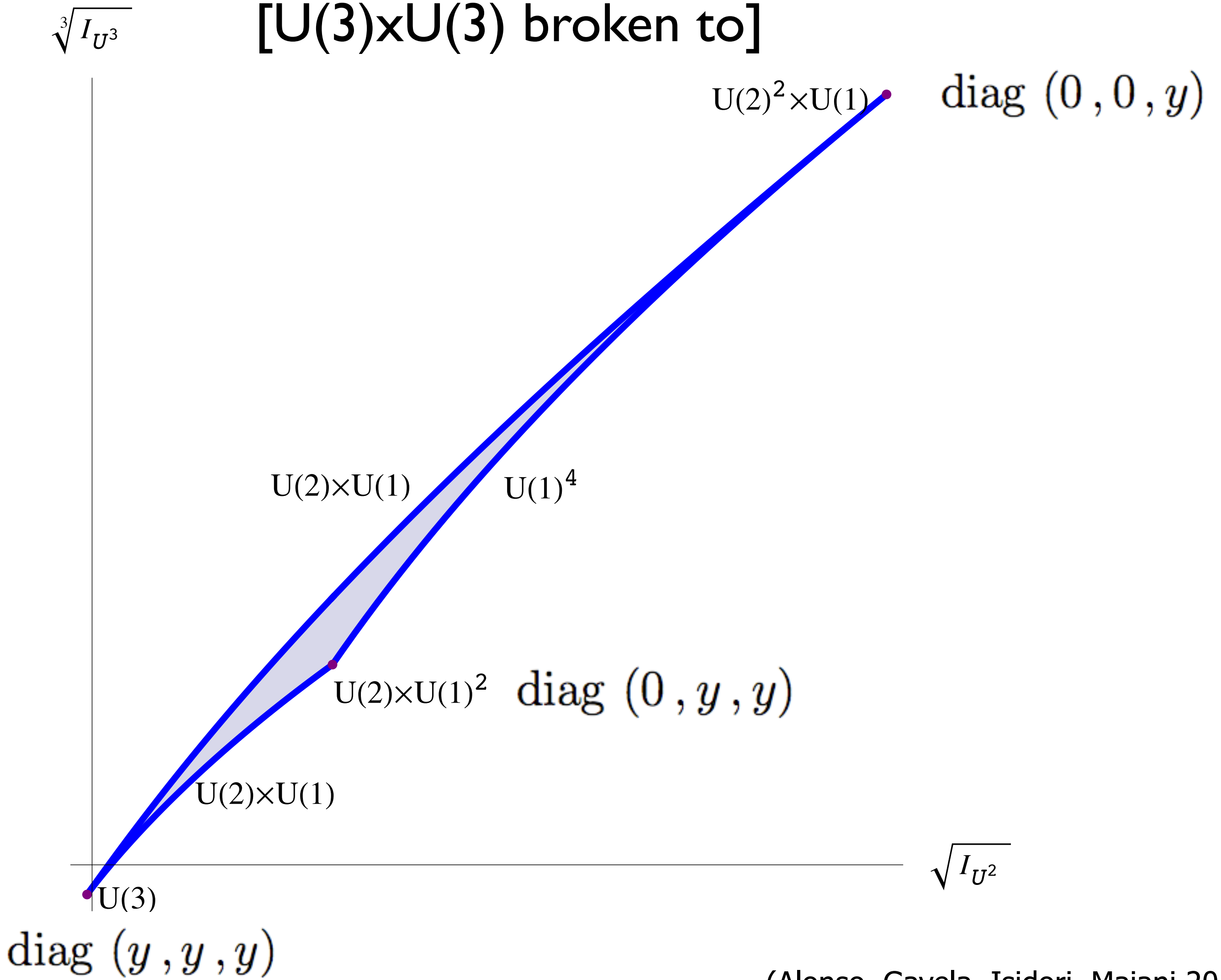
under study in the lepton sector

Jacobian Analysis: Eigenvalues



Jacobian Analysis: Masses

[U(3)xU(3) broken to]



(Alonso, Gavela, Isidori, Maiani 2013)

Jacobian Analysis: [40 years ago...]

Breaking of $SU(3) \times SU(3)$ [Cabibbo, Maiani]

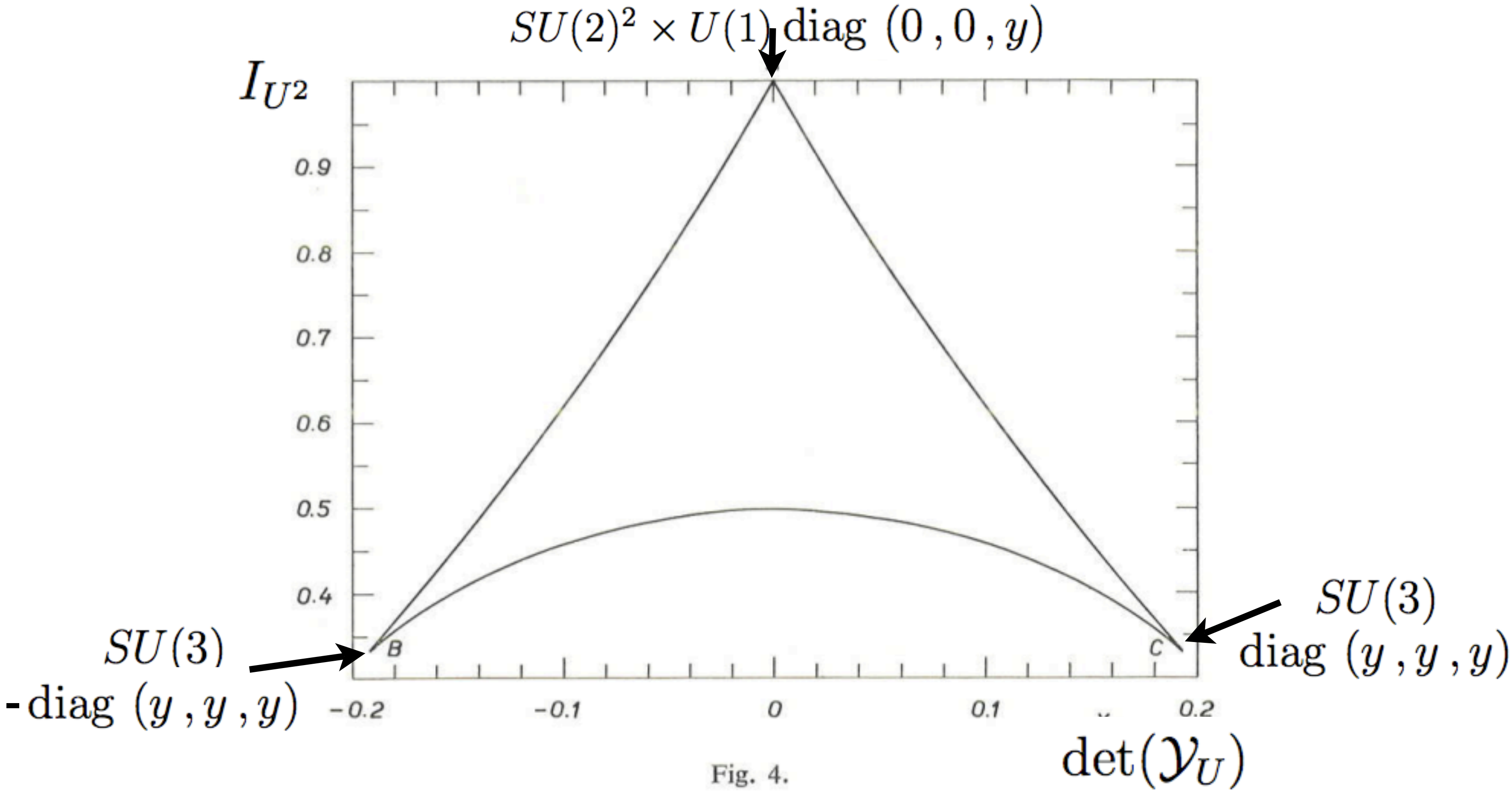


Fig. 4.

Lepton Natural Flavour Pattern

Summarizing, a possible and natural breaking pattern:

$$\not{G}^{l}_{\mathcal{F}} \quad : \quad U(3)^2 \times O(3) \to U(2) \times U(1)$$

brings along hierarchical charged leptons

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} y_{\nu_1} & 0 & 0 \\ 0 & y_{\nu_2}/\sqrt{2} & -iy_{\nu_2}/\sqrt{2} \\ 0 & y_{\nu_3}/\sqrt{2} & iy_{\nu_3}/\sqrt{2} \end{pmatrix},$$

and (at least) two degenerate neutrinos and maximal angle and Majorana phase

$\theta_{23} = 45°$;

Majorana Phase Pattern (1,1,i)

& Mass degeneracy: $m_{\nu 2} = m_{\nu 3}$

Renormalizable Potential

Invariants at the Renormalizable Level

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\right] , \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D\mathcal{Y}_D^\dagger\right] ,$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)^2\right] , \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right] ,$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\right)^3\right] , \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^3\right] ,$$

$$I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right] , \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right] ,$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U\mathcal{Y}_U^\dagger\left(\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right] , \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U\mathcal{Y}_U^\dagger\mathcal{Y}_D\mathcal{Y}_D^\dagger\right)^2\right] .$$

Renormalizable Potential

with the definition

$$X \equiv (I_U, I_D)^T = \left(\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right), \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)\right)^T ,$$

the potential

$$V^{(4)} = -\mu^2 \cdot X + X^T \cdot \lambda \cdot X + g\,\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) + h_U \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_U \mathcal{Y}_U^\dagger\right) + h_D \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) ,$$

mass spectrum

which contains 8 parameters

Renormalizable Potential

with the definition

$$X \equiv (I_U, I_D)^T = \left(\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right), \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)\right)^T ,$$

the potential

mixing

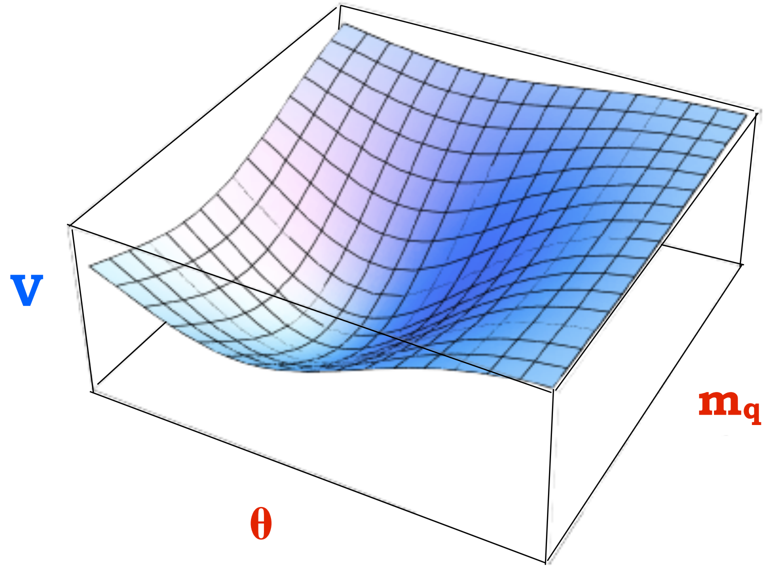
$$V^{(4)} = -\mu^2 \cdot X + X^T \cdot \lambda \cdot X + g\,\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) + h_U \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_U \mathcal{Y}_U^\dagger\right) + h_D \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) ,$$

which contains 8 parameters

e.g. for the case of two families:

$$\mathrm{Tr}(\mathcal{Y}_u \mathcal{Y}_u^+ \mathcal{Y}_d \mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

at the minimum: $\boxed{(m_c^2 - m_u^2)(m_s^2 - m_d^2)\sin 2\theta = 0}$



-> NO MIXING

same conclusion for 3 families

Berezhiani-Rossi; Anselm, Berezhiani; Alonso, Gavela, Merlo, Rigolin

Renormalizable Potential, mixing **three families**

Von Neumann Trace Inequality

$$y_u^2 y_b^2 + y_s^2 y_c^2 + y_d^2 y_t^2 \leq \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) \leq y_u^2 y_d^2 + y_s^2 y_c^2 + y_b^2 y_t^2 \, .$$

So the Potential selects:

coefficient in the potential

"normal" Hierarchy

$$g < 0\,, \quad V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} ;$$

"inverted" Hierarchy

$$g > 0\,, \quad V_{CKM} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} .$$

No mixing, independently of the mass spectrum

Quark Natural Flavour Pattern

Summarizing, a possible and natural breaking pattern arises:

$$\not{G}^q_{\mathcal{F}} \quad : \quad U(3)^3 \to U(2)^3 \times U(1)$$

giving a hierarchical mass spectrum **without mixing**

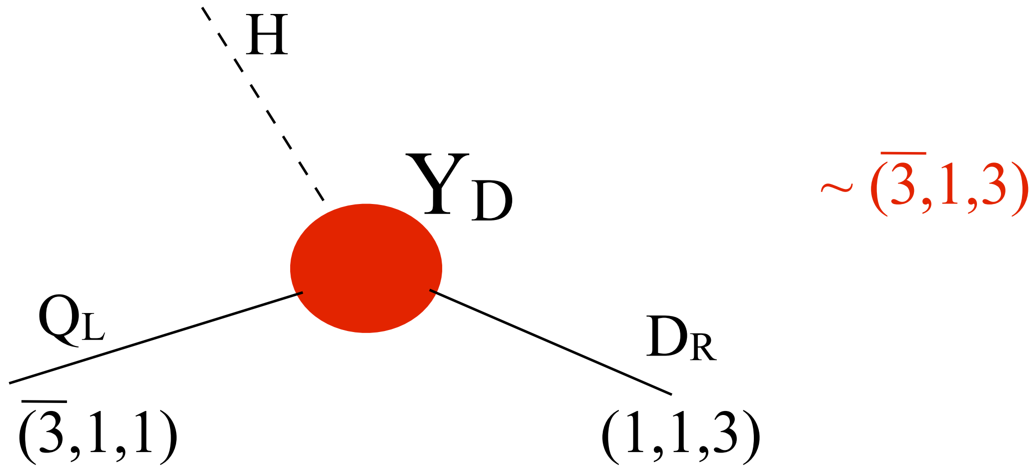
$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix} , \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix} ,$$

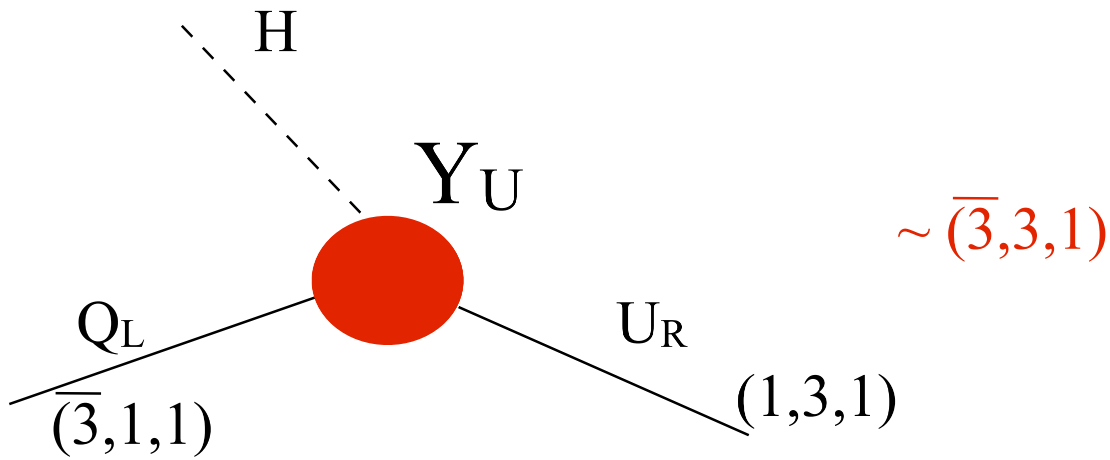
a good approximation to the observed Yukawas to order $(\lambda_C)^2$

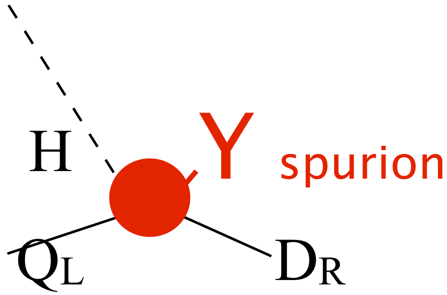
The non-abelian part of the flavour symmetry of the SM:

$$G_f = SU(3)_{Q_L} \text{ x } SU(3)_{U_R} \text{ x } SU(3)_{D_R}$$

broken by Yukawas:







Some good ideas:

Minimal Flavour Violation:

- Use the flavour symmetry of the SM in the limit of massless fermions (**Chivukula+ Georgi**)

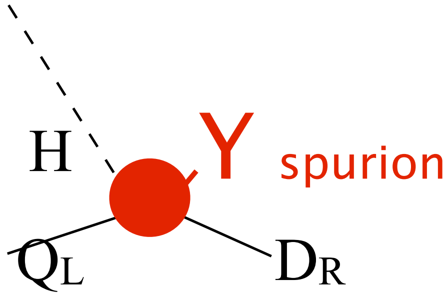
quarks: $G_{flavour} = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$

- Assume that Yukawas are the only source of flavour in the SM and beyond

$$\frac{Y_{\alpha\beta}^{+} Y_{\delta\gamma}}{\Lambda_{flavour}^{2}} \overline{Q}_{\alpha} \gamma_{\mu} Q_{\beta} \overline{Q}_{\gamma} \gamma^{\mu} Q_{\delta}$$

... agrees with flavour data being aligned with SM
... allows to bring down $\Lambda_{flavour}$ --> TeV

D'Ambrosio+Giudice+Isidori+Strumia;
Cirigliano+Isidori+Grinstein+Wise



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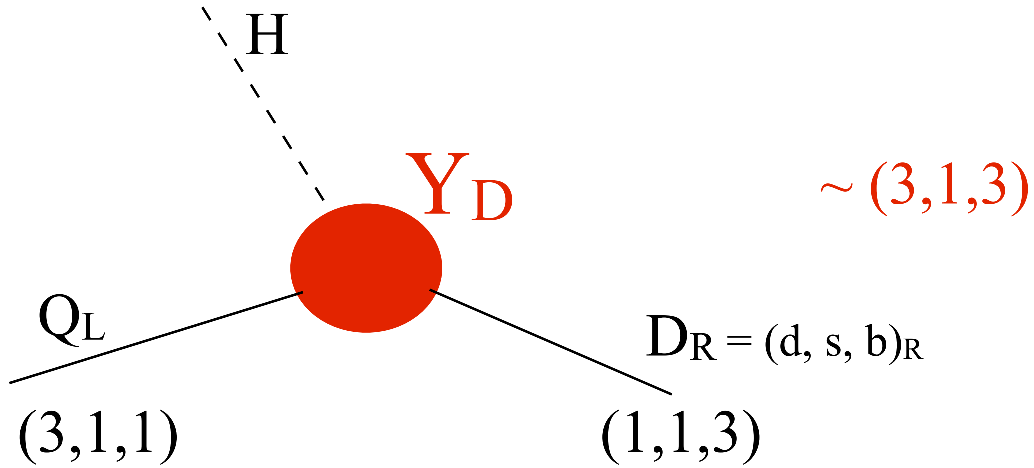
(Chivukula+Georgi 87; Hall+Randall; D'Ambrosio+Giudice+Isidori+Strumia; Cirigliano+Isidori+Grisnstein +Wise; Davidson+Pallorini; Kagan+G. Perez + Volanski+Zupan,...)

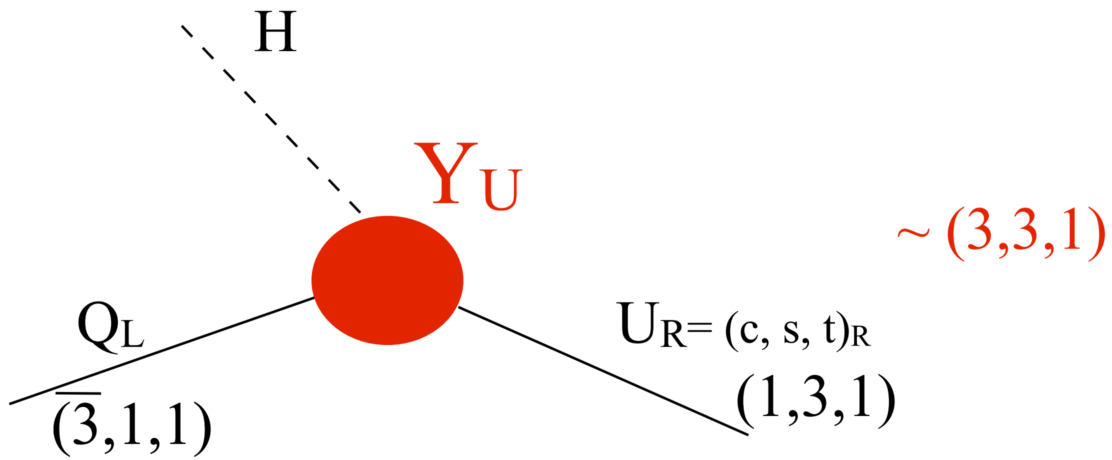
Lalak, Pokorski, Ross; Fitzpatrick, Perez, Randall; Grinstein, Redi, Villadoro

Use the flavour symmetry of the SM with masless fermions:

$$G_f = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$$

which is broken by Yukawas:

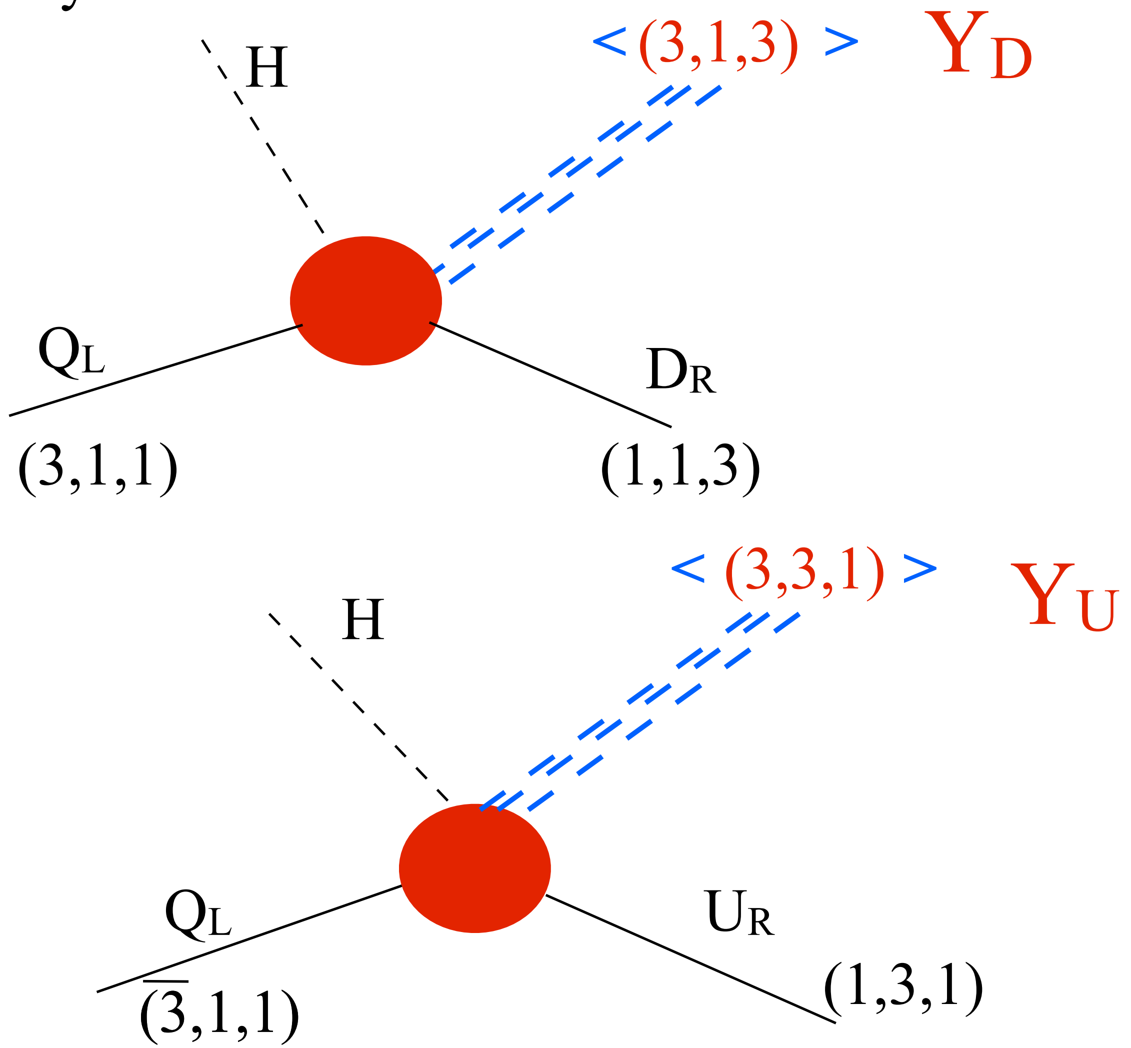




Use the flavour symmetry of the SM with masless fermions:

$$G_f = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}$$

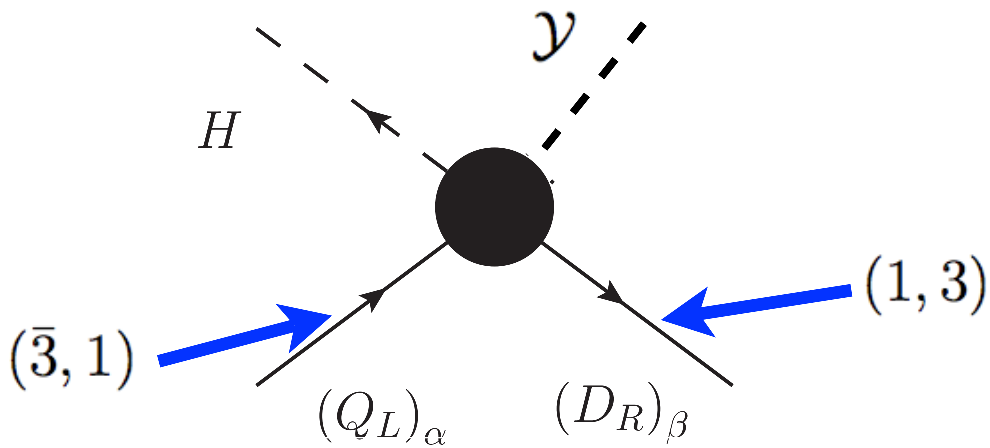
replace Yukawas by fields:



Flavour Fields

The Yukawa Operator has to be explicitly flavour invariant
at high energies

$$\overline{Q}_L \frac{\mathcal{Y}}{\Lambda_f} U_R \tilde{H}$$



$n = 1$ (d=5)

A single and therefore
"bi-fundamental" field

$$\mathcal{Y} \sim (3, \bar{3})$$

Bi-fundamental Flavour Fields

Physical parameters
=Independent Invariants

$$\#\,\text{d.o.f. in } \mathcal{Y}_{U,D} \text{ - } \left(\dim(\mathcal{G}^q_{\mathcal{F}}) - 1_{U(1)_B}\right) = 10$$

$$2 \times 18 \qquad\qquad 3 \times 9 - 1$$

These are (proportional to):

3 masses in de up sector,
3 masses in de down sector,
4 mixing parameters in V_{CKM}

$$\sum_j \frac{\partial I_j}{\partial y_i}\frac{\partial V}{\partial I_j} \equiv J_{ij}\frac{\partial V}{\partial I_j} = 0\,,$$

Jacobian Analysis

$$J = \begin{pmatrix} \partial_{\mathbf{y}_U} I_{U^n} & 0 & \partial_{\mathbf{y}_U} I_{UD} \\ 0 & \partial_{\mathbf{y}_D} I_{D^n} & \partial_{\mathbf{y}_D} I_{UD} \\ 0 & 0 & \partial_{\theta_c} I_{UD} \end{pmatrix} \equiv \begin{pmatrix} J_U & 0 & \partial_{\mathbf{y}_U} I_{UD} \\ 0 & J_D & \partial_{\mathbf{y}_D} I_{UD} \\ 0 & 0 & J_{UD} \end{pmatrix}\,.$$

for the sub-Jacobian which involves only masses
we can identify the shape of the *I-manifold*

(Alonso, Gavela, Isidori, Maiani 2013)

Renormalizable Potential

Invariants at the Renormalizable Level

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger\right] , \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D \mathcal{Y}_D^\dagger\right] ,$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^2\right] , \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] ,$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^3\right] , \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^3\right] ,$$

$$I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right] , \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] ,$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] , \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] .$$

Renormalizable Potential

with the definition

$$X \equiv (I_U, I_D)^T = \left(\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right), \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)\right)^T ,$$

the potential

$$V^{(4)} = -\mu^2 \cdot X + X^T \cdot \lambda \cdot X + g\,\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) + h_U \mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_U \mathcal{Y}_U^\dagger\right) + h_D \mathrm{Tr}\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right) ,$$

mass spectrum

which contains 8 parameters

Renormalizable Potential

with the definition

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the potential

mixing

$$V^{(4)} = -\mu^2 \cdot X + X^T \cdot \lambda \cdot X + g\,\mathrm{Tr}\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right)$$
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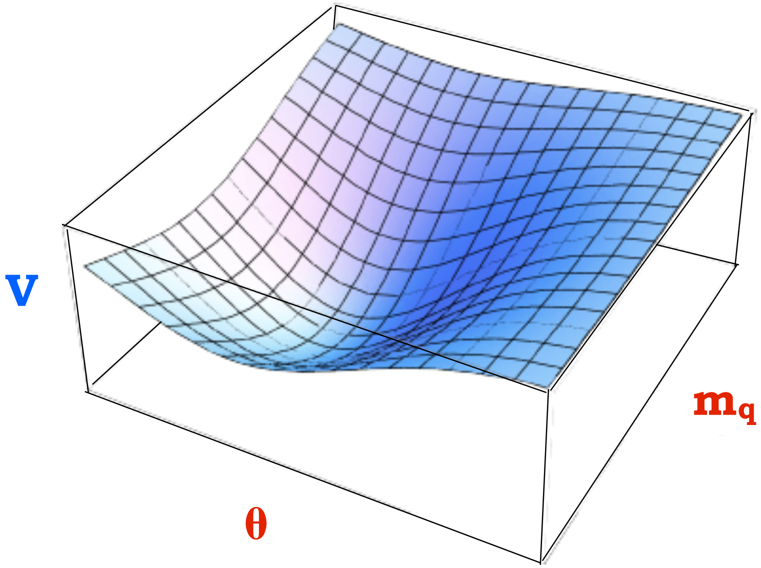
which contains 8 parameters

e.g. for the case of two families:

$$\mathrm{Tr}(\mathcal{Y}_u \mathcal{Y}_u^+ \mathcal{Y}_d \mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

at the minimum: $$\boxed{(m_c^2 - m_u^2)(m_s^2 - m_d^2)\sin 2\theta = 0}$$



-> NO MIXING

same conclusion for 3 families

Berezhiani-Rossi; Anselm, Berezhiani; Alonso, Gavela, Merlo, Rigolin

Renormalizable Potential, mixing **three families**

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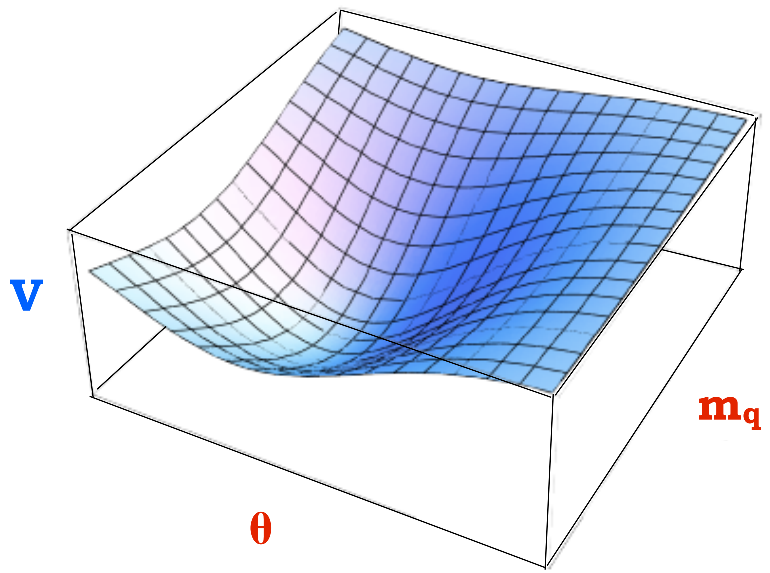

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Berezhiani-Rossi; Anselm, Berezhiani; Alonso, Gavela, Merlo, Rigolin

2 families, leptons; let us analyze the mixing invariant

diagonal eigenvalues

Using Casas-Ibarra parametrization $\mathbf{Y_\nu = U_{PMNS}\, m_\nu^{1/2}\, R\, M_N^{1/2}}$

complex orthogonal; it encodes our ignorance of the high energy theory

it follows that:

$$\mathrm{Tr}(\mathcal{Y}_E\, \mathcal{Y}_E^{+}\, \mathcal{Y}_\nu\, \mathcal{Y}_\nu^{+}) = \mathrm{Tr}(m_i^{1/2}\, U^{+}\, m_l^{2}\, U\, m_i^{1/2}\, R^{+}\, M_N\, R)$$

* In degenerate limit of heavy neutrinos $M_{N1}=M_{N2}=M$

$$\mathbf{R} = \begin{pmatrix} \mathrm{ch}\,\omega & -i\,\mathrm{sh}\,\omega \\ i\,\mathrm{sh}\,\omega & \mathrm{ch}\,\omega \end{pmatrix}$$ with ω real,

for 2 generations, the mixing terms in $\mathbf{V}(\mathcal{Y}_E, \mathcal{Y}_\nu)$ is :

Leptons

$$\mathrm{Tr}(\mathcal{Y}_E\, \mathcal{Y}_E^+\, \mathcal{Y}_\nu\, \mathcal{Y}_\nu^+) \propto$$

$$(m_\mu^2 - m_e^2)\left[\cos 2\omega(m_{\nu_2} - m_{\nu_1}) \cos 2\theta + 2 \sin 2\omega \sqrt{m_{\nu_2} m_{\nu_1}}\, sin 2\alpha \sin 2\theta\right]$$

where $U_{PMNS} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$

Quarks

$$\mathrm{Tr}(\mathcal{Y}_u\, \mathcal{Y}_u^+\, \mathcal{Y}_d\, \mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2) \cos 2\theta$$

e.g., **for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_\mathbf{E}, \mathcal{Y}_\mathbf{\nu})$ is :

Leptons

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This mixing term unphysical if either "up" or "down" fermions degenerate

Mixing physical even with degenerate neutrino masses, if Majorana phase non-trivial

Quarks

$$\mathrm{Tr}(\mathcal{Y}_u \mathcal{Y}_u^+ \mathcal{Y}_d \mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2)\cos 2\theta$$

e.g., **for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_E, \mathcal{Y}_\nu)$ is :

Minimisation (for non trivial sin2ω)

$\mathrm{Tr}(\mathcal{Y}_E \mathcal{Y}_E^+ \mathcal{Y}_\nu \mathcal{Y}_\nu^+)$

* $\sin 2\omega \sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\theta \cos 2\alpha = 0$ → $\boxed{\alpha = \pi/4 \text{ or } 3\pi/4}$

Maximal Majorana phase

* $\mathrm{tg} 2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}} \; \mathbf{tgh}\, \mathbf{2\omega}$

Large angles correlated with degenerate masses

Example: 2 families; consider the renormalizable set of invariants:

The flavour symmetry is G_f = $U(2)_L \times U(2)_{E_R} \times O(2)_{N_R}$

which adds a new invariant for the lepton sector. In total:

$$\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^{+}) \qquad \mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^{+})^2$$

$$\mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^{+}) \qquad \mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^{+})^2$$

$$\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^{+}\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^{+}) \longleftarrow \text{mixing}$$

$$\mathrm{Tr}\,(\mathcal{Y}_\nu^{+}\mathcal{Y}_\nu\,\mathcal{Y}_\nu^{T}\,\mathcal{Y}_\nu^{*})$$ <-- **$O(2)_N$**

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$$\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)$$ ← **mixing**

$$\mathrm{Tr}\,(\mathcal{Y}_\nu^+\mathcal{Y}_\nu\,(\mathcal{Y}_\nu^+\,\mathcal{Y}_\nu)^T)$$ <-- **$O(2)_N$**

e.g., **for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_E, \mathcal{Y}_\nu)$ is :

Minimisation of $\mathrm{Tr}(\mathcal{Y}_E \mathcal{Y}_E^{+} \mathcal{Y}_\nu \mathcal{Y}_\nu^{+})$

$\alpha = \pi/4$ or $3\pi/4$

Maximal Majorana phase

$$tg2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}} \; \mathrm{tgh}\, 2\omega$$

Large angles correlated with degenerate masses

where $U_{PMNS} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$

Jacobian

$$J = \begin{pmatrix} \partial_{\mathbf{y}_E} I_{E^n} & 0 & 0 & \partial_{\mathbf{y}_E} I_{L^n} & \partial_{\mathbf{y}_E} I_{LR} \\ 0 & \partial_{\mathbf{y}_\nu} I_{\nu^n} & \partial_{\mathbf{y}_\nu} I_{R^n} & \partial_{\mathbf{y}_\nu} I_{L^n} & \partial_{\mathbf{y}_\nu} I_{LR} \\ 0 & 0 & \partial_{\mathcal{U}_R} I_{R^n} & 0 & \partial_{\mathcal{U}_R} I_{LR} \\ 0 & 0 & 0 & \partial_{\mathcal{U}_L} I_{L^n} & \partial_{\mathcal{U}_L} I_{LR} \\ 0 & 0 & 0 & 0 & \partial_{\mathcal{U}_L \mathcal{U}_R} I_{LR} \end{pmatrix},$$

$$\mathrm{Diag}(J) \equiv (J_E\,,\ J_\nu\,,\ J_{\mathcal{U}_R}\,,\ J_{\mathcal{U}_L}\,,\ J_{LR})$$

Jacobian Analysis: Mixing

What is the symmetry in this boundary?

$$Y_\nu = \begin{pmatrix} y_1 & 0 & 0 \\ 0 & \frac{y_2}{\sqrt{2}} & -i\frac{y_2}{\sqrt{2}} \\ 0 & \frac{y_3}{\sqrt{2}} & i\frac{y_3}{\sqrt{2}} \end{pmatrix} \qquad \lambda'_3 Y_\nu - Y_\nu \lambda_7 = 0;\ \lambda'_3 = \mathrm{diag}(0,1,-1)\ ,$$

$$U(1)_{diag}$$

which is extended if the eigenvalues are degenerate

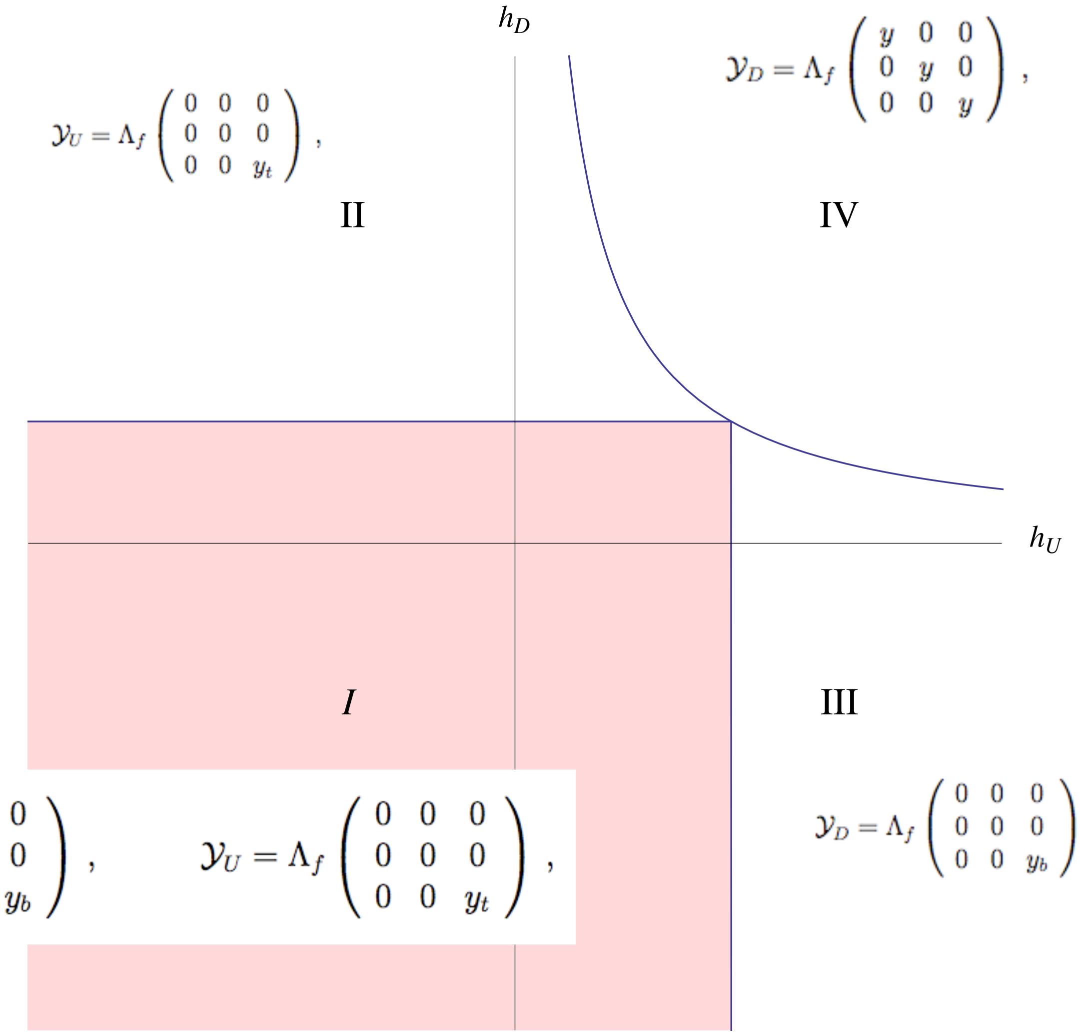
$$Y_\nu \to y \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -i\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & i\frac{1}{\sqrt{2}} \end{pmatrix} = yV\ , \qquad Y_\nu \to (V\mathcal{O}V^\dagger) Y_\nu \mathcal{O}^T = Y_\nu\ .$$

$$O(3)_{diag}$$

[Alonso, Gavela, G. Isidori, L. Maiani]

Renormalizable Potential

Renormalizable Potential, masses



II:

$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y_b \end{pmatrix}, \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix},$$

IV:

$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y \end{pmatrix}, \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} y' & 0 & 0 \\ 0 & y' & 0 \\ 0 & 0 & y' \end{pmatrix},$$

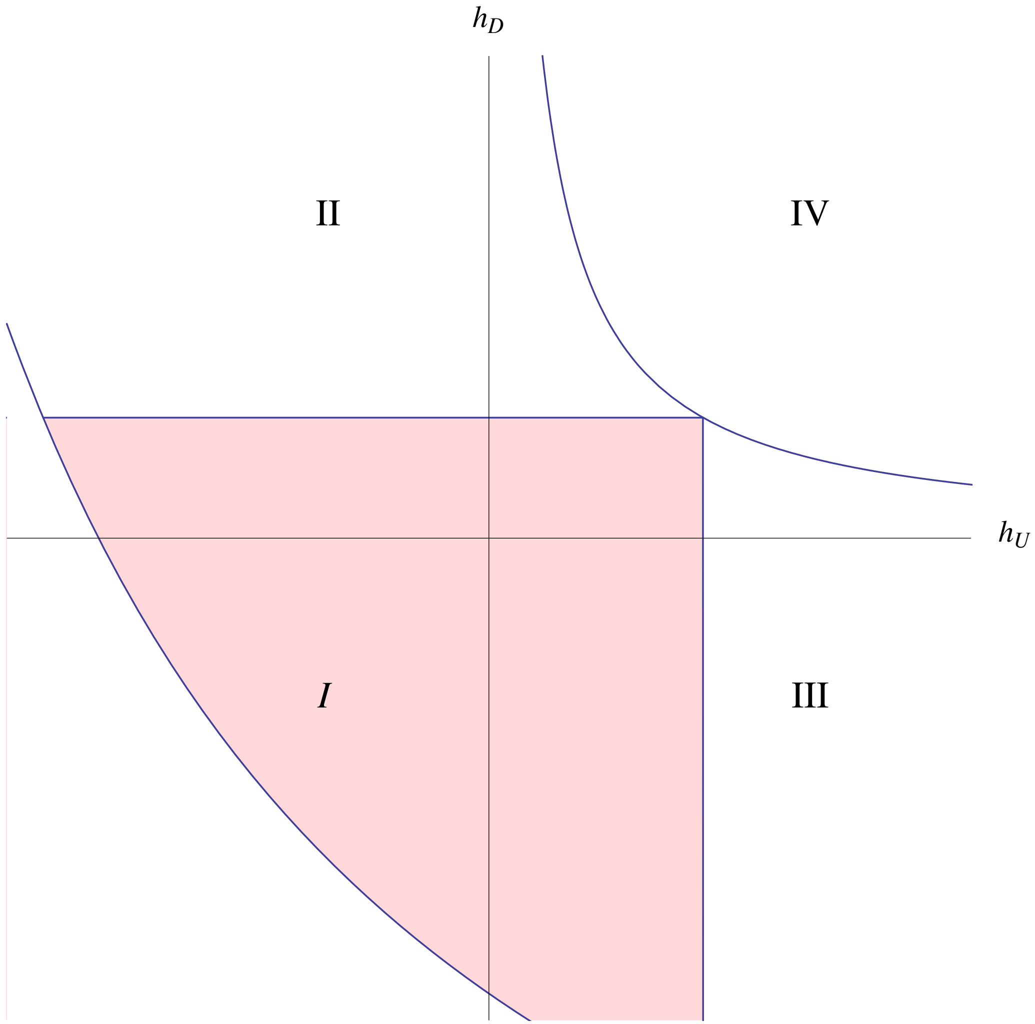
I:

$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix},$$

III:

$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}, \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} y' & 0 & 0 \\ 0 & y' & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

Renormalizable Potential, Stability



This region's size nonetheless depends on the rest of paramters (λ,g)

Renormalizable Potential

defining

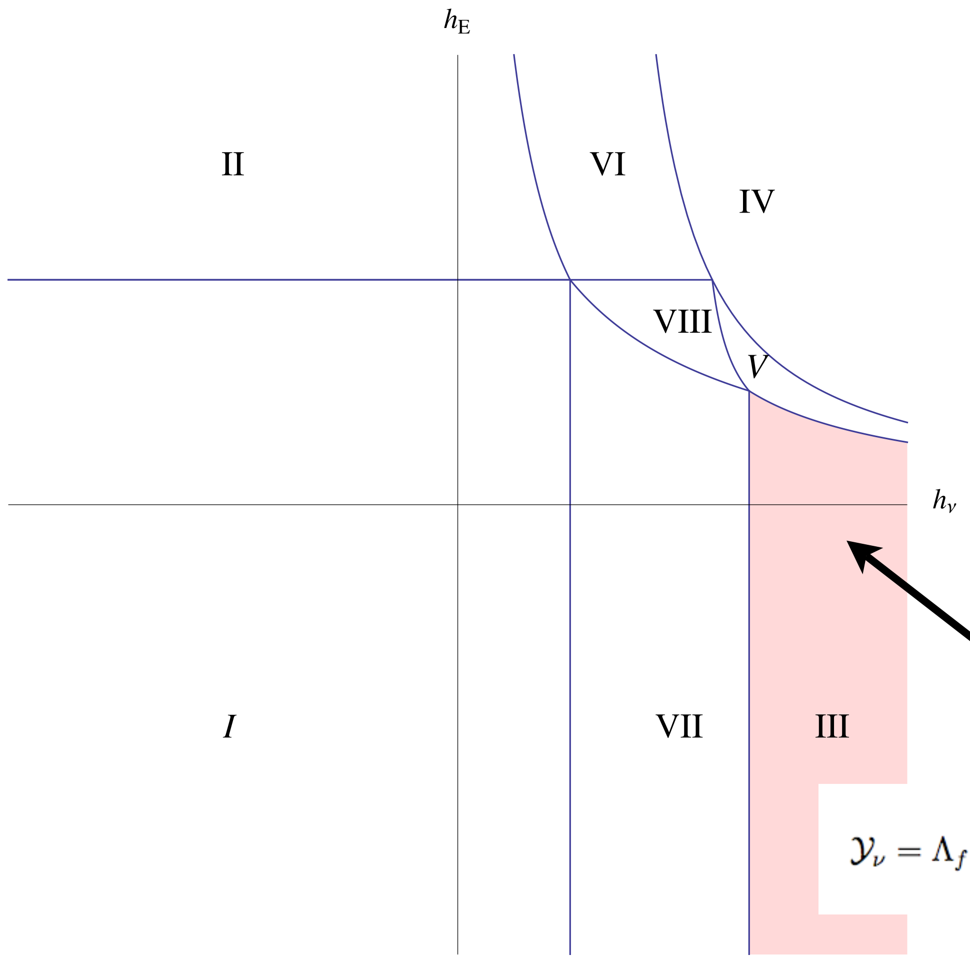
$$\mathbf{X} \equiv \left(\mathrm{Tr}\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right), \mathrm{Tr}\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)\right)^T ,$$

the potential reads:

$$V = -\mu^2 \cdot \mathbf{X} + \mathbf{X}^T \cdot \lambda \cdot \mathbf{X} + h_E \,\mathrm{Tr}\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right) + g \,\mathrm{Tr}\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)$$
$$+ h_\nu \,\mathrm{Tr}\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right) + h'_\nu \,\mathrm{Tr}\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger\right) .$$

9 parameters

Renormalizable Potential: Masses



$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix},$$

$$\mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} y_{\nu_1}/\sqrt{2} & 0 & i y_{\nu_1}/\sqrt{2} \\ 0 & y_{\nu_2} & 0 \\ -y_{\nu_3}/\sqrt{2} & 0 & i y_{\nu_3}/\sqrt{2} \end{pmatrix}$$

Renormalizable Potential

defining

$$\mathbf{X} \equiv \left(\mathrm{Tr}\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right), \mathrm{Tr}\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)\right)^T ,$$

the potential reads:

$$V = -\mu^2 \cdot \mathbf{X} + \mathbf{X}^T \cdot \lambda \cdot \mathbf{X} + h_E \, \mathrm{Tr}\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right) + g \, \mathrm{Tr}\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)$$
$$+ h_\nu \, \mathrm{Tr}\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right) + h'_\nu \, \mathrm{Tr}\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger\right) .$$

9 parameters

Renormalizable Potential: Mixing

One maximal angle again but not quite in the right place

$$h'_\nu > 0\,, \qquad U_{PMNS} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \end{pmatrix},$$

The solution with a maximal θ_{23} , may arise in a Non-Renormalizable Potential or could be a Local Minima of the Renormalizable Potential

* What is the role of the neutrino flavour group?

Leptons: $G_{flavour} = U(2)_L \times U(2)_{E_R} \times$?

O(2), SU(n), O(n) .... ?

Inmediate results using for both quark and leptons

$Y = U_L \, y^{diag} \, U_R$

* What is the role of the neutrino flavour group?

To analyze this in general, use common parametrization for quarks and leptons:

$$\mathbf{Y} = U_L \; y^{diag.} \, U_R$$

* **Quarks**, for instance: U_R unphysical, $U_L \dashrightarrow U_{CKM}$

$$\mathbf{Y_D} = \mathbf{U_{CKM}} \, \mathrm{diag}(y_d, y_s, y_b) \quad ; \quad \mathbf{Y_U} = \mathrm{diag}(y_u, y_c, y_t)$$

* **Leptons**:

$$\mathbf{Y_E} = \mathrm{diag}(y_e, y_\mu, y_\tau) \quad ; \quad \mathbf{Y_\nu} = U_L \; y^{diag.} \, U_R$$

$\mathbf{U_{PMNS}}$ diagonalize

$$m_\nu \sim \mathbf{Y_\nu} \frac{v^2}{M} \mathbf{Y_\nu}^{\mathbf{T}} = U_L \, y_\nu^{diag.} \, U_R \frac{v^2}{M} U_R^T \, y_\nu^{diag.} \, U_L^T$$

* What is the role of the neutrino flavour group?

U(n)

*** What is the role of the neutrino flavour group?**

$$U(n)$$

i.e.: $U(3)_L \times U(3)_{E_R} \times U(2)_{N_R}$

or: $U(3)_L \times U(3)_{E_R} \times U(3)_{N_R}$

* What is the role of the neutrino flavour group?

e.g. $U(n)_{N_R}$ ... leptons

e.g. generic seesaw

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\partial\!\!\!/ N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R}\mathbf{M} N_R{}^c + h.c.\right]$$

with **M** carrying flavour → **M** spurion

More invariants in this case:

$Tr\,(\mathcal{Y}_E\,\mathcal{Y}_E^+)$ $\quad Tr\,(\mathcal{Y}_E\,\mathcal{Y}_E^+)^2$ $\quad Tr\,(\mathcal{Y}_E\,\mathcal{Y}_E^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)$

$Tr\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)$ $\quad Tr\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)^2$

$Tr\,(M_N\,M_N^+)$ $\quad Tr\,(M_N\,M_N^+)^2$ $\quad Tr\,(M_N\,M_N^+\,\mathcal{Y}_\nu^+\mathcal{Y}_\nu)$

Result: no mixing for flavour groups U(n)

SU(n)

* What is the role of the neutrino flavour group?

e.g. $SU(n)_{NR}$ ... leptons

e.g. generic seesaw

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\partial\!\!\!/ N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R}\mathbf{M}N_R{}^c + h.c.\right]$$

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$Tr(\mathcal{Y}_\nu \mathcal{Y}_\nu^+)$ $\quad Tr(\mathcal{Y}_\nu \mathcal{Y}_\nu^+)^2$

$Tr(\mathcal{M}_N \mathcal{M}_N^+)$ $\quad Tr(\mathcal{M}_N \mathcal{M}_N^+)^2$ $\quad Tr(\mathcal{M}_N \mathcal{M}_N^+ \mathcal{Y}_\nu^+ \mathcal{Y}_\nu)$

At the minimum:

* $Tr(\mathcal{Y}_\nu \mathcal{Y}_\nu^+ \mathcal{Y}_E \mathcal{Y}_E^+) = Tr(U_L\, y_\nu^{diag.\,2}\, U_L^+\, y_l^{diag.\,2})$ → $U_L=1$
* $Tr(\mathcal{M}_N \mathcal{M}_N^+ \mathcal{Y}_\nu \mathcal{Y}_\nu^+) = Tr(U_R\, y_\nu^{diag.\,2}\, U_R^+\, M_i^{diag.\,2})$ → $U_R=1$

same conclusion for 3 families of quarks:

$$\mathbf{Y} = U_L \; y^{diag.} \; U_R$$

* **Quarks**, for instance: U_R unphysical, U_L --> U_{CKM}

$$\mathbf{Y_D} = \mathbf{U_{CKM}} \, \mathrm{diag}(y_d, y_s, y_b) \quad ; \quad \mathbf{Y_U} = \mathrm{diag}(y_u, y_c, y_t)$$

$$\mathrm{Tr}\,(\, \mathcal{Y}_u \, \mathcal{Y}_u^+ \, \mathcal{Y}_d \, \mathcal{Y}_d^+ \,) = \mathrm{Tr}\,(\, U_L \, y_u^{diag.\,2} \, U_L^{\,+} \, y_d^{diag.\,2})$$

⟶ $U_L = U_{CKM} \sim 1$ at the minimum

NO MIXING

$O(n)$

Can its minimum correspond <u>naturally</u> to the observed masses and mixings?

i.e. with all dimensionless λ's ~ 1

and dimensionful μ's $\leqq \Lambda_f$

Y --> one single field Σ

Spectrum for flavons Σ in the bifundamental:

* **3 generations: for the largest fraction of the parameter space, the stable solution is a degenerate spectrum**

$$\begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix} \sim \begin{pmatrix} y & & \\ & y & \\ & & y \end{pmatrix}$$

instead of the observed hierarchical spectrum, i.e.

$$\begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix} \sim \begin{pmatrix} 0 & & \\ & 0 & \\ & & y \end{pmatrix}$$

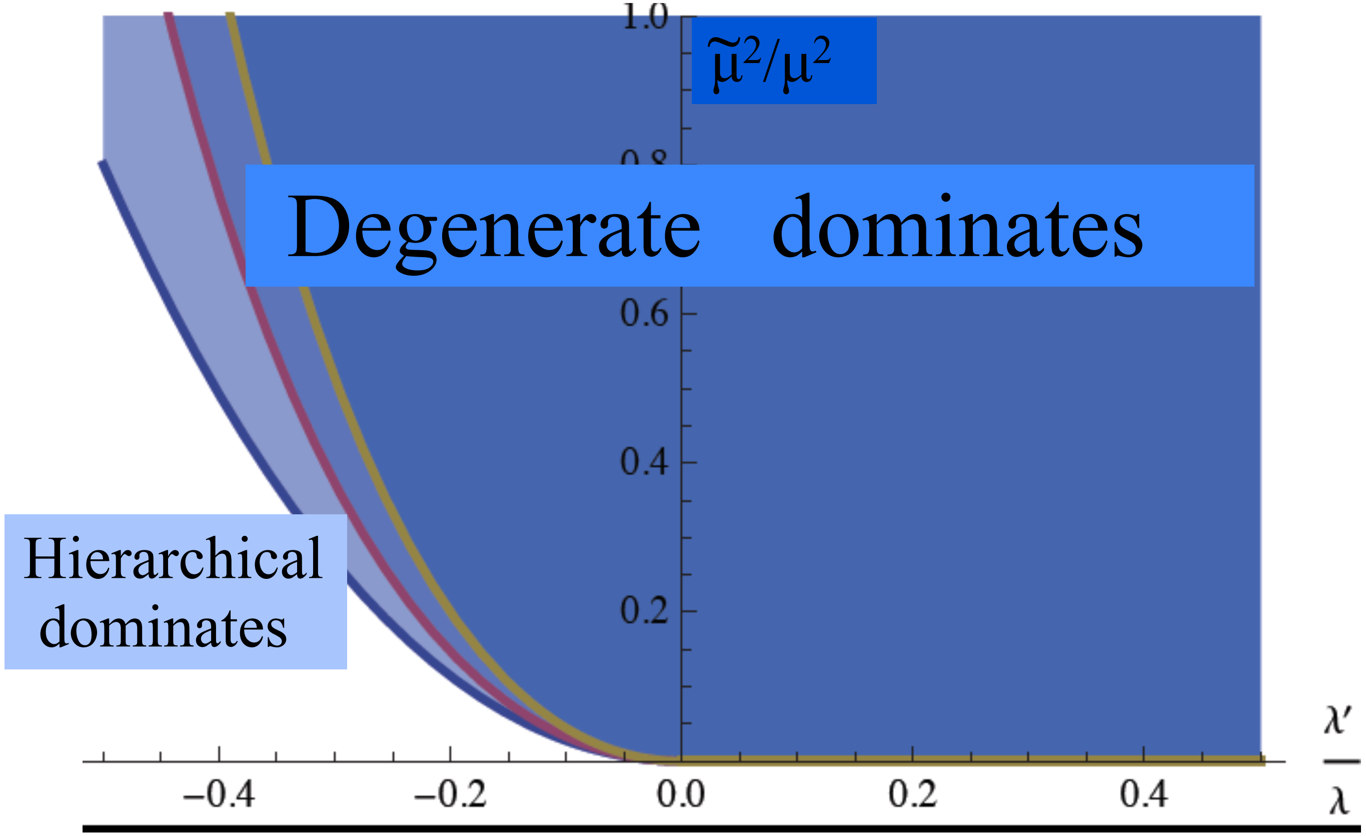
(at leading order)

Spectrum: the hierarchical solution is unstable in most of the parameter space.

Stability: $\frac{\tilde{\mu}^2}{\mu^2} < \frac{2\lambda'^2}{\lambda}$

$$V^{(4)} = \sum_{i=u,d} \left(-\mu_i^2 A_i + \tilde{\mu}_i B_i + \lambda_i A_i^2 + \lambda_i' A_{ii}\right) + g_{ud} A_u A_d + \lambda_{ud} A_{ud} \,.$$

ie, the u-part: $V^{(4)} = -\mu_u^2 A_u + \tilde{\mu}_u B_u + \lambda_u A_u^2 + \lambda_u' A_{uu}$



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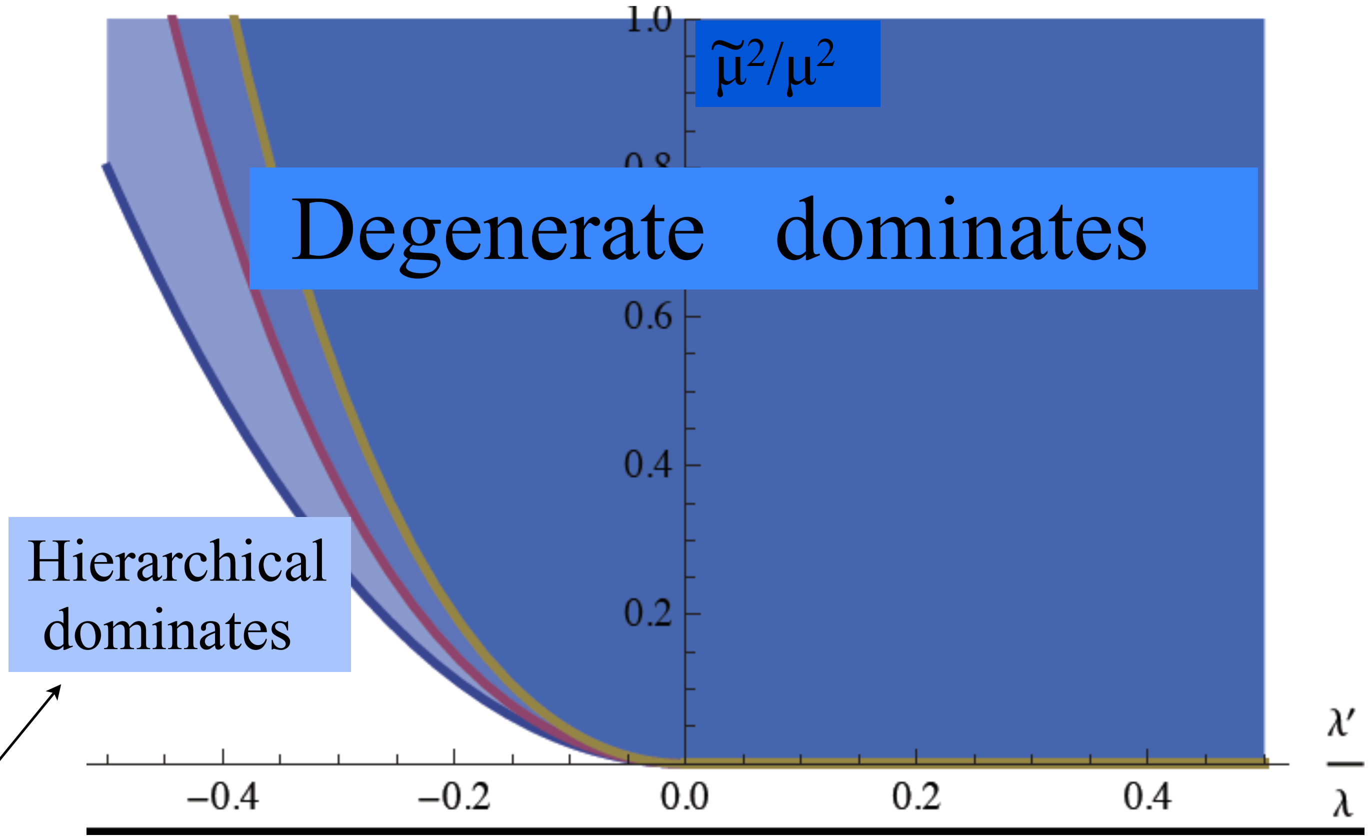


Nardi emphasized this solution (and extended the analysis to include also U(1) factors)

Normal hierarchy:

Up to terms of $\mathcal{O}(\sqrt{r}, s_{13})$, we find

$$r = \frac{|\Delta m^2_{12}|}{|\Delta m^2_{13}|}$$

$$Y_N^T \simeq y \begin{pmatrix} e^{i\delta} s_{13} + e^{-i\alpha} s_{12} r^{1/4} \\ s_{23}\left(1 - \frac{\sqrt{r}}{2}\right) + e^{-i\alpha} r^{1/4} c_{12} c_{23} \\ c_{23}\left(1 - \frac{\sqrt{r}}{2}\right) - e^{-i\alpha} r^{1/4} c_{12} s_{23} \end{pmatrix} .$$

Inverted hierarchy:

$$Y_N^T \simeq \frac{y}{\sqrt{2}} \begin{pmatrix} c_{12} e^{i\alpha} + s_{12} e^{-i\alpha} \\ c_{12}\left(c_{23} e^{-i\alpha} - s_{23} s_{13} e^{i(\alpha-\delta)}\right) - s_{12}\left(c_{23} e^{i\alpha} + s_{23} s_{13} e^{-i(\alpha+\delta)}\right) \\ -c_{12}\left(s_{23} e^{-i\alpha} + c_{23} s_{13} e^{i(\alpha-\delta)}\right) + s_{12}\left(s_{23} e^{i\alpha} - c_{23} s_{13} e^{-i(\alpha+\delta)}\right) \end{pmatrix}$$

Y --> one single field Σ

The invariants can be written in terms of masses and mixing

* two families:

$$<\Sigma_d> = \Lambda_f \,.\, \mathrm{diag}\,(y_d)\,; \qquad <\Sigma_u> = \Lambda_f \,.V_{Cabibbo}\, \mathrm{diag}(y_u)$$

$$Y_D = \begin{pmatrix} y_d & 0 \\ 0 & y_s \end{pmatrix}, \qquad Y_U = \mathcal{V}_C^\dagger \begin{pmatrix} y_u & 0 \\ 0 & y_c \end{pmatrix} \qquad V_{Cabibbo} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$<Tr\,(\Sigma_u \Sigma_u^+)> = \Lambda_f^2\ (y_u^2 + y_c^2)\,; \quad <\det\,(\Sigma_u)> = \Lambda_f^2\, y_u y_c$$

$$<Tr\,(\Sigma_u \Sigma_u^+ \Sigma_d \Sigma_d^+)> = \Lambda_f^4\, [(y_c^2 - y_u^2)\,(y_s^2 - y_d^2)\, \cos 2\theta + \ldots]/2$$

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

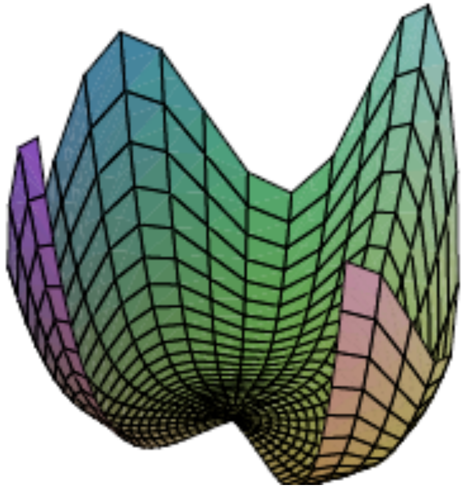
Y --> one single field Σ

Minimum of the Potential

Dimension 5 Yukawa Operator

The minimum of the Potential is given by:

$$\frac{\partial V}{\partial y_i} = 0 \qquad \frac{\partial V}{\partial \theta_i} = 0$$



Take the angle for example:

$$\frac{\partial V}{\partial \theta_c} \propto (y_c^2 - y_u^2)\,(y_s^2 - y_d^2)\,\sin 2\theta_c = 0$$

Non-degenerate masses $\longrightarrow$ $\sin 2\theta_c = 0$ No mixing !

Notice also that $$\frac{\partial V^{(4)}}{\partial \theta} \sim \sqrt{J}$$ (Jarlskog determinant)

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

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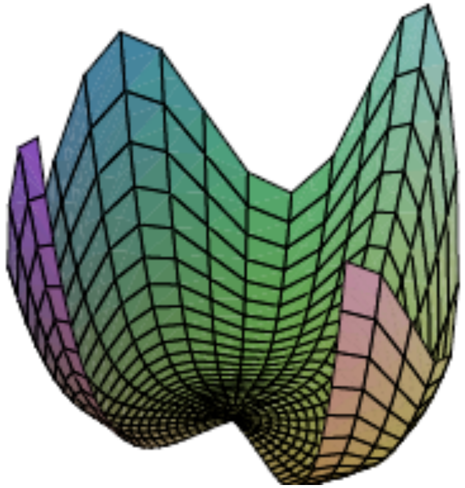

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Can the actual masses and mixings fit naturally in the minimum of the Potential? e.g. adding non-renormalizable terms... **NO**

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

Y --> one single field Σ

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Dimension 5 Yukawa Operator

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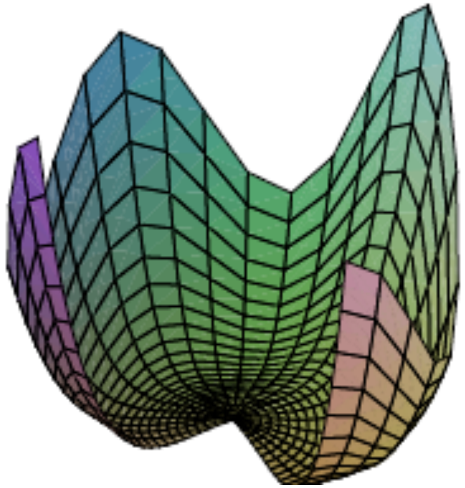


Non-degenerate masses $\sin 2\theta_c = 0$ No mixing !

Can the actual masses and mixings fit naturally in the minimum of the Potential? e.g. adding non-renormalizable terms...

NO

* Without fine-tuning, for two families the spectrum is degenerate
* To accomodate realistic mixing one must introduce wild fine tunnings of $O(10^{-10})$ and nonrenormalizable terms of dimension 8

(Alonso, Gavela, Merlo, Rigolin)

Y --> one single field Σ

three families

* at renormalizable level: 7 invariants instead of the 5 for two families

$$\mathrm{Tr}\left(\Sigma_u\Sigma_u^\dagger\right) \overset{vev}{=} \Lambda_f^2\left(y_t^2+y_c^2+y_u^2\right)\,, \qquad Det\left(\Sigma_u\right) \overset{vev}{=} \Lambda_f^3 y_u y_c y_t\,,$$

$$\mathrm{Tr}\left(\Sigma_d\Sigma_d^\dagger\right) \overset{vev}{=} \Lambda_f^2\left(y_b^2+y_s^2+y_d^2\right)\,, \qquad Det\left(\Sigma_d\right)\Big| \overset{vev}{=} \Lambda_f^3 y_d y_s y_b\,,$$

$$\mathrm{Tr}\left(\Sigma_u\Sigma_u^\dagger\Sigma_u\Sigma_u^\dagger\right) \overset{vev}{=} \Lambda_f^4\left(y_t^4+y_c^4+y_u^4\right)\,,$$

$$\mathrm{Tr}\left(\Sigma_d\Sigma_d^\dagger\Sigma_d\Sigma_d^\dagger\right) \overset{vev}{=} \Lambda_f^4\left(y_b^4+y_s^4+y_d^4\right)\,,$$

$$\mathrm{Tr}\left(\Sigma_u\Sigma_u^\dagger\Sigma_d\Sigma_d^\dagger\right) \overset{vev}{=} \Lambda_f^4\left(P_0+P_{int}\right)\,,$$

Interesting angular dependence:

$$P_0 \equiv -\sum_{i<j}\left(y_{u_i}^2-y_{u_j}^2\right)\left(y_{d_i}^2-y_{d_j}^2\right)\sin^2\theta_{ij}\,,$$

$$\begin{aligned}P_{int} \equiv &\sum_{i<j,k}\left(y_{d_i}^2-y_{d_k}^2\right)\left(y_{u_j}^2-y_{u_k}^2\right)\sin^2\theta_{ik}\sin^2\theta_{jk}+\\ &-\left(y_d^2-y_s^2\right)\left(y_c^2-y_t^2\right)\sin^2\theta_{12}\sin^2\theta_{13}\sin^2\theta_{23}+\\ &+\frac{1}{2}\left(y_d^2-y_s^2\right)\left(y_c^2-y_t^2\right)\cos\delta\,\sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13}\,,\end{aligned}$$

The real, unavoidable, problem is again mixing:

* Just one source:

$$\mathrm{Tr}\,(\Sigma_u \Sigma_u^+ \Sigma_d \Sigma_d^+) = \Lambda_f^4 (P_0 + P_{int})$$

P_0 and P_{int} encode the angular dependence,

$$P_0 \equiv -\sum_{i<j} \left(y_{u_i}^2 - y_{u_j}^2\right)\left(y_{d_i}^2 - y_{d_j}^2\right) \sin^2\theta_{ij}\,,$$

$$P_{int} \equiv \sum_{i<j,k} (y_{d_i}^2 - y_{d_k}^2)\left(y_{u_j}^2 - y_{u_k}^2\right) \sin^2\theta_{ik} \sin^2\theta_{jk} +$$

$$-\,(y_d^2 - y_s^2)\,(y_c^2 - y_t^2) \sin^2\theta_{12} \sin^2\theta_{13} \sin^2\theta_{23} +$$

$$+\,\frac{1}{2}\,(y_d^2 - y_s^2)\,(y_c^2 - y_t^2) \cos\delta\, \sin 2\theta_{12} \sin 2\theta_{23} \sin\theta_{13}$$

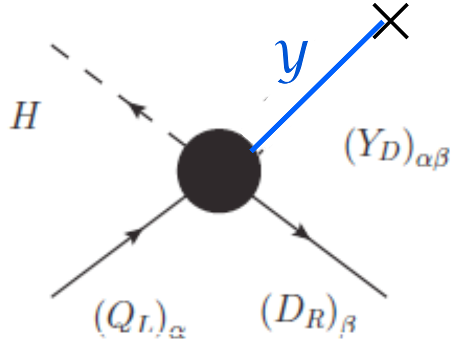
Sad conclusions as for 2 families:

needs non-renormalizable + super fine-tuning

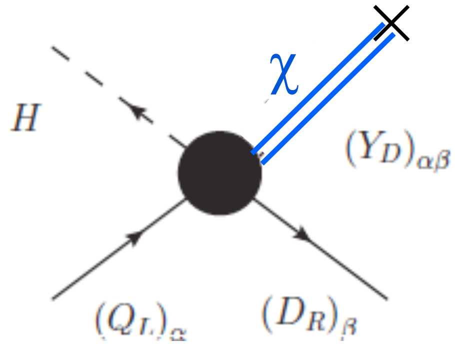
*a good possibility for the other angles :

Yukawas --> add fields in the fundamental of the flavour group

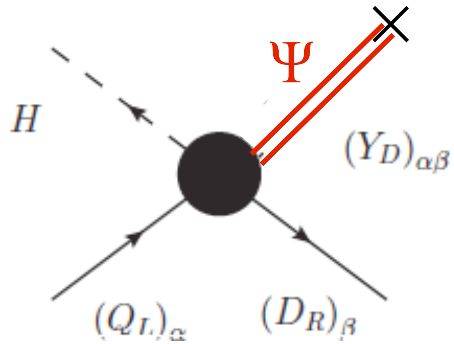
1) Y -- > one single scalar $\mathcal{Y} \sim (3, 1, \bar{3})$



2) Y -- > two scalars $\chi\, \chi^{+} \sim (3, 1, \bar{3})$



3) Y -- > two fermions $\bar{\Psi}\Psi \sim (3, 1, 3)$



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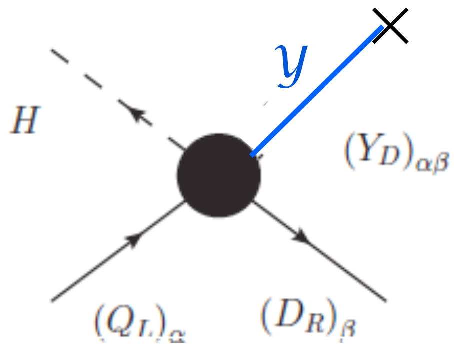
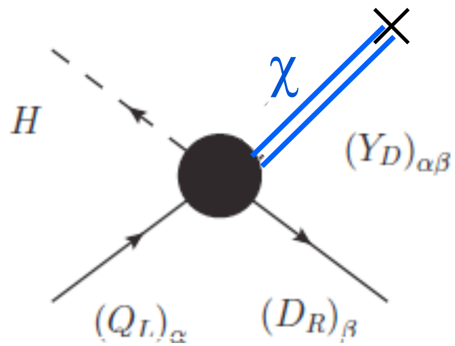


2) Y -- > two scalars $\chi\, \chi^+ \sim (3, 1, \bar{3})$

$\chi \sim (3, 1, 1)$



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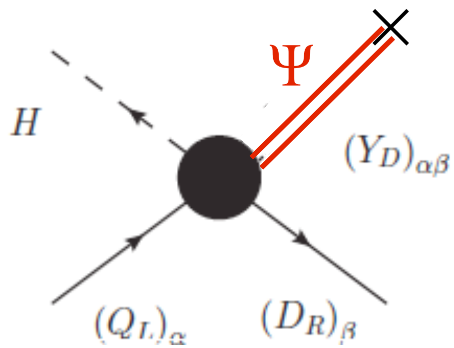
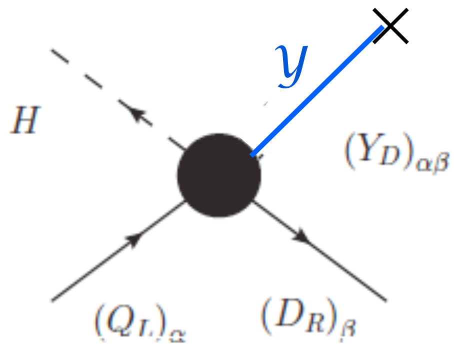


1) Y --> one single scalar $\mathcal{Y} \sim (3, 1, \bar{3})$

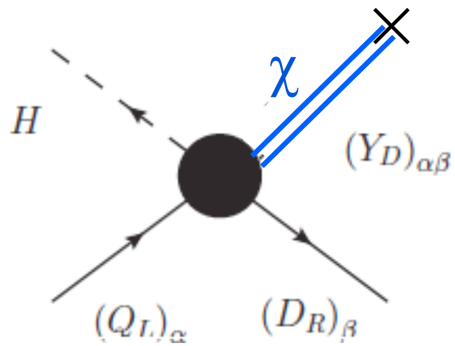
d=5 operator



2) Y --> two scalars $\chi\, \chi^{+} \sim (3, 1, \bar{3})$

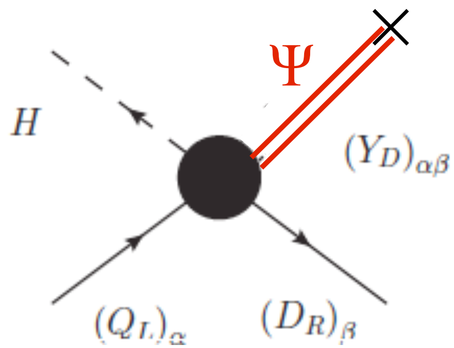
d=6 operator

$\chi \sim (3, 1, 1)$



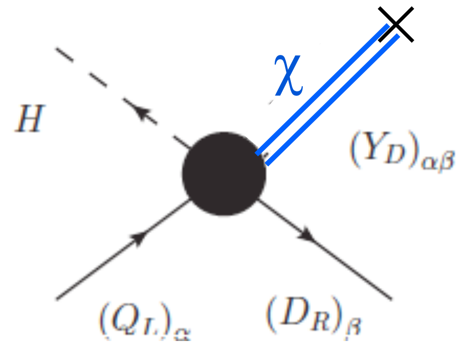
3) Y --> two fermions $\bar{\Psi}\Psi \sim (3, 1, 3)$

d=7 operator



Y --> quadratic in fields χ

$$Y \sim \frac{\langle \chi \chi^{\dagger} \rangle}{\Lambda_f^2}$$

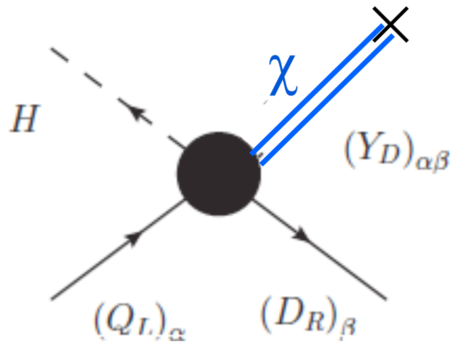


Automatic strong mass hierarchy and one mixing angle already at the renormalizable level

Holds for 2 and 3 families !

2) Y --> quadratic in fields χ

$$Y \sim \frac{< \chi \chi^{\dagger} >}{\Lambda_f^2}$$



i.e. $Y_D \sim \dfrac{\chi^{L}{}_{d}\ (\chi^{R}{}_{d})^{+}}{\Lambda_f^{2}} \sim (3, 1, 1)\ (1, 1, \overline{3}) \sim (3, 1, \overline{3})$

Y --> quadratic in fields χ

It is very simple:

- a square matrix built out of 2 vectors

$$\begin{pmatrix} d \\ e \\ f \\ \vdots \end{pmatrix} (a, b, c \, \ldots\ldots)$$

has only one non-vanishing eigenvalue

→ strong mass hierarchy at leading order:

-- only 1 heavy "up" quark

-- only 1 heavy "down" quark

only $|\chi|$'s relevant for scale

Y --> quadratic in fields χ

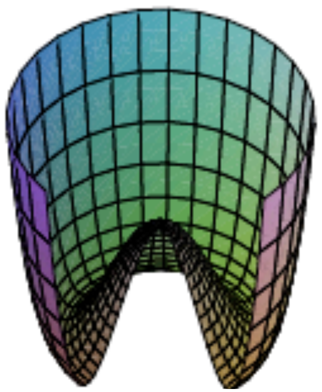
Minimum of the Potential

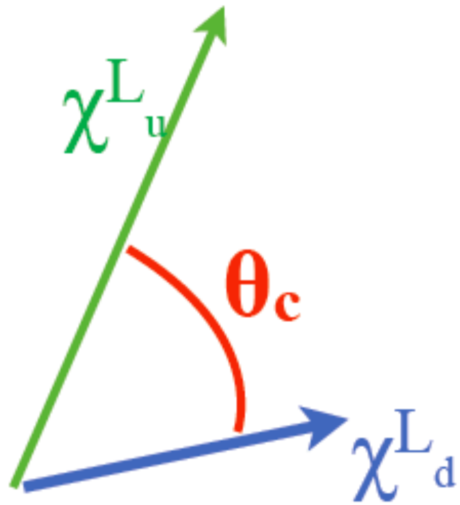
Dimension 6 Yukawa Operator

The invariants are:

$$\chi_u^{L\dagger}\chi_u^L\,,\qquad \chi_u^{R\dagger}\chi_u^R\,,\qquad \chi_d^{L\dagger}\chi_d^L\,,$$

$$\chi_d^{R\dagger}\chi_d^R\,,\qquad \chi_u^{L\dagger}\chi_d^L = \left|\chi_u^L\right|\left|\chi_d^L\right|\cos\theta_c\,.$$





θ_c is the angle between up and down L vectors

Y --> quadratic in fields χ

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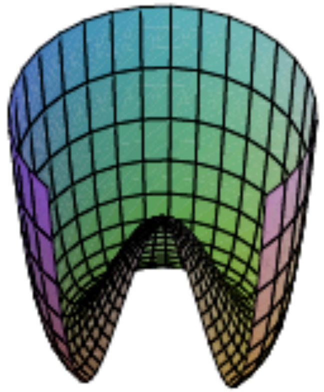


We can fit the angle and the masses in the Potential; as an example:

$$V' = \lambda_u\left(\chi_u^{L\dagger}\chi_u^L - \frac{\mu_u^2}{2\lambda_u}\right)^2 + \lambda_d\left(\chi_d^{L\dagger}\chi_d^L - \frac{\mu_d^2}{2\lambda_d}\right)^2$$
$$+\lambda_{ud}\left(\chi_u^{L\dagger}\chi_d^L - \frac{\mu_{ud}^2}{2\lambda_{ud}}\right)^2 + \cdots$$

Whose minimum sets (2 generations):

$$y_c^2 = \frac{\mu_u^2}{2\lambda_u\Lambda_f^2}\qquad y_s^2 = \frac{\mu_d^2}{2\lambda_d\Lambda_f^2}\qquad \cos\theta = \frac{\mu_{ud}^2\sqrt{\lambda_u\lambda_d}}{\mu_u\mu_d\lambda_{ud}}$$

Y --> quadratic in fields χ

Towards a realistic 3 family spectrum

e.g. replicas of χ^L , χ^R_u , χ^R_d

???

Suggests sequential breaking:

$$SU(3)^3 \xrightarrow{\text{mt, mb}} SU(2)^3 \xrightarrow{\text{mc, ms, } \theta_C} \cdots\cdots\cdots$$

$$Y_u \equiv \frac{\langle \chi^L \rangle \langle \chi_u^{R\dagger} \rangle}{\Lambda_f^2} + \frac{\langle \chi_u'^L \rangle \langle \chi_u'^{R\dagger} \rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & \sin\theta\, y_c & 0 \\ 0 & \cos\theta\, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

$$Y_d \equiv \frac{\langle \chi^L \rangle \langle \chi_d^{R\dagger} \rangle}{\Lambda_f^2} + \frac{\langle \chi_d'^L \rangle \langle \chi_d'^{R\dagger} \rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} .$$

* From bifundamentals: $<y_u> = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}$

$$<y_d> = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}$$

* From fundamentals χ : y_c, y_s and θ_C

Y --> quadratic in fields χ

Towards a realistic 3 family spectrum

e.g. replicas of χ^{L} , χ^{R}_{u} , χ^{R}_{d}

???

Suggests sequential breaking:

$SU(3)^3$ —→ **$SU(2)^3$** —→ ....

mt, mb   **mc, ms, θ_C**

$$\frac{\langle\chi^{L}\rangle\,\langle\chi^{R\dagger}\rangle}{\Lambda_f^2} + \frac{\langle\chi'^{L}\rangle\,\langle\chi'^{R\dagger}\rangle}{\Lambda_f^2} \quad \begin{pmatrix} 0 & \sin\theta\, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

Maybe some connection to: Berezhiani+Nesti; Ferretti et al., Calibbi et al. ??

$$Y_d \equiv \frac{\langle\chi^{L}\rangle\,\langle\chi_d^{R\dagger}\rangle}{\Lambda_f^2} + \frac{\langle\chi_d'^{L}\rangle\,\langle\chi_d'^{R\dagger}\rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} .$$

i.e. for quarks, a possible path:

*** At leading (renormalizable) order:**

$$Y_u \equiv \frac{\langle \mathcal{Y}_u \rangle}{\Lambda_f} + \frac{\langle \chi_u^L \rangle \langle \chi_u^{R\dagger} \rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & \sin\theta_c \, y_c & 0 \\ 0 & \cos\theta_c \, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} ,$$

$$Y_d \equiv \frac{\langle \mathcal{Y}_d \rangle}{\Lambda_f} + \frac{\langle \chi_d^L \rangle \langle \chi_d^{R\dagger} \rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} .$$

without unnatural fine-tunings

*** The masses of the first family and the other angles from non-renormalizable terms or other corrections or replicas ?**

....and analogously for leptonic mixing ?

Y --> linear + quadratic in fields

Towards a realistic 3 family spectrum

Combining fundamentals and bi-fundamentals

i.e. combining d=5 and d =6 Yukawa operators

$$\Sigma_u \sim (3,\overline{3},1)\ , \qquad \Sigma_d \sim (3,1,\overline{3})\ , \qquad \Sigma_R \sim (1,3,\overline{3})\ ,$$

$$\chi_u^L \in (3,1,1)\ , \qquad \chi_u^R \in (1,3,1)\ , \qquad \chi_d^L \in (3,1,1)\ , \qquad \chi_d^R \in (1,1,3)\ .$$

The Yukawa Lagrangian up to the second order in $1/\Lambda_f$ is given by:

$$\mathscr{L}_Y = \overline{Q}_L \left[\frac{\Sigma_d}{\Lambda_f} + \frac{\chi_d^L \chi_d^{R\dagger}}{\Lambda_f^2} \right] D_R H + \overline{Q}_L \left[\frac{\Sigma_u}{\Lambda_f} + \frac{\chi_u^L \chi_u^{R\dagger}}{\Lambda_f^2} \right] U_R \tilde{H} + \text{h.c.}\ ,$$

LHC is more competitive for concrete seesaw models:

Low M, large Y is typical of seesaws with approximate Lepton Number conservation

$U(1)_{LN}$

(-> ~ degenerate heavy neutrinos)

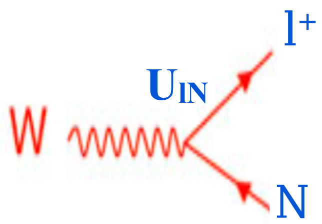
These models separate the flavor and the lepton number scale

Wyler+Wolfenstein 83, Mohapatra+Valle 86, Branco+Grimus+Lavoura 89, Gonzalez-Garcia+Valle 89, Ilakovac+Pilaftsis 95, Barbieri+Hambye+Romanino 03, Raidal+Strumia+Turzynski 05, Kersten+Smirnov 07, Abada+Biggio+Bonnet+Gavela+Hambye 07, Shaposhnikov 07, Asaka+Blanchet 08, Gavela+Hambye+D. Hernandez+ P. Hernandez 09

For instance, in the minimal seesaw I, Lepton number scale and flavour scale linked:

$$\mathscr{L}_{M_\nu} = \begin{pmatrix} 0 & Y^T v \\ Y v & M \end{pmatrix}$$

$$-\mathcal{L}_{\text{seesaw I}} = \overline{L} H Y_E E_R + \overline{L} \tilde{H} Y N + M \overline{N} N^c + h.c.$$



$$m_\nu = Y \frac{v^2}{M} Y^T \qquad U_{lN} \sim \frac{Y v}{M}$$

* What is the role of the neutrino flavour group?

e.g. $O(2)_{NR}$ ... leptons

e.g. seesaw with approximately conserved lepton number

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\, \bar{N}^c\,,\, \bar{N}'^c) \begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M}^T \\ vY'^T & \mathbf{M} & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

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e.g. $O(2)_{NR}$ ... leptons

e.g. seesaw with approximately conserved lepton number

$$\mathcal{L}_{mass} = \bar{\ell}_L \phi Y_E E_R + \bar{\ell}_L \tilde{\phi} \tilde{Y}_\nu (N_1, N_2)^T + M(\overline{N}_1 N_1^c + \overline{N}_2 N_2^c) + h.c.$$

$$\tilde{Y}_\nu = \frac{1}{\sqrt{2}} U_{PMNS}\, f_{m_\nu} \begin{pmatrix} y + y' & -i(y - y') \\ i(y - y') & y + y' \end{pmatrix}$$

$$U(3)_{\ell_L} \times U(3)_{E_R} \times O(2)_N$$

$$Y_E = \frac{< \mathcal{Y}_E >}{\Lambda_f} \sim (3, \bar{3}, 1); \quad (Y, Y') = \frac{< \mathcal{Y}_\nu >}{\Lambda} \sim (3, 1, 2)$$

$$< \mathcal{Y}_E > \propto \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \quad < \mathcal{Y}_\nu > \propto U_{PMNS} \begin{pmatrix} 0 & 0 \\ \sqrt{m_{\nu_2}} & 0 \\ 0 & \sqrt{m_{\nu_3}} \end{pmatrix} \begin{pmatrix} -iy & iy' \\ y & y' \end{pmatrix}$$

*In the O(2)model used before: $\mathrm{tgh}\, 2\omega = \frac{y^2 - y'^2}{y^2 - y'^2}$ and

$$\mathrm{tg}2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}} \frac{y^2 - y'^2}{y^2 - y'^2}$$

$\alpha = \pi/4$ or $3\pi/4$

*If we had used instead a flavor SU(2)model $\sinh 2\omega = 0$ -->NO MIXING

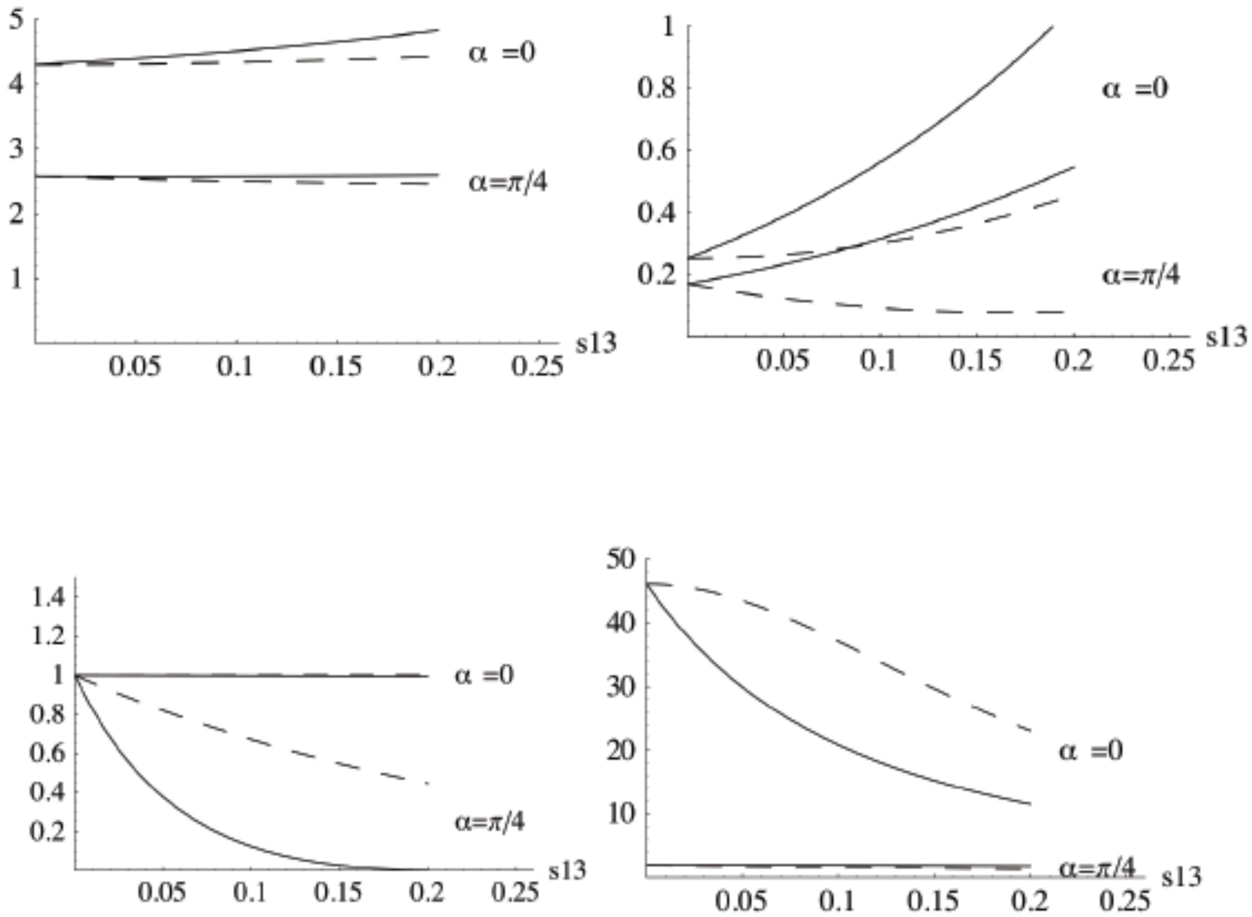
* e-μ, μ-τ etc. oscillations and rare decays studied:

Gavela, Hambye, Hernandez[2] ; .....

$Br(\mu \to e\gamma)/Br(\tau \to e\gamma)$ $Br(\mu \to e\gamma)/Br(\tau \to \mu\gamma)$

NH

IH



Gavela, Hambye, Hernandez[2] ;

Degeneracy in the Majorana phase α

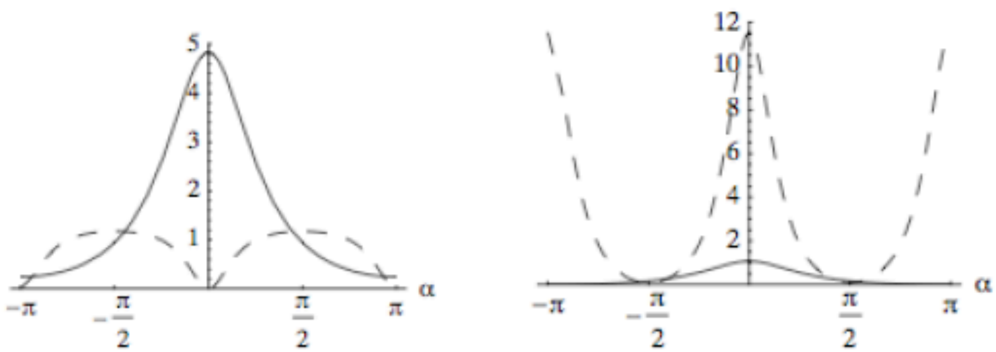
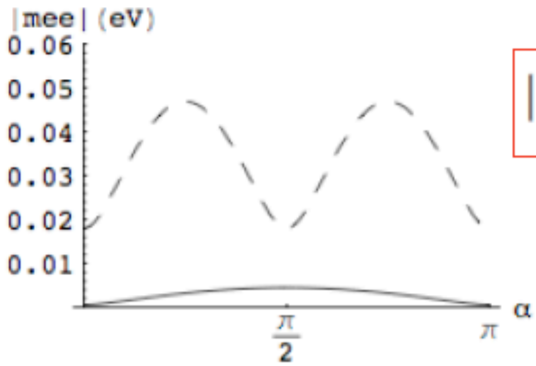


Figure 3: Left: Ratio $B_{e\mu}/B_{e\tau}$ for the normal hierarchy (solid) and the inverse hierarchy (dashed) as a function of α for $(\delta, s_{13}) = (0, 0.2)$. Right: the same for the ratio $B_{e\mu}/B_{\mu\tau}$.



$$|m_{ee}|_{IH} \simeq |s_{12}^2 e^{-2i\alpha} - c_{12}^2 e^{2i\alpha}|$$

Figure 5: m_{ee} as a function of α for the normal (solid) and inverted (dashed) hierarchies, for $(\delta, s_{13}) = (0, 0.2)$.

Gavela, Hambye, Hernandez[2] ;

cancellations for large θ_{13}

$$B_{\mu\to e\gamma} \propto |Y_{N_e} Y_{N_\mu}|^2$$

i.e.

$$Y_N^T \simeq y \begin{pmatrix} e^{i\delta} s_{13} + e^{-i\alpha} s_{12} r^{1/4} \\ s_{23}\left(1-\frac{\sqrt{r}}{2}\right) + e^{-i\alpha} r^{1/4} c_{12} c_{23} \\ c_{23}\left(1-\frac{\sqrt{r}}{2}\right) - e^{-i\alpha} r^{1/4} c_{12} s_{23} \end{pmatrix} \qquad r = \frac{|\Delta m^2_{12}|}{|\Delta m^2_{13}|}$$

Normal hierarchy

* Alonso + Li, 2010, MINSIS report:

possible suppresion of μ-e transitions for large θ_{13}

* e-μ, μ-τ etc. oscillations and rare decays studied:

Gavela, Hambye, Hernandez[2] 09 ; .....

* Alonso + Li, 2010: possible suppresion of μ-e transitions

->important impact of ν_μ - ν_τ at a near detectors

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Normal hierarchy

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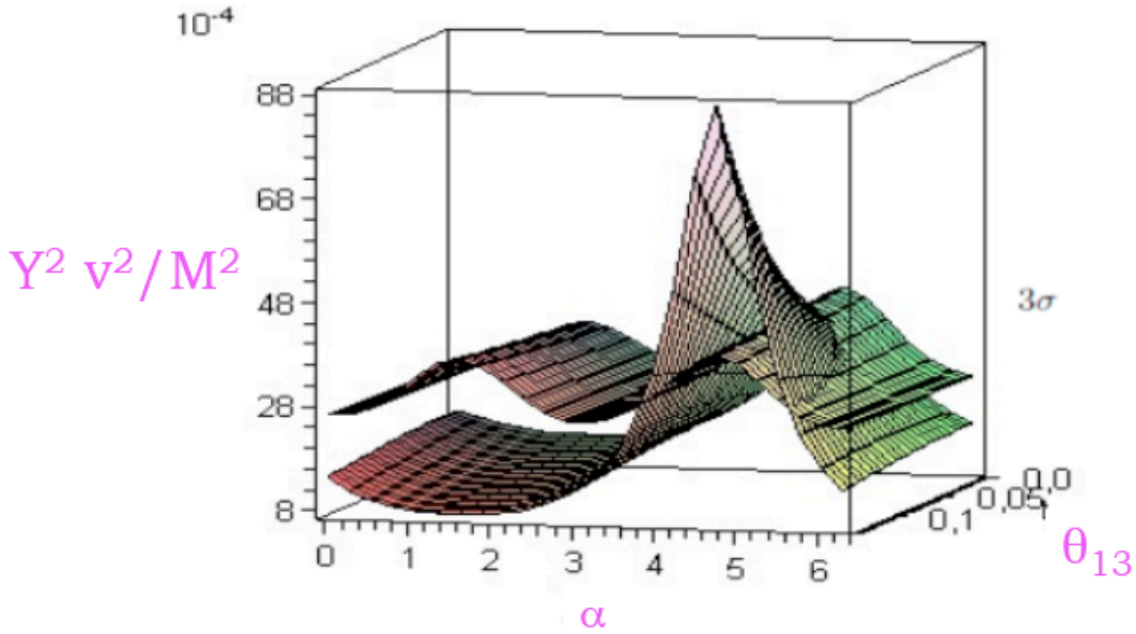
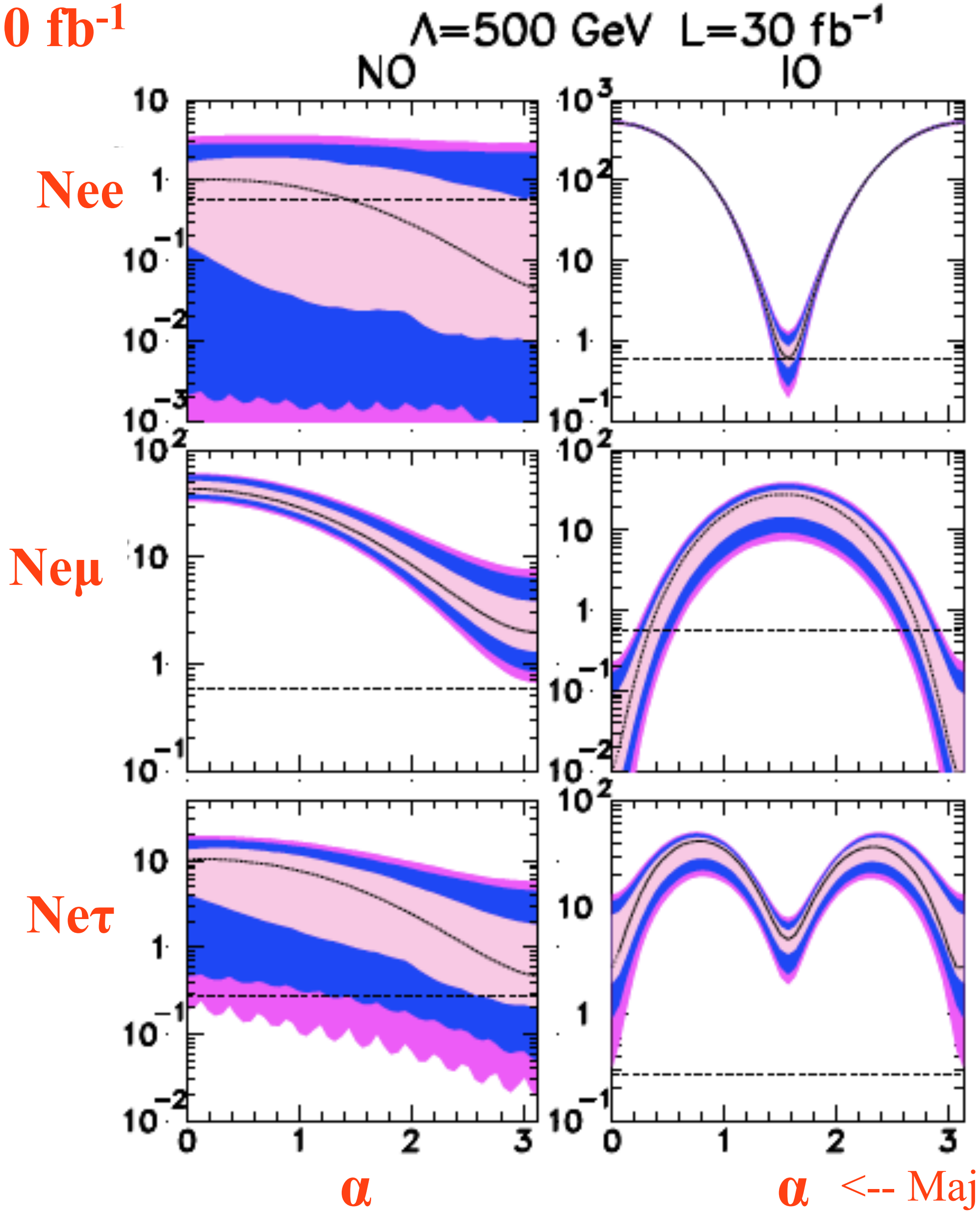


We find that there are regions where an experiment as MINSIS
would improve the present bounds on our Model

For type III version of our 2 N model, signals observable at LHC up to $\Lambda \sim$ 500 GeV for 30 fb^{-1}

Eboli,
Gonzalez-Fraile,
Gonzalez-Garcia

$$pp \rightarrow \ell_a^\pm \ell_b^\mp jjjj$$



<-- Majorana phase

Some good ideas:

“Partial compositeness”:

D.B. Kaplan-Georgi in the 80s proposed a composite Higgs:

* **Higgs light because the whole Higgs doublet is multiplet of goldstone bosons**

They explored **SU(5)--> SO(5)**.

Explicit breaking of SU(2)xU(1) symmetry via external gauged U(1)
(Kaplan, Georgi, Dimopoulos, Banks, Dugan, Galison)

Nowadays **SO(5)--> SO(4)** and explicit breaking via SM weak interaction
(Contino, Nomura, Pomarol; Agashe, Contino, Pomarol; Giudice, Pomarol, Ratazzi, Grojean; Contino, Grojean, Moretti; Azatov, Galloway, Contino...)

SO(6) --> SO(5) to get also DM (Frigerio, Pomarol, Riva, Urbano)

Anarchy: alive with not so small θ_{13} and not θ_{23} not maximal

no symmetry in the lepton sector, just random numbers

$$m_\nu \sim \begin{pmatrix} \sim 1 & \sim 1 & \sim 1 \\ \sim 1 & \sim 1 & \sim 1 \\ \sim 1 & \sim 1 & \sim 1 \end{pmatrix}$$

- Does not relate mixing to spectrum
- Does not address both quarks and leptons

(Hall, Murayama, Weiner; Haba, Murayama; De Gouvea, Murayama...
Going towards hierarchy: Altarelli, Feruglio, Masina, Merlo)

*3 families with $O(2)_{N_R}$:

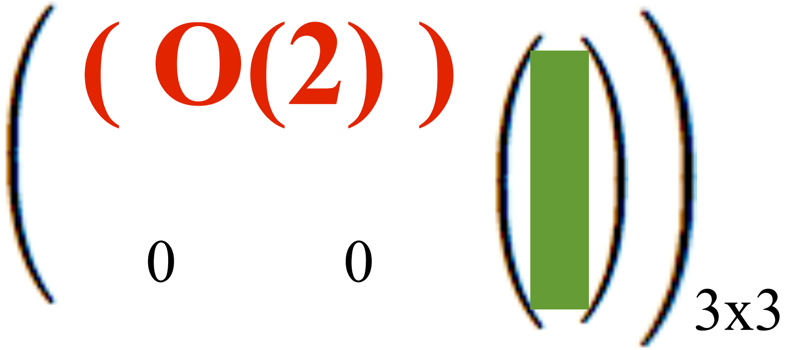
- 3 light + 2 heavy N degenerate: bad θ_{12} quadrant. It cannot accomodate data!

- 3 light + 3 heavy N : **OK for θ_{23} maximal and spectrum**

experimentally $\sin 2\theta_{23} =$ 0.41 +-0.03 or 0.59+-0.02

Gonzalez-Garcia, Maltoni, Salvado, Schwetz Sept. 2012

*What about the other angles?



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Moriond this morning, T2K best fit point $\sin^2 2\theta_{23}=1.00_{-0.068}$ 90%CL -> 45º !

*What about the other angles?

BSM electroweak

* **HIERARCHY PROBLEM**

Fine-tuning issue: if BSM physics, why Higgs so light

Interesting mechanisms to solve it: SUSY,
strong-int. light Higgs, extra-dim….

In practice, none without further fine-tunings

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* **FLAVOUR PUZZLE:** ~no theoretical progress

New B physics data AND neutrino masses and mixings

Understanding of the underlying physics stalled since 30 years. BSM theories tend to make it worse.

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In practice, none without further fine-tunings

→ $\Lambda_{electroweak} \sim$ **1 TeV** ?

* **FLAVOUR PUZZLE :** no progress

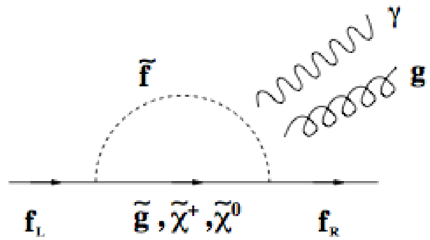
New B physics data AND neutrino masses and mixings

Understanding of the underlying physics stalled since 30 years. BSM theories tend to make it worse.

→ $\Lambda_f \sim$ **100's TeV** ???

The FLAVOUR WALL for BSM

i) Typically, BSMs have **electric dipole moments** at one loop
i.e susy MSSM:

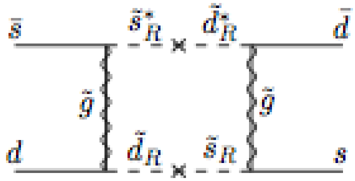


< 1 loop in SM ---> **Best (precision) window of new physics**

ii) **FCNC**

i.e susy MSSM:

$K^0 - \overline{K}^0$ mixing



$\mu \to e\gamma$

competing with SM at one-loop

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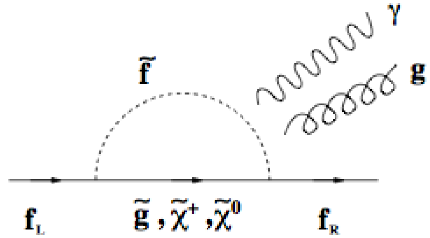

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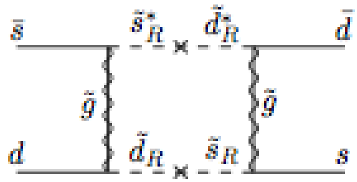

$\mu \to e$ conversion (MEG, μ2e...)

competing with SM at one-loop

What happens if we add non-renormalizable terms to the potential?

In fact one should consider as many invariants as physical variables

seesaw I with **Just TWO heavy neutrinos**

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\,\bar{N}^c\,,\,\bar{N}'^c) \begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

Lepton number scale and flavour scale distinct

Raidal, Strumia, Turszynski
Gavela, Hambye, Hernandez[2]

Just TWO heavy neutrinos

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$$m_\nu = \mathbf{Y} \frac{v^2}{M} \mathbf{Y}'^T \qquad\qquad U_{lN} \sim \frac{\mathbf{Y}}{M}$$

--> Lepton number conserved conserved if either Y or Y' vanish:

Raidal, Strumia, Turszynski
Gavela, Hambye, Hernandez[2]

Just TWO heavy neutrinos

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\, \bar{N}^c\,,\, \bar{N}'^c) \begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

--> One massless neutrino and only one Majorana phase α

the Yukawas are determined up to their overal magnitude

$$\text{N.H.} \qquad Y = \frac{y}{\sqrt{m_{\nu_2} + m_{\nu_3}}} U_{PMNS} \begin{pmatrix} 0 \\ -i\sqrt{m_{\nu_2}}e^{-i\alpha} \\ \sqrt{m_{\nu_3}}e^{i\alpha} \end{pmatrix}$$

Gavela, Hambye, Hernandez[2]
Raidal, Strumia, Turszynski

Comparing the scales reached by

Neutrino Oscillations **vs** **μ-e experiments** **vs** **LHC**

e.g. in Seesaw type I scales (heavy singlet fermions)

* **ν-oscillations:** 10^{-3}eV - $M_{GUT} \sim 10^{15}$ GeV, because interferometry

* **μ-e conversion:** 2MeV - 6000 GeV

* **LHC:** ~ # TeV

The flavour symmetry is $G_f = U(3)_{\ell_L} \times U(3)_{E_R} \times O(2)_N$

adds a new invariant for the lepton sector, in total:

$$\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+) \qquad \mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+)^2$$

$$\mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+) \qquad \mathrm{Tr}\,(\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+)^2$$

$$\mathrm{Tr}\,(\mathcal{Y}_E\,\mathcal{Y}_E^+\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^+) \longleftarrow \textbf{mixing}$$

$$\mathrm{Tr}\,(\mathcal{Y}_\nu\,\sigma_2\,\mathcal{Y}_\nu^+)^2 \;\text{<--}\; \mathbf{O(2)_N}$$

$O(2)_N$ is simply associated to Lepton Number

Leptons

Just TWO heavy neutrinos

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\, \bar{N}^c\,,\, \bar{N}'^c) \begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

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(Alonso, Gavela, D. Hernandez, Merlo, Rigolin)

Leptons

Just TWO heavy neutrinos

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(Alonso, Gavela, D. Hernandez, Merlo, Rigolin)

Jacobian Analysis: Mixing

What is the symmetry in this boundary?

$$Y_\nu = \begin{pmatrix} y_1 & 0 & 0 \\ 0 & \frac{y_2}{\sqrt{2}} & -i\frac{y_2}{\sqrt{2}} \\ 0 & \frac{y_3}{\sqrt{2}} & i\frac{y_3}{\sqrt{2}} \end{pmatrix} \qquad \lambda'_3 Y_\nu - Y_\nu \lambda_7 = 0;\ \lambda'_3 = \mathrm{diag}(0,1,-1)\ ,$$

$$\boxed{U(1)_{diag}}$$

related to the O(2) substructure

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\omega} & 0 \\ 0 & 0 & e^{i\omega} \end{pmatrix} \mathcal{Y}_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{pmatrix}$$

[Alonso, Gavela, D. Hernández, L. Merlo;
[Alonso, Gavela, D. Hernández, L. Merlo, S. Rigolin]

In many BSM the Yukawas do not come from dynamical fields:

Some good ideas:

D.B. Kaplan-Georgi in the 80's proposed a light SM scalar because being a (quasi) goldstone boson: ***composite Higgs***

(D.B. Kaplan, Georgi, Dimopoulos, Banks, Dugan, Galison.......Contino, Nomura, Pomarol; Agashe, Contino, Pomarol; Giudice, Pomarol, Ratazzi, Grojean; Contino, Grojean, Moretti; Azatov, Galloway, Contino... Frigerio, Pomarol, Riva, Urbano...)

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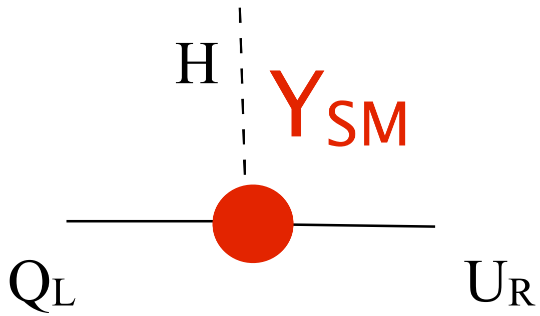
Flavour "Partial compositeness" D.B Kaplan 91:

A sort of **"seesaw for quarks"**

(nowadays sometimes justified from extra-dim physics)



$m_q = v\, Y_{SM}$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

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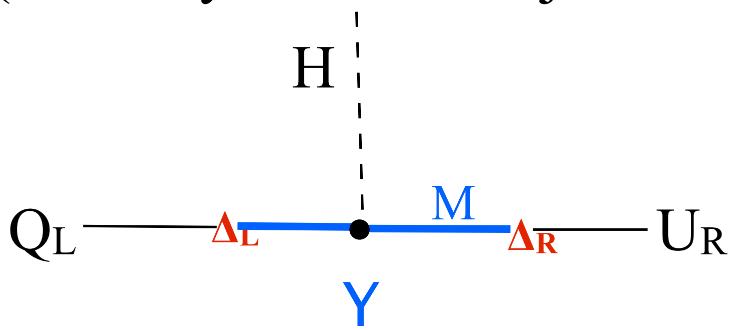

$$Y_{SM} = Y \Delta_L \Delta_R / M^2$$

$$m_q = v\, Y_{SM}$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

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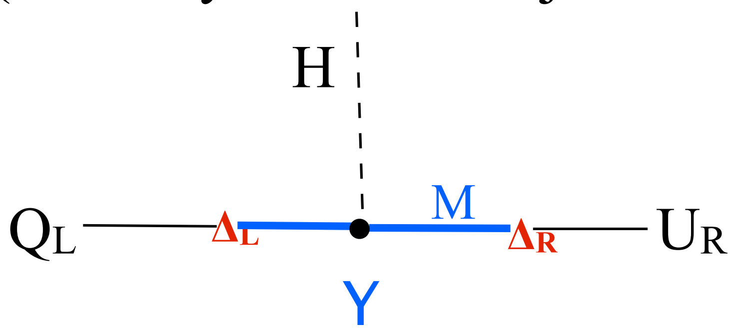
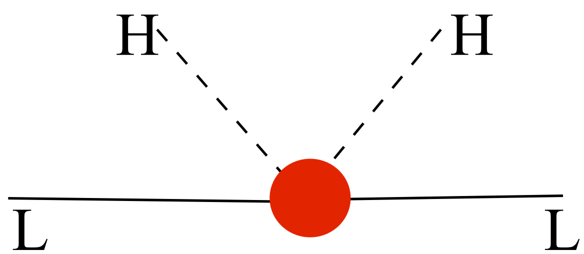


Neutrino masses:



d=5 Weinberg operator

$$Y_{SM} = Y \Delta_L \Delta_R / M^2$$

$$m_q = v\, Y_{SM}$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

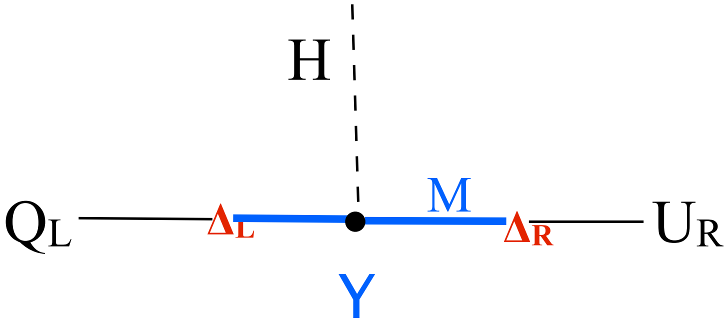
Some good ideas:

D.B. Kaplan-Georgi in the 80's proposed a Higgs light because being a (quasi) goldstone boson: ***composite Higgs***

"Partial compositeness":

A sort of **"seesaw for quarks"**

(nowadays sometimes justified from extra-dim physics)



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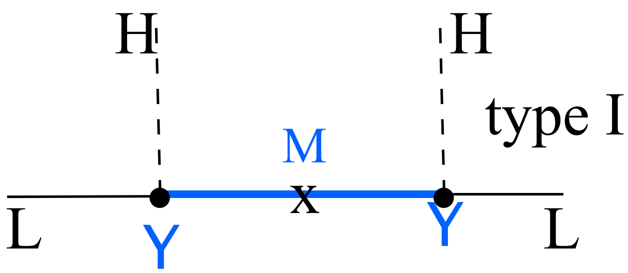
Neutrino masses:



$$m_\nu = Y\, v^2/M\, Y^T$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

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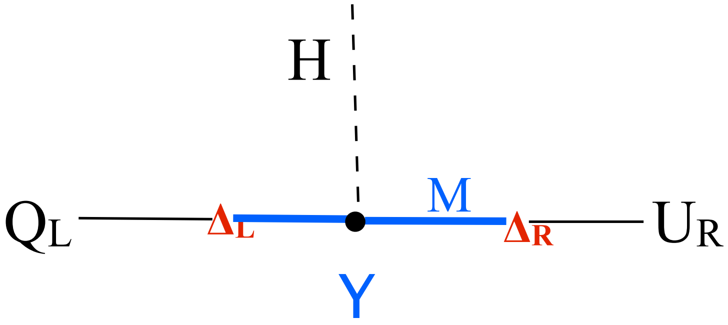

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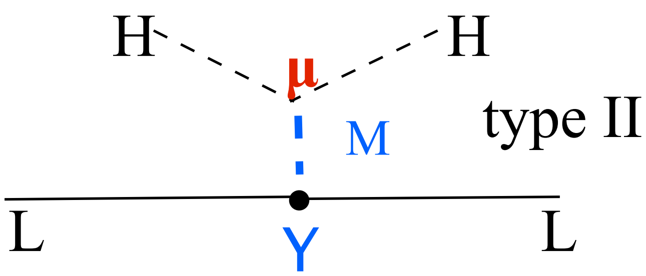


$$m_\nu = Y \mu\, v^2/M^2$$

(D.B Kaplan 91; Redi, Weiler; Contino, Kramer, Son, Sundrum; da Rold, Delauney, Grojean, G. Perez; Contino, Nomura, Pomarol, Agashe, Giudice, Perez, Panico, Redi, Wulzer...)

For instance, in discrete symmetry ideas:

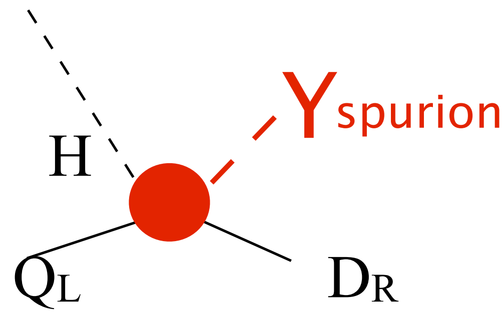
The Yukawas are indeed explained in terms of dynamical fields. And they do not need to worry about goldstone bosons.

In spite of θ_{13} not very small, there is activity.
For instance, combine generalized CP (Bernabeu, Branco, Gronau 80s) with discrete Z_2 groups in the neutrino sector : maximal θ_{23}, strong constraints on values of CP phases
(Feruglio, Hagedorn and Ziegler 2013; Holthausen, Lindner and Schmidt 2013)

They were popular mainly because they can lead easily to large mixings (tribimaximal, bimaximal...)

But:
- Discrete approaches do not relate mixing to spectrum
- Difficulties to consider both quarks and leptons

Some good ideas:



Minimal Flavour Violation:

- Use the flavour symmetry of the SM in the limit of massless fermions (Chivukula+ Georgi)

quarks: $G_{flavour} = U(3)_{QL} \times U(3)_{UR} \times U(3)_{DR}$

Hybrid dynamical-non-dynamical Yukawas:

U(2) (Pomarol, Tomasini; Barbieri, Dvali, Hall, Romanino...)....

$U(2)^3$ (Craig, Green, Katz; Barbieri, Isidori, Jones-Peres, Lodone, Straub.. ..Sala)

Sequential ideas (Feldman, Jung, Mannel; Berezhiani+Nesti; Ferretti et al., Calibbi et al. ...)

For this talk:

each Y_{SM} -- >one single field $\mathcal{Y}$

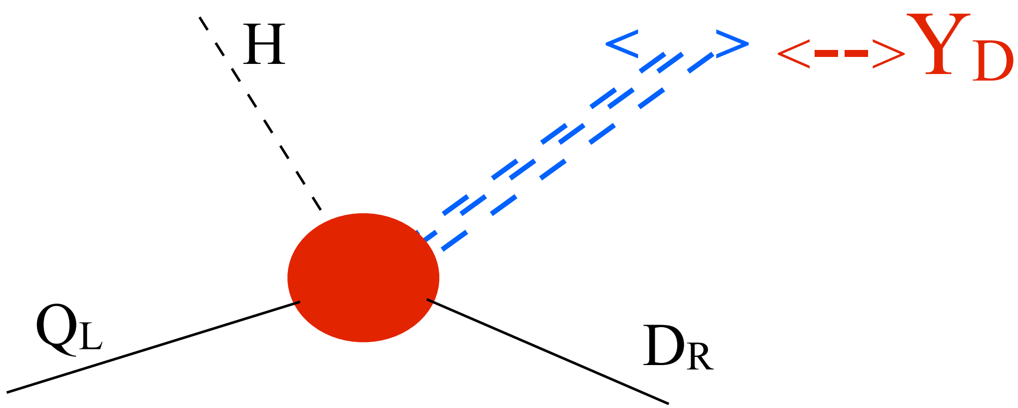
$$Y_{SM} \sim \frac{< \mathcal{Y} >}{\Lambda_{fl}}$$

Can it shed light on why quark and neutrino mixings are so different?

Alonso, B.G., D. Hernandez, L. Merlo, Rigolin

Assume that the Yukawa couplings correspond to dynamical fields at high energies ....

$Y_{SM} \sim < \varphi >$ or $Y_{SM} \sim 1/< \varphi >$ or ...... $< (\varphi\ \chi)^n >$



[Cabibbo,
Michel,+Radicati, Cabibbo+Maiani ...
C. D. Froggat, H. B. Nielsen
Anslem+Berezhiani, Berezhiani+Rossi]

(Alonso+Gavela+Merlo+Rigolin 11) ...

For this talk:

each Y_{SM} -- >one single field $\mathcal{Y}$

$$Y_{SM} \sim \frac{<\mathcal{Y}>}{\Lambda_f}$$

transforming under the SM flavour group

Anselm+Berezhiani 96; Berezhiani+Rossi 01... Alonso+Gavela+Merlo+Rigolin 11...

Generalization to any seesaw model

the effective Weinberg Operator

$$\bar{\ell}_L \tilde{H} \frac{Y_\nu Y_\nu^T}{M} \tilde{H}^T \ell_L^c$$

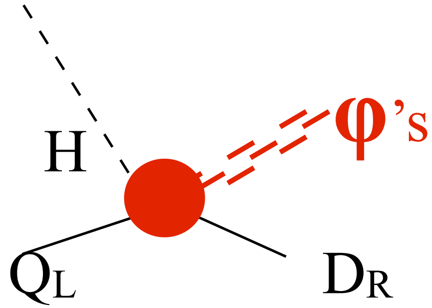
shall have a flavour structure that breaks $U(3)_L$ to O(3)

$$\frac{Y_\nu v^2 Y_\nu^T}{M} = \frac{y_\nu v^2}{M} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

then the results apply to any seesaw model

This did not need any ad-hoc discrete symmetries, but simply using the in-built continuous flavour symmetry of the SM + seesaw, $U(3)^5 \times O(3)$

Also, note that often people working with “flavons” invents a “texture” that goes well with data, and then tries to design a potential that leads to it. In our case, the inevitable potential minima encompass the different patterns of quarks and leptons.



Some good ideas, based on continuous symmetries:

Frogatt-Nielsen '79: $U(1)_{flavour}$ symmetry

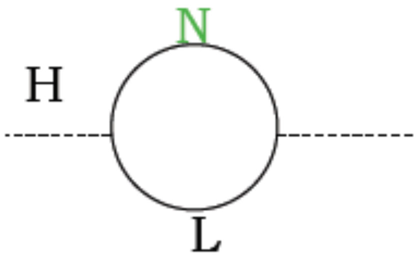
- Yukawa couplings are effective couplings,
- Fermions have $U(1)_{flavour}$ charges

$$\left(\frac{<\varphi>}{\Lambda}\right)^n Q\, H\, q_R \quad , \quad Y \sim \left(\frac{<\varphi>}{\Lambda}\right)^n$$

e.g. n=0 for the top, n large for light quarks, etc.

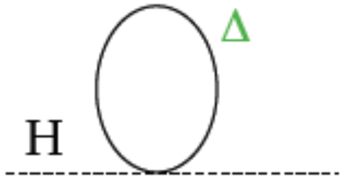
--> FCNC ?

M~1 TeV is suggested by electroweak hierarchy problem



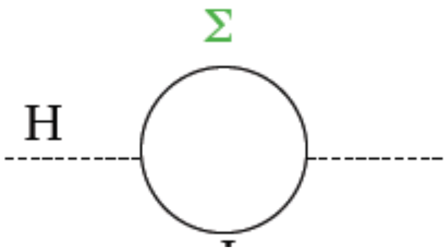
$$\delta m_H^2 = -\frac{Y_N^\dagger Y_N}{16\pi^2}\left[2\Lambda^2 + 2M_N^2 \log\frac{M_N^2}{\Lambda^2}\right]$$

(Vissani, Casas et al., Schmaltz)



$$\delta m_H^2 = -3\frac{\lambda_3}{16\pi^2}\left[\Lambda^2 + M_\Delta^2\left(\log\frac{M_\Delta^2}{\Lambda^2} - 1\right)\right] - \frac{\mu_\Delta^2}{2\pi^2}\log\left(\left|\frac{M_\Delta^2 - \Lambda^2}{M_\Delta^2}\right|\right)$$

(Abada, Biggio, Bonnet, Hambye, M.B.G.)



$$\delta m_H{}^2 = -3\frac{Y_\Sigma^\dagger Y_\Sigma}{16\pi^2}\left[2\Lambda^2 + 2M_\Sigma^2 \log\frac{M_\Sigma^2}{\Lambda^2}\right]$$

In some BSM theories, Yukawas do correspond to dynamical fields:

- for instance in discrete symmetry scenarios

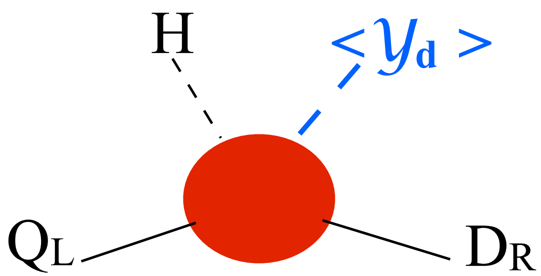
- also with continuous symmetries

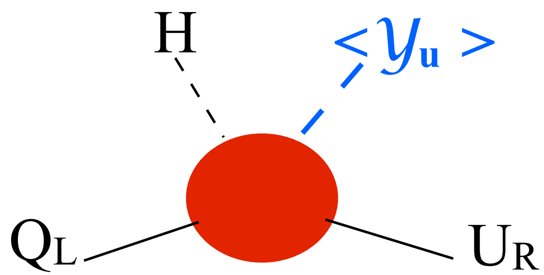
For this talk:

each Y_{SM} -- > one single field $\mathcal{Y}$

$$Y_{SM} \sim \frac{<\mathcal{Y}>}{\Lambda_f}$$

quarks:





Before the electroweak model, for masses: Cabibbo + Louis Michel and Radicati, Cabibbo and Maiani !!!

$$G_{flavour} = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \; ....$$