SUSY Naturalness and its limits.

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arXiv:1302.5262, 1304.1193, 1208.0837, 1203.0569,

in collaboration with Graham Ross, Hyun Minn Lee, Myeonghun Park.

- Outline

I. Naturalness: review & questions.

II. Naturalness without prejudice: likelihood and emergent fine-tuning.

III. CMSSM, NUGM, NUHM, NMSSM, GNMSSM,....

IV. Conclusions.

(I). Naturalness: review & questions

L. Susskind PRD 20 (1979), 2619

• why SM EW scale $v \sim M_Z \ll M_P$ stable under quantum corrections. $\left[\frac{G_f h^2}{G_N c^2} = 1.7\times 10^{33}\right]$

$\delta v^2 \sim \delta m_h^2 \sim f(\alpha_j)\,\Lambda^2$, if $\Lambda \sim M_P$: tune coupling: $1:10^{33}\,(!)$.

Hierarchy Problem $\Leftrightarrow$ Fine tuning. Unnatural. Ways out?

A physical parameter $\rho(\mu)$ is naturally v.small if $\rho(\mu)=0$ increases the symmetry.

"Naturalness dogma": 't Hooft (1979)

- Solutions:

1. Scale (conformal) symmetry. see for example Bardeen 1995; talk by M. Lindner

2. SUSY: $\delta m_h^2 \sim m_S^2 \ln \Lambda/m_S$, $m_S \sim$TeV... no SUSY seen, $m_S \gg \mathrm{TeV}$ $\rightarrow$ back to SM fine-tuning

.....why worry? worse (unrelated?) fine tunings: cosmological const. $\left[\frac{\rho_v}{\rho} \approx \frac{(2.3\times 10^{-12}\mathrm{GeV})^4}{(10^{19}\,\mathrm{GeV})^4}\right]$

.....because softly broken TeV-scale SUSY solves h.p.

3. Forget it. Use an EFT approach, enforce consistency at quantum level (given loop order).

• SUSY models:

$$\begin{aligned} V &= m_1^2\,|H_1|^2 + m_2^2\,|H_2|^2 - (B_0\,\mu_0\,H_1\cdot H_2 + h.c.) + \lambda_1/2\,|H_1|^4 + \lambda_2/2\,|H_2|^4 + \lambda_3\,|H_1|^2\,|H_2|^2 \\ &+ \lambda_4\,|H_1\cdot H_2|^2 + \Big[\lambda_5/2\,(H_1\cdot H_2)^2 + \lambda_6\,|H_1|^2\,(H_1\cdot H_2) + \lambda_7\,|H_2|^2\,(H_1\cdot H_2) + h.c.\Big] \end{aligned}$$

$$m^2 \equiv m_1^2\cos^2\beta + m_2^2\sin^2\beta - B_0\,\mu_0\,\sin 2\beta, \quad \text{UV}: m_{1,2}^2 = m_0^2 + \mu_0^2$$

$$\lambda \equiv \frac{\lambda_1}{2}\cos^4\beta + \frac{\lambda_2}{2}\sin^4\beta + \frac{\lambda_{345}}{4}\sin^2 2\beta + \sin 2\beta\,\big(\lambda_6\cos^2\beta + \lambda_7\sin^2\beta\big)$$

• The Problem: scales vs. couplings "tension" ("fine-tuning"):

$$v^2 = -m^2/\lambda, \ \text{with}\ v = \mathcal{O}(100\ \text{GeV}), \quad \lambda < 1, \quad \textbf{but} \quad m_{1,2}, B_0 \ \text{and}\ m \sim \mathcal{O}(1\,\text{TeV}).$$

- $m_h > m_Z$ ↔ large loop effects ↔ large $m_{1,2}, B_0$...; also a problem of couplings (λ small)

⇒ Solution: - increase λ by 1.- quantum corrections

2.- (susy) corrections from "new physics" beyond MSSM.

- live with fine-tuning? (cl. nonlinear systems); fundamental concept or UV ignorance?

- Questions.....

$$\Delta_{max} \equiv \max_{\gamma} \big|\Delta_\gamma\big|, \qquad \Delta_\gamma \equiv \frac{\partial \ln v^2}{\partial \ln \gamma^2}, \quad \gamma = \{m_0, m_{1/2}, \mu_0, A_0, B_0\}, \quad v = \text{EW scale}$$

Ellis, Enqvist, Nanopoulos, Zwirner (1986)

Barbieri, Giudice (1988)

a). based on physical intuition. Mathematical support?

b). "small" Δ wanted (?); what is "small"? ≤ 20? 1000?

c). what definition? $\Delta_q = \Big[\sum_\gamma \Delta_\gamma^2\Big]^{1/2}$; also should $\gamma \supset y_t, y_b$?

see: Anderson, Castano, hep-ph/9409419

Dreiner et al arXiv:1204.4199

Baer et al 2012.

d). different definitions $\rightarrow$ different results?

e). Take: 2 models: A: v.good χ^2/ndf but large $\Delta = 1000$ $\rightarrow$ model v. likely but...unnatural!

B: good χ^2/ndf but small $\Delta = 10$. $\rightarrow$ model less likely, but ...natural!

$\chi^2 = \sum_i (O_i(\gamma) - O_i^{exp})^2/\sigma_i^2$, χ^2/ndf≈ 1. Then what model is better?

f). Is the fine-tuning "cost" to fix m_Z related to χ^2 "cost" to fit m_Z regarded as observable?

$\Rightarrow \Delta$: tune γ_i to fix EW scale $v \sim m_Z$ (same for observables); $\Rightarrow \Delta$, likelihood related!?

• A closer look: Lagrangian $\mathcal{L}$ of UV parameters $\gamma_i : m_0, \mu_0, A_0, B_0, m_{1/2}, \cdots$.

$$\frac{(g_1^2+g_2^2)\,v^2}{8} = -\mu^2 + \frac{m_1^2 - m_2^2\,\tan^2\beta}{\tan^2\beta - 1} + \ldots,$$
$$2\,m_3^2 = (m_1^2 + m_2^2 + 2\mu^2)\,\sin 2\beta + \cdots,$$

so $v = v(\gamma,\beta)$, $\beta = \beta_0(\gamma) \Rightarrow \quad m_Z = m_Z(\gamma, \beta_0(\gamma))$. Taylor expansion

$$m_Z = m_Z^0 + \left(\frac{\partial m_Z}{\partial \gamma_i}\right)_{\gamma_i=\gamma_i^0}(\gamma_i - \gamma_i^0) + \cdots, \qquad m_Z^0 = 91.187\,\mathrm{GeV}$$

$$\Rightarrow \frac{\delta m_Z}{m_Z^0} = \Delta_q\, n^i\, \frac{\delta\gamma_i}{\gamma_i^0} + \mathcal{O}((\delta\gamma_i)^2), \qquad \vec{n} \text{ normal to } m_Z(\gamma_i^0, \beta_0(\gamma_i^0)) = m_Z^0.$$

$$\text{with notation} \quad \Delta_q \equiv \left\{ \sum_i \left(\frac{\partial \ln m_Z}{\partial \ln \gamma_i}\right)^2_{\gamma_i=\gamma_i^0} \right\}^{1/2}$$

$$\frac{\delta m_Z}{m_Z^0} = 4.6 \times 10^{-5} \;\; (2\sigma), \text{if } \; \Delta_q \approx 1000 \;\to \frac{\delta\gamma_i}{\gamma_i^0} \approx 4.6 \times 10^{-8} \to \gamma_i = 1\,\mathrm{TeV} \to \delta\gamma_i = 46\,\mathrm{keV}!$$

(II). Naturalness without prejudice: likelihood and emergent fine-tuning.

D.G., G. Ross, arXiv:1208:0837, arXiv:1304.1193, arXiv:1302.5262

- Max of likelihood to fit EW observables O_j (γ_i UV parameters), factorization:

$$L(\text{data}|\gamma_i; v, \beta) = \prod_{j\geq 1} L(O_j|\gamma_i; v, \beta), \qquad \gamma_i = m_0, m_{1/2}, A_0, B_0, \mu_0, ...(\text{susy}) \;\; (y_t, y_b, ...)$$

- Also regard m_Z as an observable, with $m_Z^0 = 91.187$ GeV.

$$f_1(\gamma_i, v, \beta) = v - (-m^2/\lambda)^{1/2} = 0, \qquad f_2(\gamma_i, v, \beta) = \tan\beta - \tan\beta_0(\gamma_i) = 0$$

$$L(\text{data}, m_Z|\gamma_i) = \int dv\; d(\tan\beta)\; \delta\left(f_1(\gamma_i; v, \beta)\right)\; \delta\left(f_2(\gamma_i; v, \beta)\right)\; L(\text{data}|\gamma_i; v, \beta)\; \delta(1 - m_Z/m_Z^0)$$

$$= v_0 \left[L\big(\text{data}|\gamma_i;\, v_0,\, \beta\big)\; \delta\big[f_1\big(\gamma_i;\, v_0,\, \beta\big)\big]\right]_{\beta=\beta_0(\gamma_i)}$$

$$= \; L\big(\text{data}|\gamma_i;\, v_0,\, \beta_0(\gamma_i)\big)\; \delta\Big[1 - \frac{m_Z(\gamma_i, \beta_0(\gamma_i))}{m_Z^0}\Big],$$

$$v_0 = 246 \text{ GeV}$$
$$m_Z = gv/2$$
$$m_Z^0 = gv_0/2$$
$$g^2 = g_1^2 + g_2^2$$

- Note: factorization assumes no correlations (not true in general)!

If all γ_i fixed, except, say, μ: D.G., G.G. Ross, arXiv:1208:0837.

$$\delta\big(1 - m_Z(\mu)/m_Z^0\big) = \Big\vert\frac{\partial \ln m_Z(\mu)}{\partial \ln \mu}\Big\vert^{-1}_{\mu=\mu_0} \delta\big(1-\mu/\mu_0\big),$$

If all γ_i vary:

$$\delta\Big[1 - \frac{m_Z(\gamma_i, \beta_0(\gamma_i))}{m_Z^0}\Big] = \frac{1}{\Delta_q}\,\delta\Big[n^j\,(1 - \frac{\gamma_j}{\gamma_j^0})\Big], \qquad \Delta_q \equiv \Big[\sum_\alpha \Big(\frac{\partial \ln m_Z(\gamma_i, \beta_0(\gamma_i))}{\partial \ln \gamma_i}\Big)\Big]^{1/2}_{\gamma_i=\gamma_i^0},$$

$$\Rightarrow \quad L(\text{data}, m_Z|\gamma_i^0) = \frac{1}{\Delta_q}\, L(\text{data}|\gamma_i; v_0, \beta_0(\gamma_i))\Big\vert_{\gamma_i=\gamma_i^0}$$

$\Rightarrow$ Emergent fine-tuning Δ_q, as part of total likelihood (not put in by hand!). $\Rightarrow$ answers a), c), f).

$\Rightarrow$ maximize the ratio in the rhs!

$\Rightarrow$ answers e): how to compare (likely but unnatural) and (unlikely but natural) models!

Bayesian case: see Casas et al 2008, Allanach et al 2007, 2009, Fichet 2012, D.G., H.M. Lee, M. Park 2012

$$\delta(f(\vec{z})) = \frac{1}{|\nabla_z f|_o}\,\delta\Big[\vec{n}.(\vec{z} - \vec{z}^0)\Big], \quad n_i = \frac{\partial_{z_i} f}{|\nabla f|_o}$$

With $\chi^2 = -2\ln L$: D.G., G.G. Ross, arXiv:1208:0837.

$$\chi^2_{\rm z}(\gamma_i) = \Big[\; \chi^2(\gamma_i) + \underbrace{2\ln\Delta_q(\gamma_i)}_{\delta\chi^2}\;\Big]_{f_1=0, f_2=0}$$

$\Rightarrow$ total $\chi^2_{\rm z}$/ndf≈ 1. $\Rightarrow$ Naturalness bound/limit: $\Delta_q < \exp({\rm ndf}/2) \sim 100.$ (answers b)

$\Rightarrow$ Possible corrections: from correlations among observables and/or γ_i in UV (symmetries, etc)

Correlations effects: $\Delta_q \rightarrow \big[\det M \det(\mathcal{J}^T M^{-1}\mathcal{J})\big]^{1/2}$, $\mathcal{J}_{ij} = \frac{\partial \ln O_i}{\partial \ln \gamma_j}$, M cov. matrix. D.G. et al.

$\Rightarrow$ Implications for SUSY models:

$$\Delta_q \approx 10 \Rightarrow \delta\chi^2/{\rm ndf} \approx 4.6/9$$
$$\Delta_q \approx 100 \Rightarrow \delta\chi^2/{\rm ndf} \approx 9.2/9$$
$$\Delta_q \approx 500 \Rightarrow \delta\chi^2/{\rm ndf} \approx 12.5/9$$

• what is the value of Δ_q in SUSY models?

(III). Numerical results for Δ in SUSY models:

D.G., H. M. Lee, M. Park, arXiv:1203:0569

1) CMSSM (constrained MSSM) $\gamma = \{m_0, \mu_0, m_{1/2}, A_0, B_0\}$
2) NUHM1 (non universal Higgs Mass) $\gamma = \{m_0, \mu_0, m_{H_1}^{uv} = m_{H_2}^{uv}, m_{1/2}, A_0, B_0\}$
3) NUHM2 (non universal Higgs Mass) $\gamma = \{m_0, \mu_0, m_{H_1}^{uv}, m_{H_2}^{uv}, m_{1/2}, A_0, B_0\}$
4) NUGM (non universal gaugino masses) $\gamma = \{m_0, \mu_0, m_{\lambda_{1,2,3}}, A_0, B_0\}$

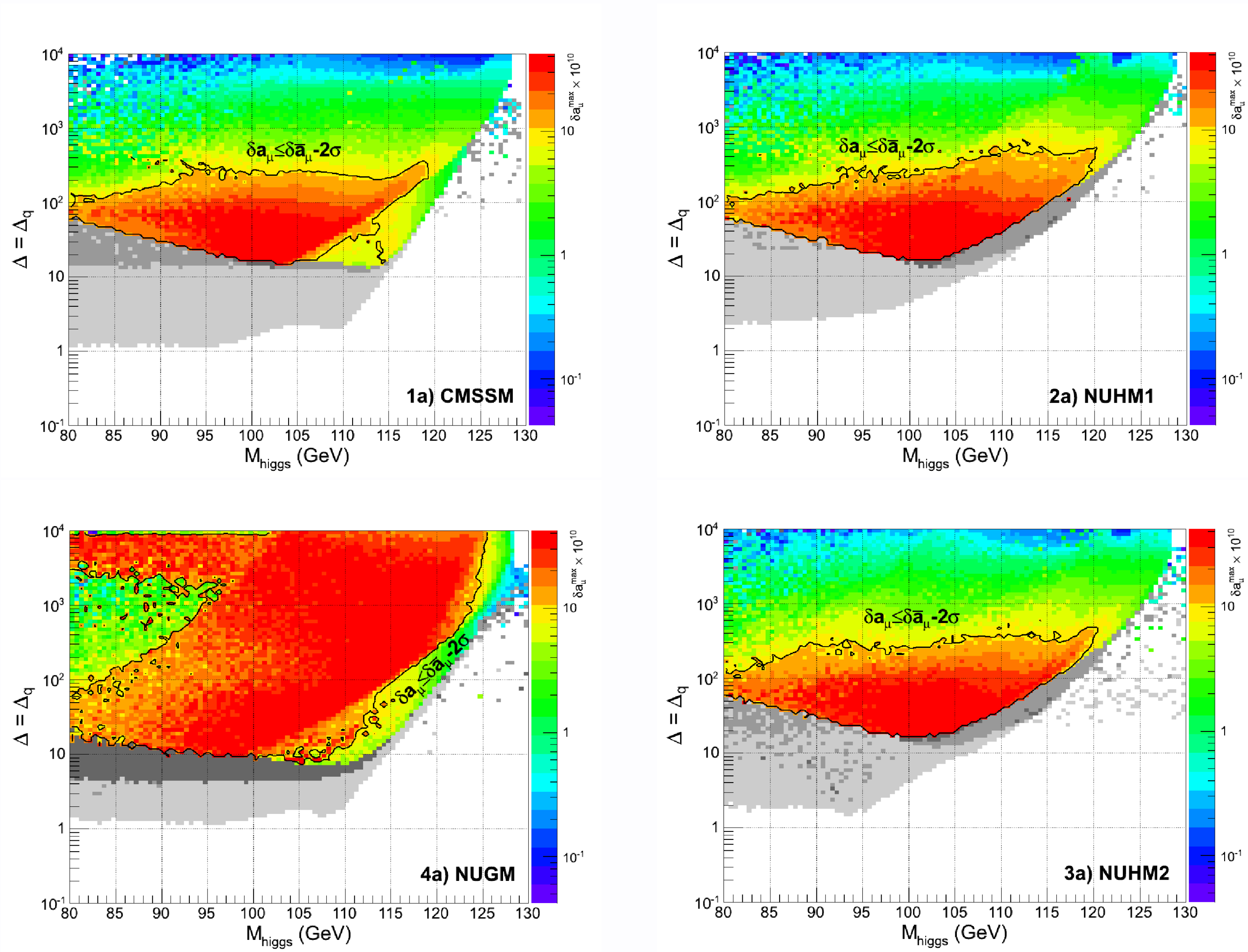
- Experimental Constraints:

- SUSY masses: micrOmegas 2.4.5, "MSSM/masslim.c"
- muon magnetic moment: $\delta a_\mu = (25.5 \pm 2 \times 8) \times 10^{-10}$ at 2σ
- $b \to s\,\gamma$ $3.03 < 10^4$ Br$(b \to s\gamma) < 4.07$ at 2σ
- $B_s \to \mu^+\mu^-$ Br$(B_s \to \mu^+\mu^-) < 1.08 \times 10^{-8}$ at 2σ
- ρ-parameter $-0.0007 < \delta\rho < 0.0033$ at 2σ
- dark matter $\Omega h^2 = 0.1099 \pm 3 \times 0.0062$ at 3σ
- ATLAS/CMS: $m_h \approx 126$ GeV and δa_μ not imposed.

- Tools: micrOMEGAs 2.4.5, SoftSUSY 3.2.4. Random scan. $\Rightarrow \Delta_q$, Δ_{max} at 2-loop LL.

- Impact of m_h, δa_μ on Δ_q. [2-loop, all $\{\gamma, \tan\beta\}$ all values]

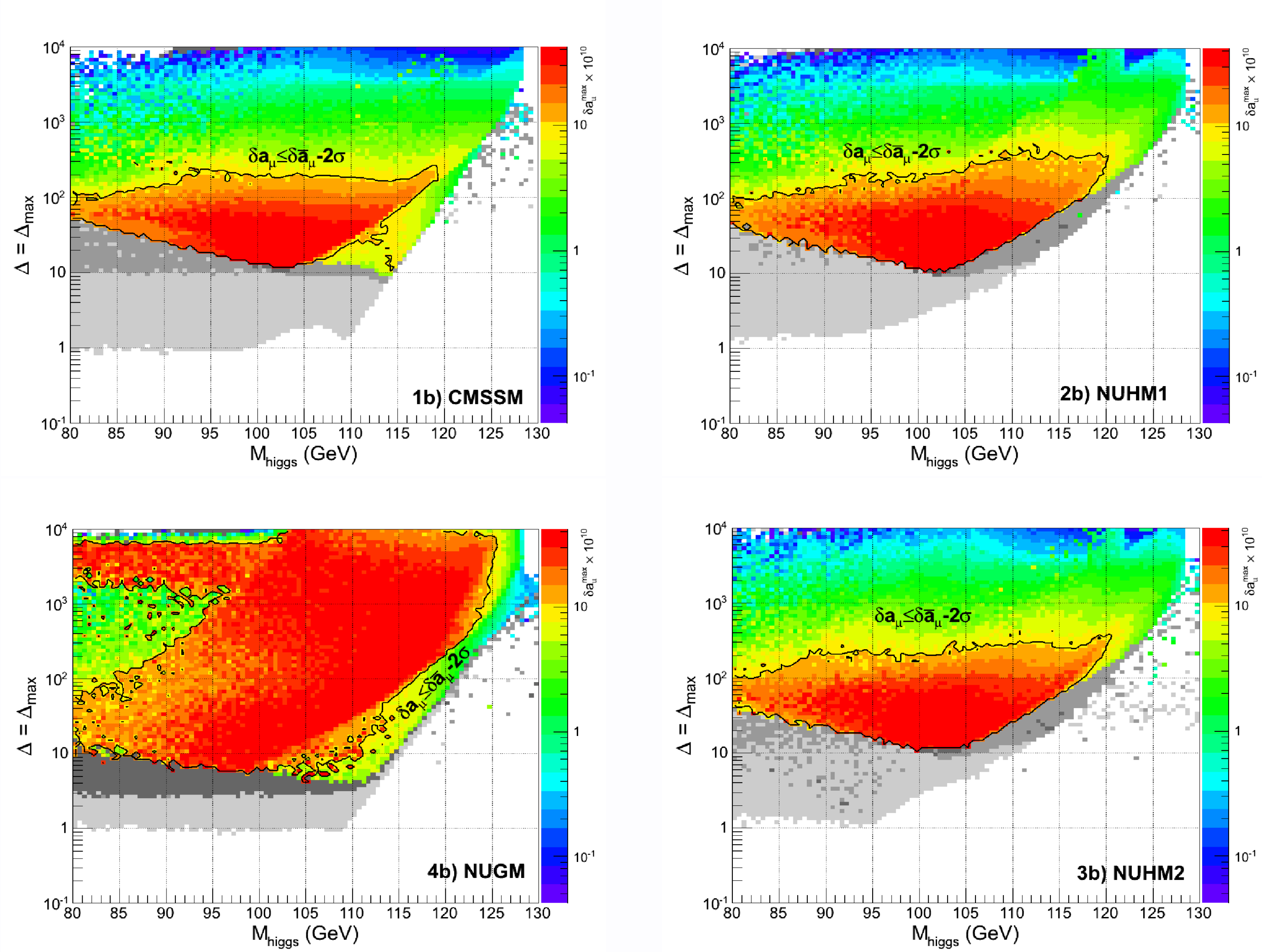
D.G., H. M. Lee, M. Park, arXiv:1203:0569



- grey 0: excluded by SUSY; grey 1: $b \to s\gamma$, $B_s \to \mu^+\mu^-$, $\delta\rho$; grey 2: excluded by $\delta a_\mu > 0$.

$\Rightarrow$ m_h strongest impact: $\Delta_q \sim \exp(m_h)$. $\Delta_q \sim 1000$ (!) near 125 GeV. NUGM better behaviour.

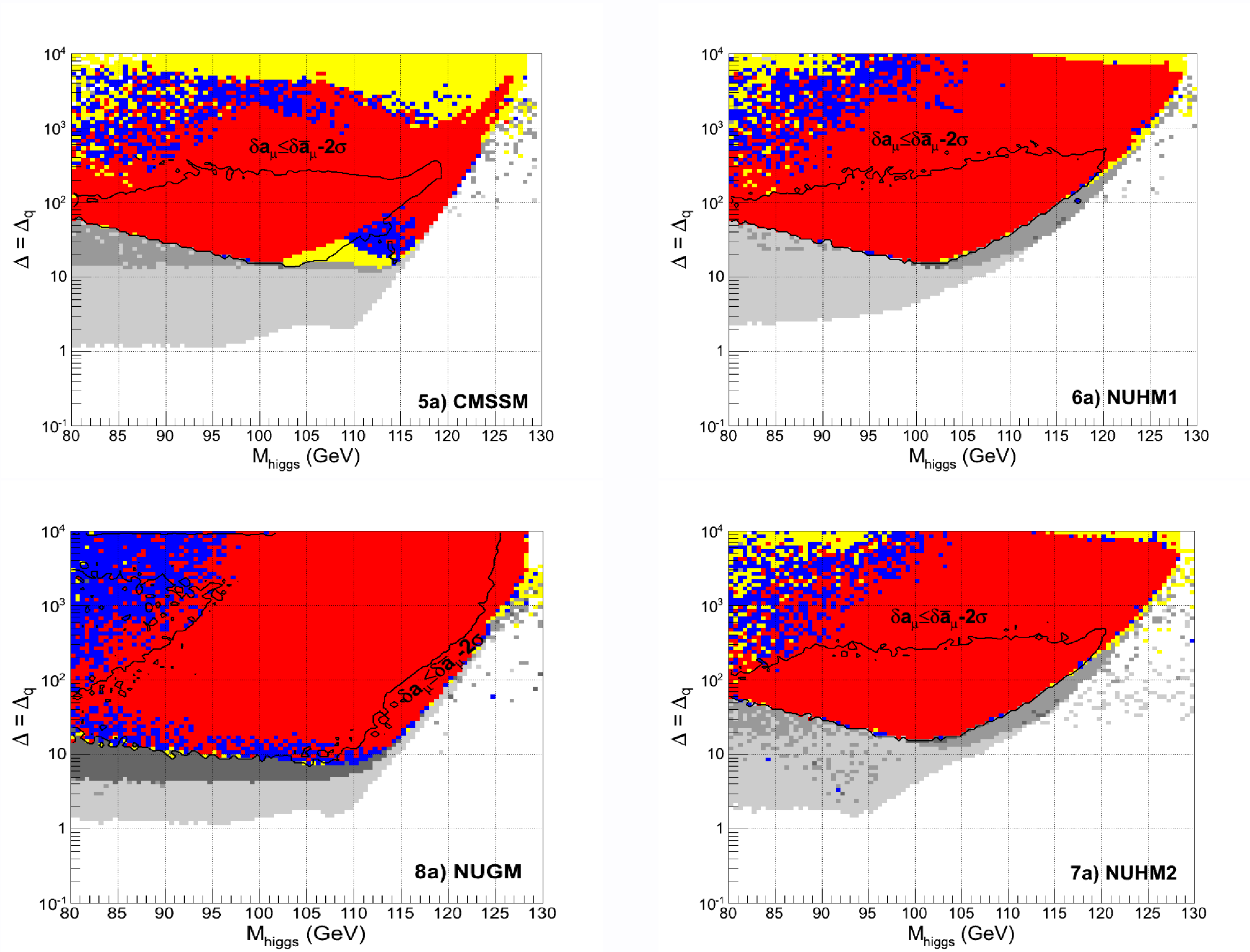
• Impact of m_h, δa_μ on Δ_{max}. [2-loop, all $\{\gamma, \tan\beta\}$ all values]. D.G., H. M. Lee, M. Park, arXiv:1203:0569



• δa_μ: 2σ contour (red) [smaller δa_μ outside]. $\Rightarrow$ Δ_{max} identical behaviour to Δ_q (marginally smaller)

answers d)

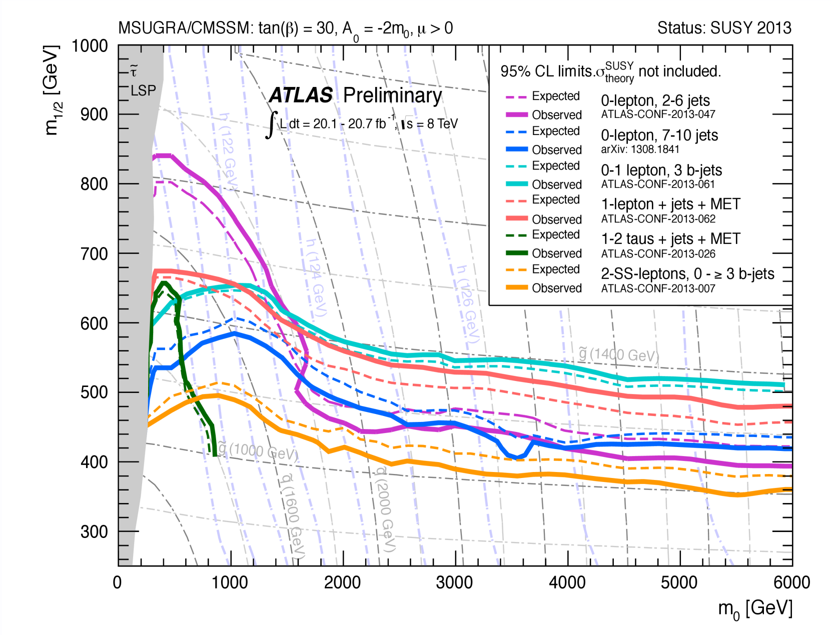
• Impact of m_h, dark matter on Δ_q: [2-loop, $\{\gamma, \tan\beta\}$ all values]. D.G., H. M. Lee, M. Park, arXiv:1203:0569

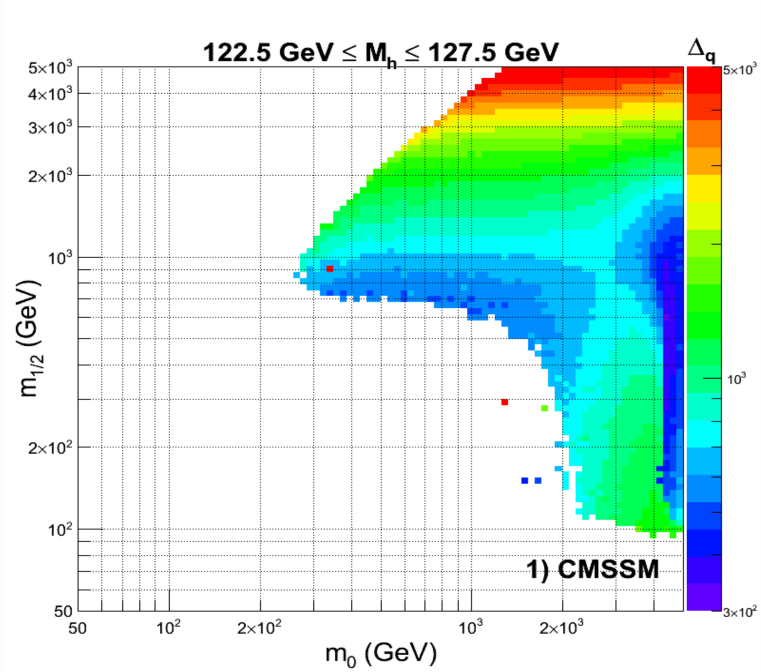


• blue: consistent with Ωh^2; Red: saturate Ωh^2 within 3σ. Grey areas ruled out by data.

$\Rightarrow \Delta_{max}$ (not shown) nearly identical plots to Δ_q, (marginally smaller for same Higgs mass).

- SUSY searches from ATLAS and impact on parameter space (CMSSM only):





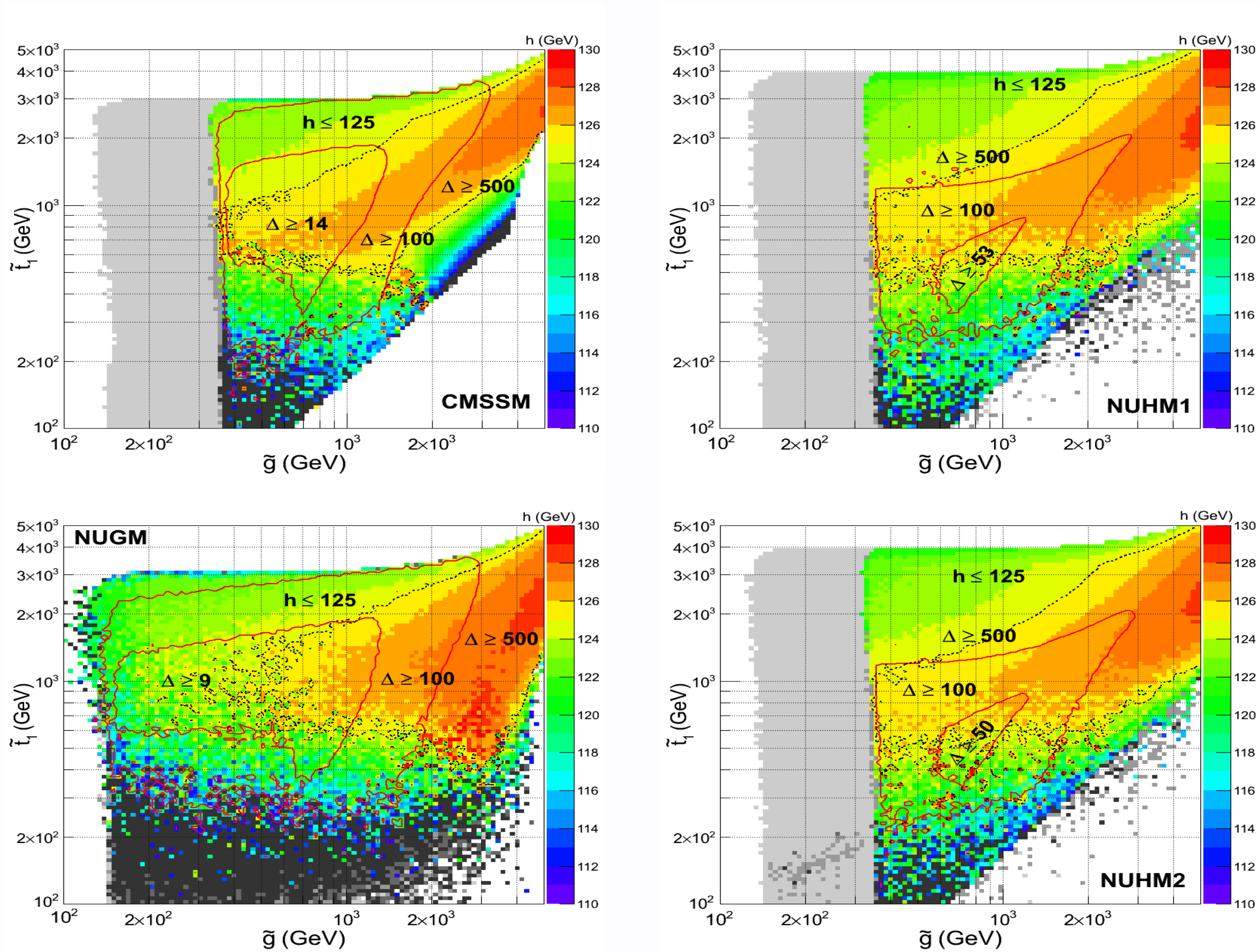
ATLAS-2013

CMSSM, $122.5 \leq m_h \leq 127.5$ GeV and Δ_q.

[all values for γ, $\tan\beta$]. $\Delta_q > 500$.

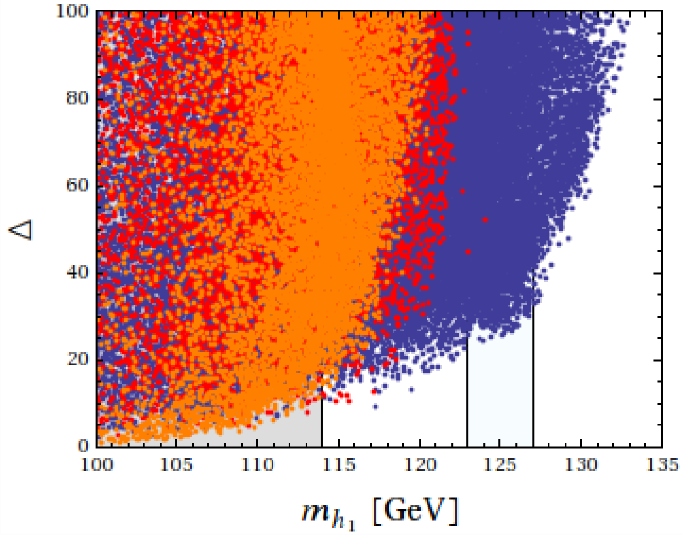
D.G., Hyun Min Lee, Myeonghun Park (2012)

• Stop vs Gluino with largest m_h and min Δ_q. [$\{\gamma, \tan\beta\}$ all values] D.G., H. M. Lee, M. Park, arXiv:1203:0569



⇒ constraints on m_h strongly reduce the viable regions.

Other models (beyond MSSM): NMSSM, GNMSSM, ...



MSSM

NMSSM: $W = W_Y + \lambda S H_1 H_2 + \kappa S^3, \qquad \Delta \sim 200.$

GNMSSM: $W = W_{NMSSM} + m_{3/2}^2 S + m_{3/2} S^2 + m_{3/2} H_1 H_2.$

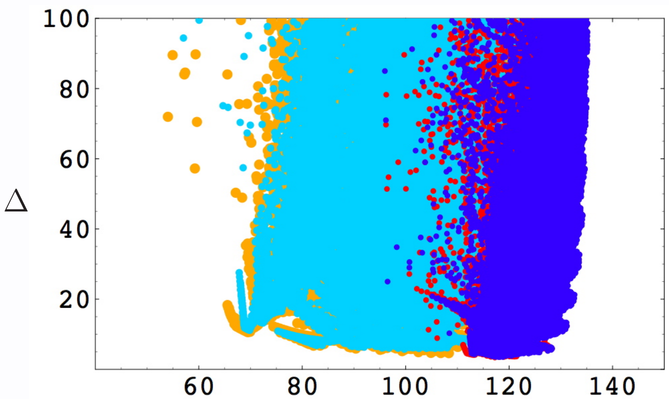
from underlying Z_4^R symmetry.

$W = W_Y + (\mu + \lambda S)\, H_1 H_2 + M_*\, S^2 + \kappa S^3$

$\Delta < 30$ for $m_h \leq 126\, GeV.$

G.G. Ross et al, arXiv:1205.1509, 1102.3595

U. Ellwanger et al arXiv:1107.2472



m_{h_1} [GeV]

GNMSSM: limit of decoupling S: MSSM + (d=5 operator)

$$W = W_Y + (\mu + \lambda S)\, H_1 H_2 + M_*\, S^2$$
$$\Rightarrow W = W_Y + \mu\, H_1 H_2 + \zeta\, (H_1 H_2)^2, \quad \zeta \sim \lambda / M_*$$
$$\Delta < 30 \text{ for } m_h \leq 126\, GeV.$$

S. Cassel, D.G., G.G. Ross, NPB 825(2010)

$\Rightarrow$ massive singlet $\Rightarrow$ reduces Δ considerably. usually: $\Delta_q(m_h) \approx \exp(-\delta m_h/\mathrm{GeV})\, \Delta_q(m_h)|_{CMSSM}$

(IV) Conclusions:

$\Rightarrow$ Δ_q emerges from the total likelihood to fit the data (including m_Z) which thus accounts for naturalness. Δ_q not a fundamental concept?.

$\Rightarrow$ large $L(\text{data}, m_Z|\gamma) = L(\text{data}|\gamma)/\Delta_q$ required (or small $\chi_z^2 = \chi^2 + 2\ln\Delta_q$). total $\chi_{\rm z}^2$/ndf≈ 1 model independent bound: $\Delta_q < \exp(\text{ndf}/2) \sim 100$.

$\Rightarrow$ not considered (!): corrections from correlations of observables or UV relations among γ_i

Numerical results:

$\Rightarrow$ Δ_q in CMSSM, NUHM1,2, NUGM: of all data, m_h strongest constraint.

$\Rightarrow$ Δ_q, $\Delta_{max} \sim \exp(m_h/\text{GeV}) \sim 500 - 1000$ in these models, for $m_h \approx 126$ GeV.

$\Rightarrow$ GNMSSM $\Delta_q = \mathcal{O}(10)$.

Δ_q can be reduced to $\mathcal{O}(10)$ in MSSM + singlet, U(1)$'$, etc (using MSSM+effective operators).

- Naturalness in the Bayesian approach. $p(a|b)\, p(b) = p(a \cap b) = p(b|a)\, p(a)$

Bayes theorem: [initial belief + data $\rightarrow$ updated belief].

Thomas Bayes (1761), Laplace (1812)

$$p(\gamma|\text{data}) = \frac{L(\text{data}|\gamma)\ p(\gamma)}{p(\text{data})}, \qquad p(\text{data}) = \int L(\text{data}|\gamma)\, p(\gamma) d\gamma, \qquad \gamma : \{m_0, m_{1/2}, \mu_0, A_0, B_0; y_t, y_b, ...\}.$$

- $p(\text{data})$: "evidence". Models $1, 2$: $p_1(\text{data})/p_2(\text{data})$. $p(\gamma)$=priors.
- EW constraints: $f_1(\gamma; v, \beta) = f_2(\gamma; v, \beta) = 0, \ \Rightarrow \ v(\gamma, ...), \tan\beta_0(\gamma ...)$.

$$p(\text{data}) = \int d\gamma\ p(\gamma)\, dv\ d(\tan\beta)\ \delta(m_Z - m_Z^0)\ \delta\left(f_1(\gamma; v, \beta)\right)\ \delta\left(f_2(\gamma; v, \beta)\right) L(\text{data}|\gamma; \beta, v),$$

$$= \int_{f_{1,2}=0} dS_\gamma\ \frac{L(\text{data}|\gamma)}{\Delta_q(\gamma)}\ p(\gamma), \qquad \Delta_q \equiv \Big[\sum_\gamma \Delta_\gamma^2\Big]^{1/2}, \ \ d\gamma \equiv \prod_i d\gamma_i, \ \ p(\gamma) = p(\gamma_1, ...\gamma_i).$$

D.G., H. M. Lee, M. Park, arXiv:1203:0569

$\Rightarrow$ Δ_q due to fixing EW scale, in addition to/independent of priors $p(\gamma)$! large L/Δ_q needed.

answers e)?

$$\int_{R^n} h(z_1, ..., z_n)\, \delta(g(z_1, ..., z_n))\, dz_1 ....dz_n = \int_{S_{n-1}} dS_{n-1}\, h(z_1, ...z_n)\, \frac{1}{|\nabla_{z_i} g|},$$

- Reducing Δ_q by physics beyond MSSM higgs sector: MSSM + d=6 operators

$$\mathcal{O}_j \;=\; \int d^4\theta\;\mathcal{Z}_j\;(H_j^\dagger\, e^{V_j}\, H_j)^2, \quad (j=1,2). \qquad \mathcal{O}_3 = \int d^4\theta\;\;\mathcal{Z}_3\;(H_1^\dagger\, e^{V_1}\, H_1)\,(H_2^\dagger\, e^{V_2}\, H_2)$$

$$\mathcal{O}_4 \;=\; \int d^4\theta\;\mathcal{Z}_4\;(H_2\, H_1)\,(H_2\, H_1)^\dagger, \qquad \mathcal{O}_k = \int d^4\theta\;\;\mathcal{Z}_k\;(H_k^\dagger\, e^{V_k}\, H_k)\, H_2 H_1 + h.c., \;\text{(k=5,6)}$$

$$\mathcal{O}_7 \;=\; \int d^2\theta\;\mathcal{Z}_7\,\mathrm{Tr}\, W_i^\alpha\, W_{i,\alpha}\,(H_2\, H_1) + h.c.,$$

where $\mathcal{Z}_j(S,S^\dagger) = \alpha_{j0} + \alpha_{j1}\, S + \alpha_{j1}^*\, S^\dagger + \alpha_{j2}\, m_0^2\, S\, S^\dagger, \qquad \alpha_{jk} \sim 1/M_*^2, \;\; S = m_0\,\theta\theta$

$\mathcal{O}_{1,2,3}$: generated by massive T, U(1); $\mathcal{O}_4$: singlet, T. $\mathcal{O}_{5,6}$: 2 D, singlet.

$$\Rightarrow \delta\, m_h^2 \;=\; -2\, v^2\, \big[(\alpha_{30}+\alpha_{40})\mu_0^2 - \alpha_{20}\, m_Z^2\big] - \frac{(2\,\zeta_0\,\mu_0)^2\;v^4}{m_A^2 - m_Z^2} + \frac{v^2\cot\beta}{m_A^2 - m_Z^2}\big[4\, m_A^2\,\mu_0^2\;(2\alpha_{50}+\alpha_{60}) - (2\alpha_{60} - 3\alpha_{70})\, m_A^2\, m_Z^2 - (2\alpha_{60}+\alpha_{70})\, m_Z^4\big] + \mathcal{O}\big(1/(M_*^2\tan^2\beta)\big)$$

$\Rightarrow \alpha_{j0}$ (choice?) $\Rightarrow$ increase m_h, reduce fine-tuning by:

$$\Delta_q(m_h) \approx \exp(-\delta m_h/\text{GeV})\,\Delta_q(m_h)\big|_{\text{CMSSM}}$$

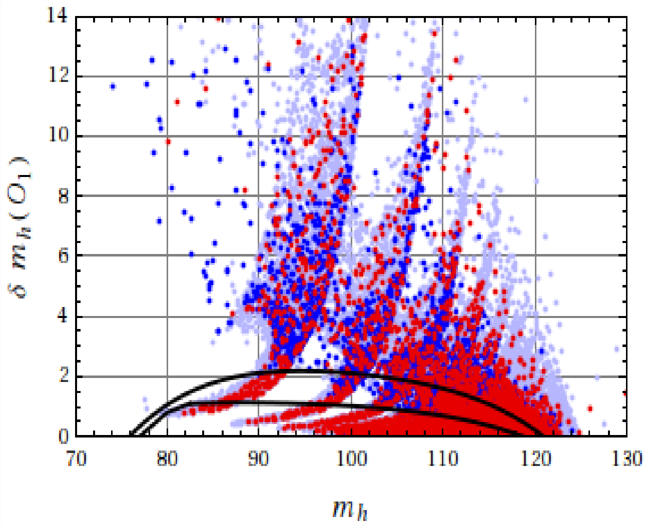
D.G. et al, NPB 848(2011), NPB 831(2010),

M. Carena et al, PRD 85(2012), PRD 81, 82(2010),

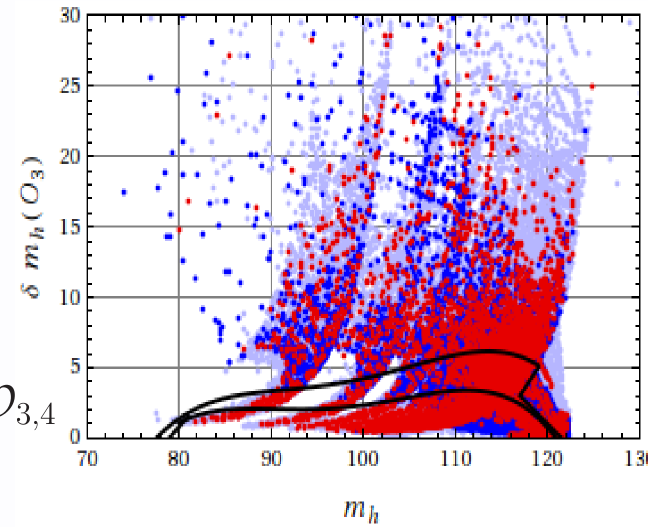
F. Boudjema et al, PRD 85 (2012)

- Corrections to m_h from CMSSM + d=6 operators   D.G., I. Antoniadis, E. Dudas, P. Tziveloglu, NPB B848(2011)1.

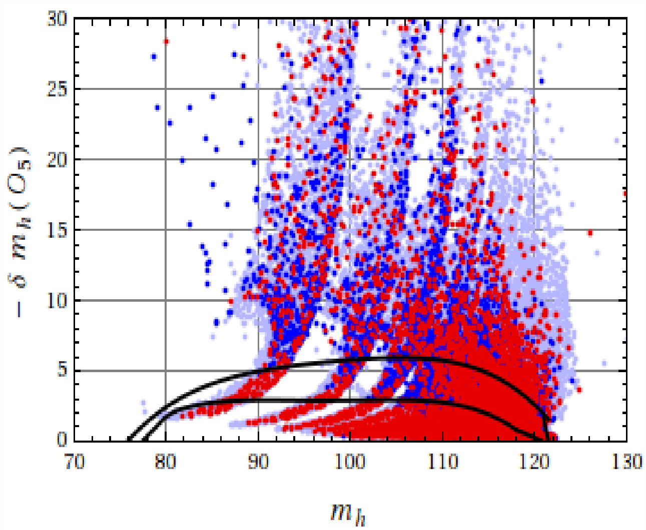
$\mathcal{O}_{1,2}$



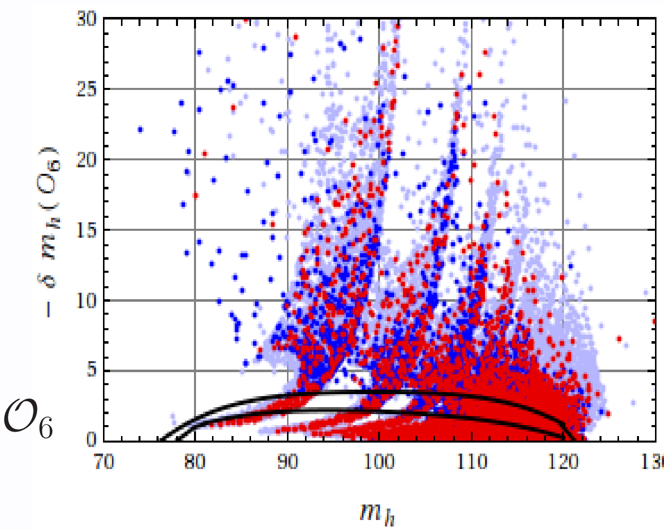
$\mathcal{O}_{3,4}$



$\mathcal{O}_5$



$\mathcal{O}_6$



$\delta m_h = \left(m_h^2 + \delta m_h^2\right)^{1/2} - m_h$, m_h: 2-loop LL CMSSM. $M_* = 8$ TeV. take: $\alpha_{j0} \leq 1/4$

$\Rightarrow$ top curve: $\Delta < 200$: $m_h < 122$ GeV, $\delta m_h < 6$ GeV. ± 1 GeV $(\delta m_h) \leftrightarrow \mp 1$ TeV (δM_*).

$\Rightarrow$ largest δm_h: $\mathcal{O}_3$, $\mathcal{O}_4$. $\Delta_q(128\,\text{GeV}) \approx e^{-6}\,\Delta_q(128\,\text{GeV})_{\text{CMSSM}} = \mathcal{O}(10)$.