Symmetry-Improved Effective Higgs Potential

Apostolos Pilaftsis

School of Physics and Astronomy, University of Manchester, Manchester M13 9PL, United Kingdom

SCALARS 2013

12–16 September 2013, Warsaw, POLAND

Based on A.P. and D. Teresi, Nucl. Phys. B **874** (2013) 594;
[arXiv:1305.3221 [hep-ph]]

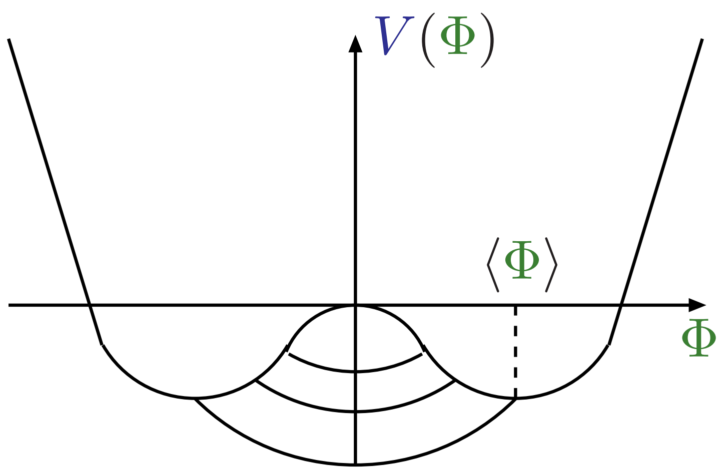
Outline:

- The Standard Theory of Electroweak Symmetry Breaking: SM
- The CJT Effective Action and Field-Theoretic Problems
- Symmetry-Improved CJT Formalism
- $\overline{\rm MS}$ Renormalization in CJT
- Finite-Width Effects within Quantum Loops
- Symmetry-Improved Effective Higgs Potential in CJT
- Conclusions and Future Directions

• The Standard Theory of Electroweak Symmetry Breaking

Higgs Mechanism in SM: $\mathrm{SU(3)}_c \otimes \mathrm{SU(2)}_L \otimes \mathrm{U(1)}_Y \to \mathrm{SU(3)}_c \otimes \mathrm{U(1)}_{\mathrm{em}}$

[P. W. Higgs '64; F. Englert, R. Brout '64; G. S. Guralnik, C. R. Hagen, T. W. B. Kibble '64]



Higgs potential $V(\Phi)$

$$V(\Phi) \;=\; -m^2\Phi^\dagger\Phi \;+\; \lambda(\Phi^\dagger\Phi)^2\,.$$

Ground state:

$$\langle\Phi\rangle \;=\; \sqrt{\frac{m^2}{2\lambda}}\begin{pmatrix}0\\1\end{pmatrix}$$

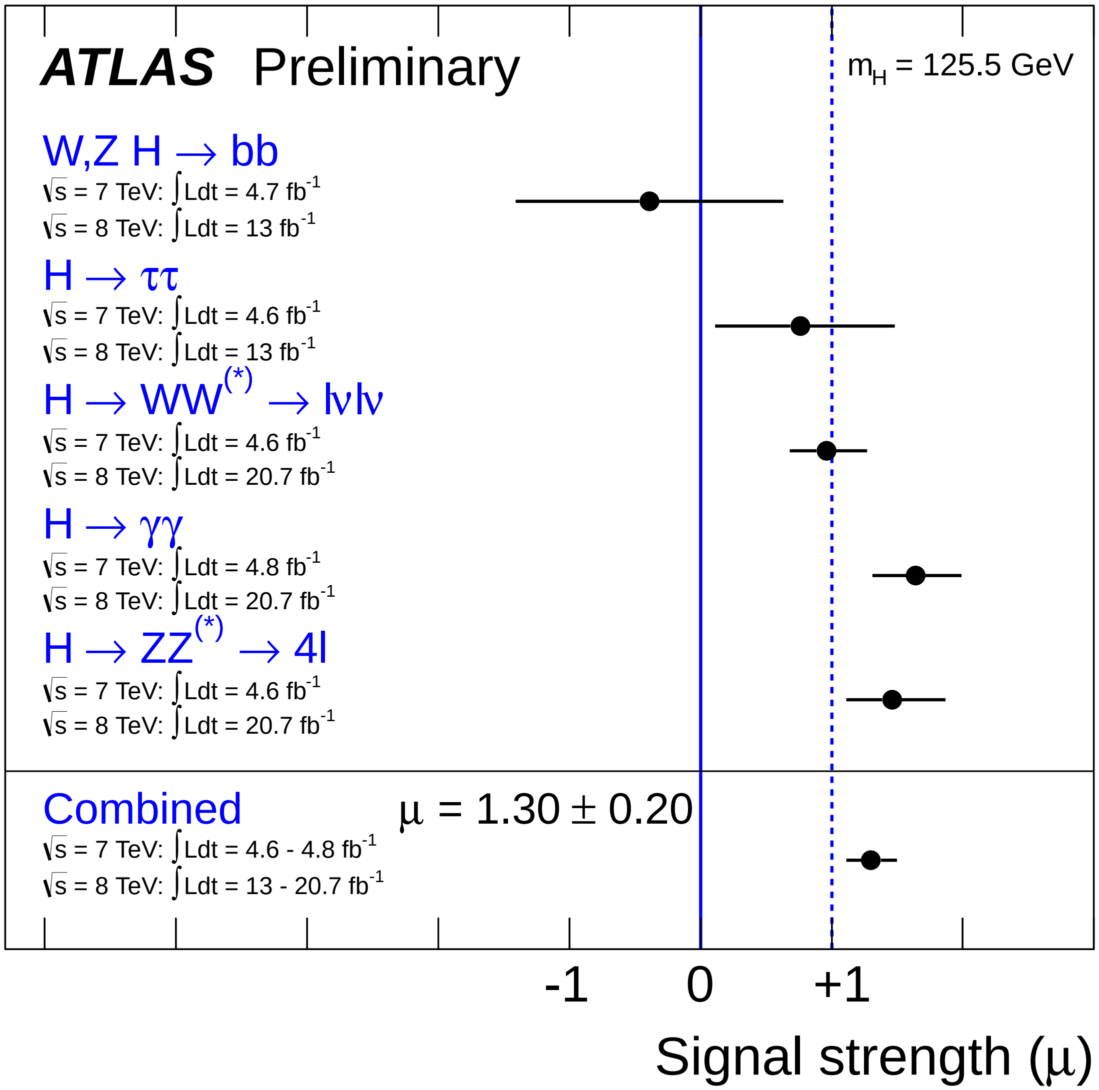
carries weak charge, but no electric charge and colour.

After Spontaneous Symmetry Breaking:

$\Rightarrow$ $W^\pm$, Z bosons and matter feel the presence of $\langle\Phi\rangle$ and become massive, but not γ and g^a, e.g. $M_W = g_w\langle\Phi\rangle$

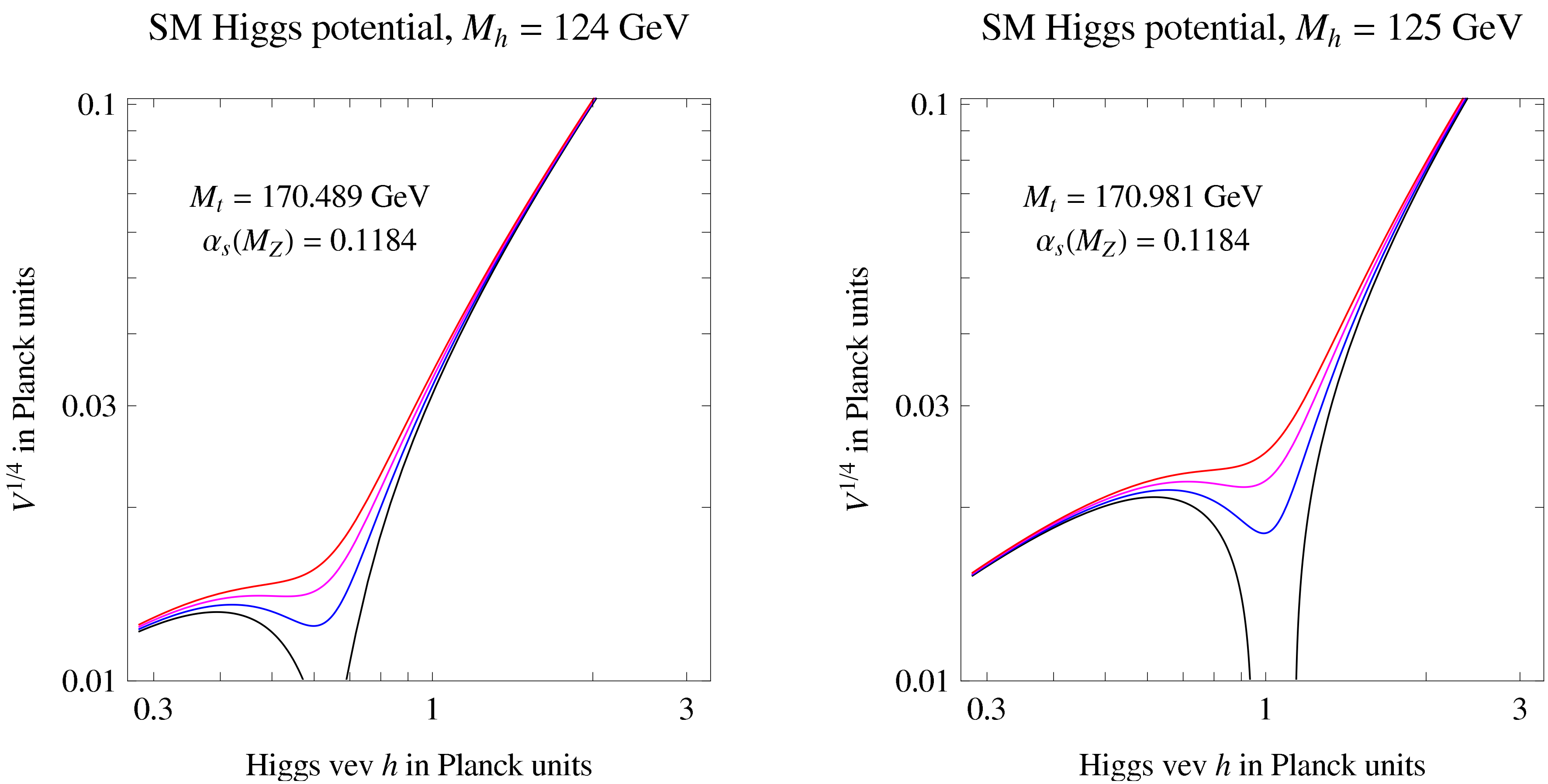
$\Rightarrow$ Quantum excitations of $\Phi = \langle\Phi\rangle + H\begin{pmatrix}0\\1\end{pmatrix}$; H is the Higgs boson.

On the SM-like Higgs-Boson Discovery at the LHC:



Uncertainties of the SM Higgs potential at NNLO

[G. Degrassi *et al*, JHEP **1208** (2012) 098]



Higgs Potential versus Variations in Top Mass M_t by **0.1 MeV**

[Analysis includes the Multi-Critical scenario: D.L. Bennett, H.B. Nielsen, IJMA9 (1994) 5155]

• The CJT Effective Action and Field-Theoretic Problems

[J.M. Cornwall, R. Jackiw, E. Tomboulis, PRD10 (1974) 2428]

Connected Generating Functional of 2PI Effective Action:

$$W[J,K] \;=\; -i\,\ln \int \mathcal{D}\phi^i \, \exp\left[i\left(S[\phi] \,+\, J^i_x\,\phi^i_x \,+\, \frac{1}{2}\,K^{ij}_{xy}\,\phi^i_x\,\phi^j_y \right)\right] \;,$$

where $S[\phi] = \int_x \mathcal{L}[\phi]$ is the classical action of a $\mathbb{O}(N)$ theory.

Legendre transform of $W[J,K]$ with respect to J and K:

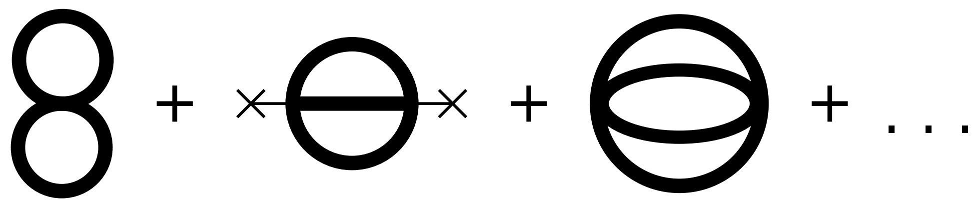
$$\frac{\delta W[J,K]}{\delta J^i_x} \;\equiv\; \phi^i_x \;, \qquad \frac{\delta W[J,K]}{\delta K^{ij}_{xy}} \;=\; \frac{1}{2}\left(i\Delta^{ij}_{xy} \,+\, \phi^i_x\,\phi^j_y \right) \;,$$

to get the 2PI effective action

$$\Gamma[\phi,\Delta] \;=\; W[J,K] \,-\, J^i_x\,\phi^i_x \,-\, \frac{1}{2}\,K^{ij}_{xy}\,\left(i\Delta^{ij}_{xy} \,+\, \phi^i_x\phi^j_y \right) \;.$$

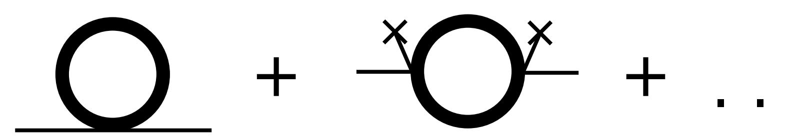
The 2PI CJT Effective Action:

$$\Gamma[\phi,\Delta] \;=\; S[\phi] + \frac{i}{2}\,\mathrm{Tr}\,\ln\Delta^{-1} + \frac{i}{2}\,\mathrm{Tr}\,(\Delta^{0^{-1}}\Delta) \;-\; i\,\Gamma^{(2)}_{\rm 2PI}[\phi,\Delta]\,,$$

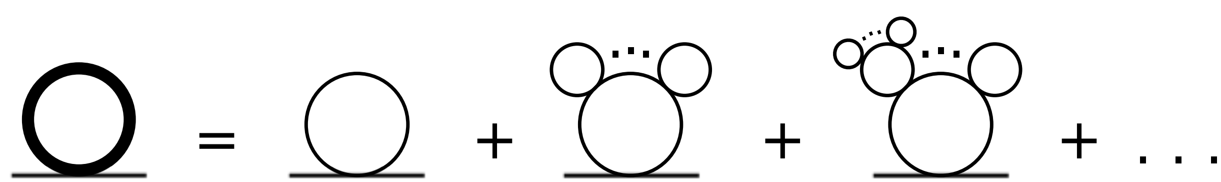
where $\Gamma^{(2)}_{\rm 2PI}[\phi,\Delta] \;=$ 

Equations of Motion:

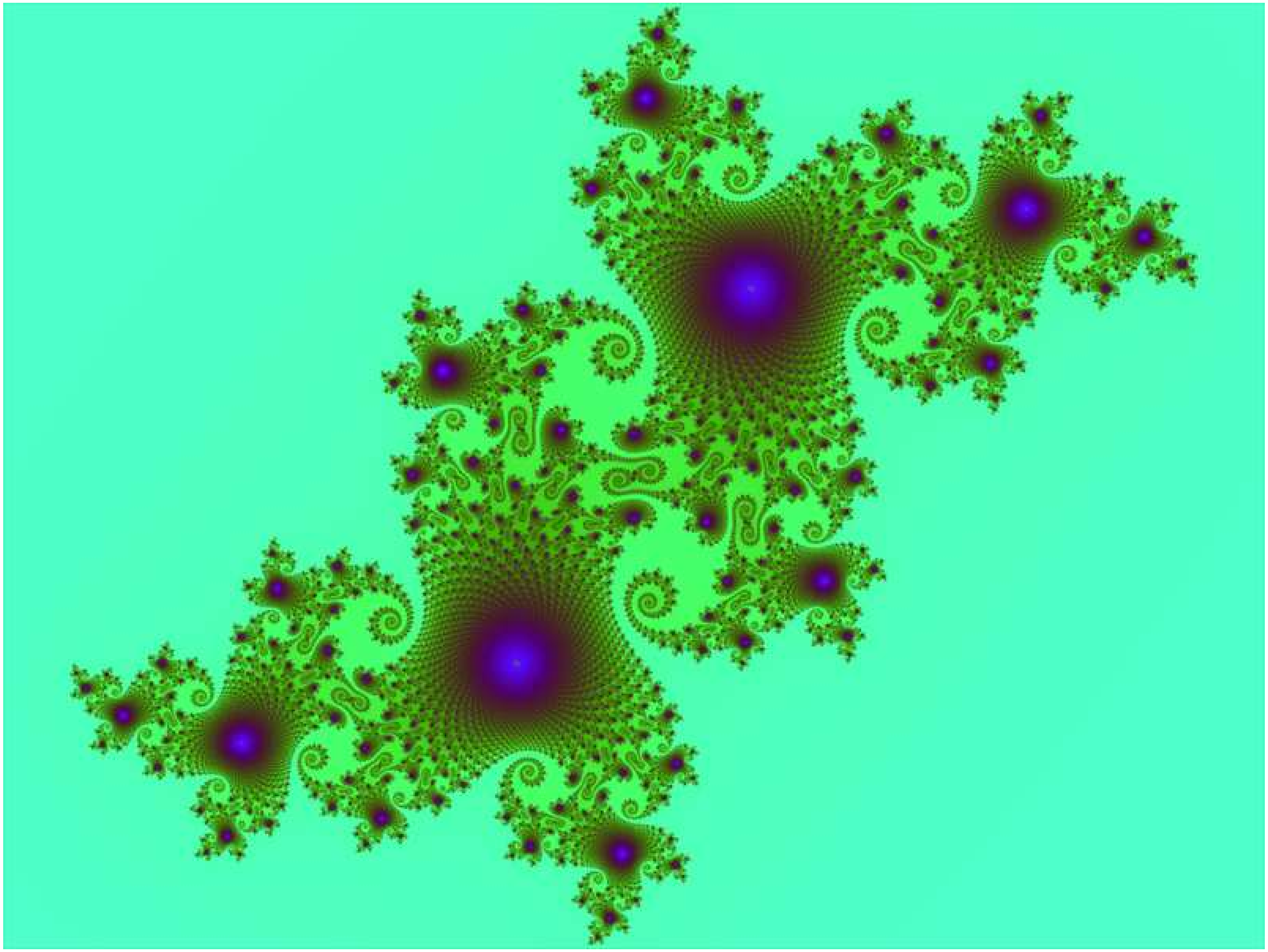
- $\dfrac{\delta\Gamma[\phi,\Delta]}{\delta\phi} \;=\; 0$

- $\dfrac{\delta\Gamma[\phi,\Delta]}{\delta\Delta} \;=\; 0 \;\Rightarrow\; \Delta^{-1} \;=\; \Delta^{0^{-1}} \;+$ 

Hartree-Fock:



Graphical impression of the infinite HF-term:



– Field-Theoretic Problems in CJT

- Systematic formal resummation of high-order graphs:
  - Rigorous Derivation of Schwinger–Dyson Equations
  - Thermal Masses in the high-T Regime
  - Finite-Width Effects within Quantum Loops [this talk]

- Non-Equilibrium QFT, through Kadanoff–Baym equations.

  [For instance, P. Millington, AP, PLB724 (2013) 56; arXiv:1211.3152, $>$102 pages, to appear in PRD]

– Field-Theoretic Problems in CJT

- Systematic formal resummation of high-order graphs:
  - – Rigorous Derivation of Schwinger–Dyson Equations
  - – Thermal Masses in the high-T Regime
  - – Finite-Width Effects within Quantum Loops [this talk]

- Non-Equilibrium QFT, through Kadanoff–Baym equations.
  [For instance, P. Millington, AP, PLB724 (2013) 56; arXiv:1211.3152, $>$102 pages, to appear in PRD]

BUT

- Truncations of CJT lead to residual violations of symmetries, e.g. global or local symmetries.

$\longrightarrow$ Erroneous First-Order Phase Transition in $\mathbb{O}(N)$ Theories
$\longrightarrow$ Goldstone Bosons become Massive
$\longrightarrow$ Erroneous Thresholds for the Resummed Higgs-Boson Propagator.
$\longrightarrow$ $\cdots$

Pertinent Literature to the Global-Symmetry Problem

- G. Baym, G. Grinstein, Phys. Rev. D **15** (1977) 2897.

⋮

- G. Amelino-Camelia, Phys. Lett. B **407** (1997) 268.
- N. Petropoulos, J. Phys. G **25** (1999) 2225.
- Y. Nemoto, K. Naito, M. Oka, Eur. Phys. J. A **9** (2000) 245.
- J. T. Lenaghan, D. H. Rischke, J. Phys. G **26** (2000) 431.
- H. van Hees, J. Knoll, Phys. Rev. D **66** (2002) 025028.
- J. Baacke, S. Michalski, Phys. Rev. D **67** (2003) 085006.
- Y. Ivanov, F. Riek, H. van Hees, J. Knoll, Phys. Rev. D **72** (2005) 036008.
- E. Seel, S. Struber, F. Giacosa, D. H. Rischke, Phys. Rev. D **86** (2012) 125010.
- G. Markó, U. Reinosa, Z. Szép, Phys. Rev. D **87** (2013) 105001.

• Symmetry-Improved CJT Formalism

[AP, D. Teresi, NPB874 (2013) 594]

Equivalence between 1PI and 2PI Effective Actions to All Orders:

$$\Gamma^{1\mathrm{PI}}[\phi] \;=\; \Gamma[\phi\,,\,\Delta(\phi)]\,, \qquad \text{with} \quad \frac{\delta\Gamma[\phi\,,\,\Delta(\phi)]}{\delta\Delta} \;=\; 0\,.$$

1PI Ward Identity (e.g. for $\mathbb{O}(2)$):

$$\frac{\delta\Gamma^{1\mathrm{PI}}[\phi]}{\delta\phi^i_x}\,T^a_{ij}\,\phi^j_x \;=\; 0 \quad\Longrightarrow\quad v\int_x \frac{\delta^2\Gamma^{1\mathrm{PI}}[\phi]}{\delta G_y\,\delta G_x} \;=\; \frac{\delta\Gamma^{1\mathrm{PI}}[\phi]}{\delta H} \;\to\; 0\,.$$

Replace:

$$\frac{\delta^2\Gamma^{1\mathrm{PI}}[\phi]}{\delta G_y\,\delta G_x} \;=\; \Delta^{-1,G}_{xy}\,,$$

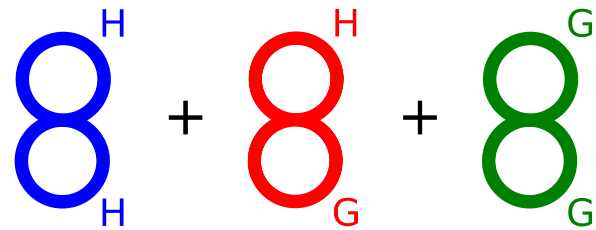
to obtain the Symmetry-Improved Equations of Motion:

$$\frac{\delta\Gamma[v\,,\,\Delta]}{\delta\Delta_{H/G}} \;=\; 0\,,$$

$$v\,\Delta^{-1}_G(k=0,v) \;=\; 0\,.$$

– $\mathbb{O}$(2) Hartree–Fock Equations of Motion:

HF Approximation: $\Gamma^{(2)}_{\rm HF}[\Delta_H\, ,\, \Delta_G] \;=$



Ansatz: $\Delta^{-1}_{H/G}(k) \;=\; k^2 \;-\; M^2_{H/G} \;+\; i\varepsilon$

Equations of Motion:

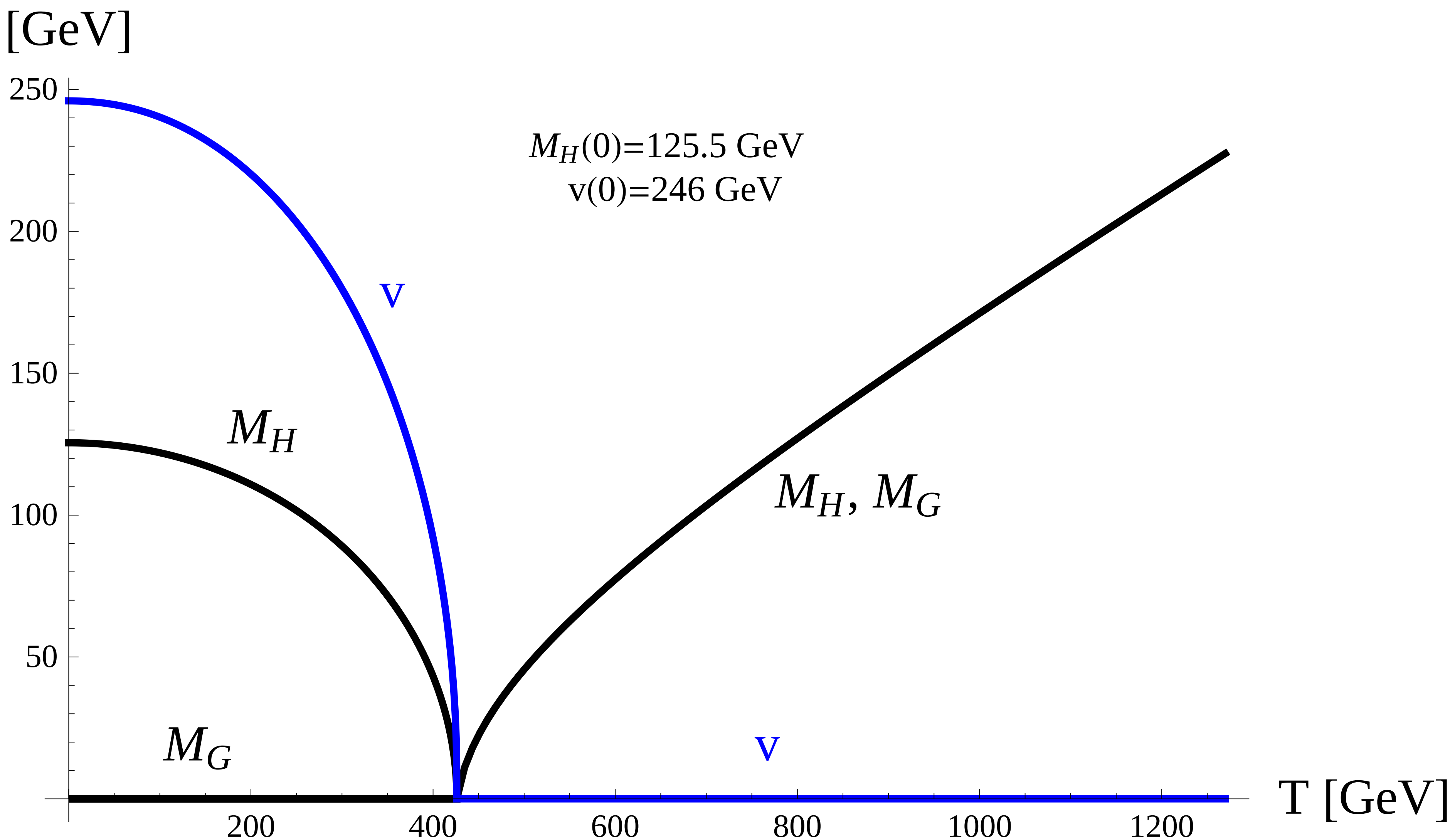
$$M^2_H \;=\; 3\lambda v^2 - m^2 \;+\; (\delta\lambda_1^A + 2\delta\lambda_1^B)v^2 \;-\; \delta m_1^2$$
$$+\;(3\lambda + \delta\lambda_2^A + 2\delta\lambda_2^B)\int_k i\Delta_H(k) \;+\; (\lambda + \delta\lambda_2^A)\int_k i\Delta_G(k)\;,$$

$$M^2_G \;=\; \lambda v^2 - m^2 \;+\; \delta\lambda_1^A v^2 \;-\; \delta m_1^2$$
$$+\;(\lambda + \delta\lambda_2^A)\int_k i\Delta_H(k) \;+\; (3\lambda + \delta\lambda_2^A + 2\delta\lambda_2^B)\int_k i\Delta_G(k)\;,$$

$$v\, M^2_G \;=\; 0\,.$$

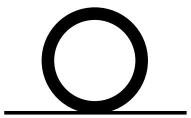
– Second-Order Phase Transition in the HF Approximation

[AP, D. Teresi, NPB874 (2013) 594]



• $\overline{\mathrm{MS}}$ Renormalization in CJT

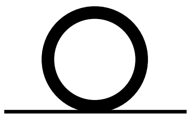
Naive Renormalization:

 $= \int_k i\Delta(k) \;\sim\; M^2\,\frac{1}{\epsilon}\,,$ Naive CT: $\delta m^2 \stackrel{?}{=} M^2\,\frac{1}{\epsilon}\,.$

But, $M^2 = M^2(T) \quad\Longrightarrow\quad$ Temperature-dependent CT?!

• $\overline{\text{MS}}$ Renormalization in CJT

Naive Renormalization:



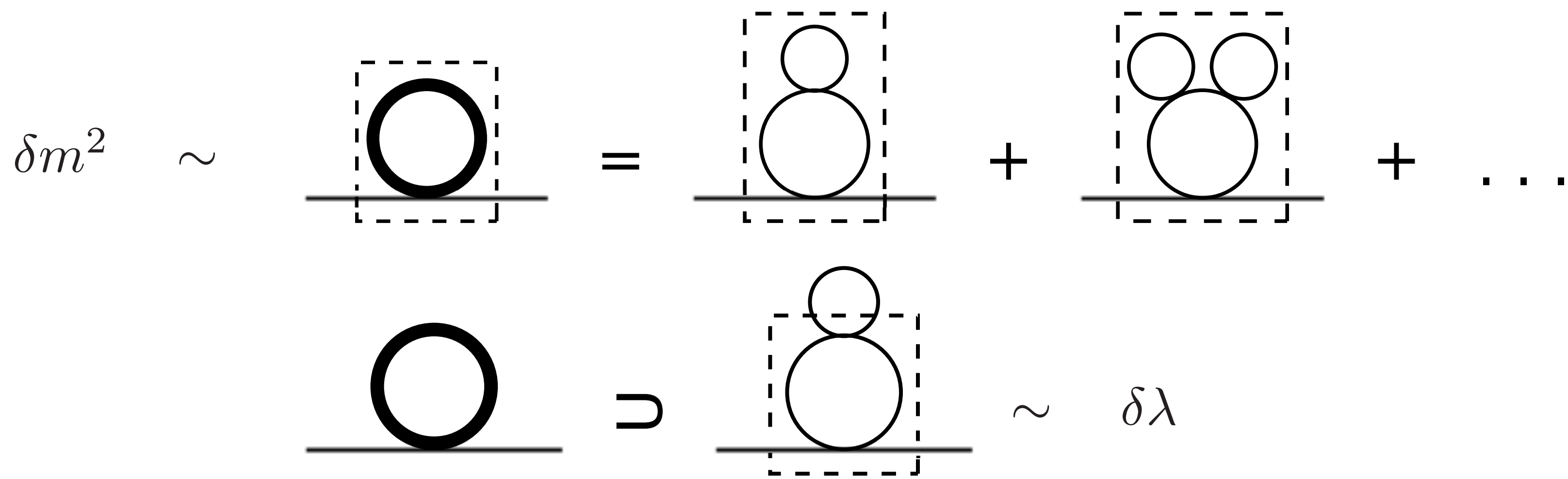
$$= \int_k i\Delta(k) \sim M^2 \frac{1}{\epsilon}\,,$$

Naive CT: $\delta m^2 \stackrel{?}{=} M^2 \frac{1}{\epsilon}$.

But, $M^2 = M^2(T) \implies$ Temperature-dependent CT?!

What has gone wrong?

[J.-P. Blaizot, E. Iancu, U. Reinosa, NPA736 (2004) 149;
J. Berges *et al*, AP320 (2005) 344]



Is there any systematic renormalization, e.g. in the $\overline{\text{MS}}$ scheme?

[W.A. Bardeen, A. Buras, D. Duke, T. Muta, PRD18 (1978) 3998]

– $\overline{\mathrm{MS}}$ Renormalization in the CJT Formalism

Procedure:

- Isolate UV infinities in EoMs, e.g. by Dimensional Regularization.
- Require that the UV-finite part of EoMs **be** UV finite:

$$(\ldots)_{\mathrm{UV}}\, \mathcal{T}_H^{\mathrm{fin}} \;+\; (\ldots)_{\mathrm{UV}}\, \mathcal{T}_G^{\mathrm{fin}} \;+\; (\ldots)_{\mathrm{UV}}\, v^2 \;+\; (\ldots)_{\mathrm{UV}}\, 1 \;\stackrel{!}{=}\; 0$$

- Cancel seperately the UV infinities $\propto\; \mathcal{T}_H^{\mathrm{fin}}(T),\;\; \mathcal{T}_G^{\mathrm{fin}}(T),\;\; v^2(T),\;$ 1.
- Check UV consistency:

$$4 \;\times\; 2 \;=\; 8 \text{ EoMs}, \quad \text{for } \; 5 \text{ CTs}:\; \delta m_1^2\,,\; \delta\lambda_1^A\,,\; \delta\lambda_1^B\,,\; \delta\lambda_2^A\,,\; \delta\lambda_2^B\;.$$

This is a non-trivial check!

– T-independent Resummed Counterterms in the HF approximation:

[AP, D. Teresi, NPB874 (2013) 594]

$$\delta\lambda_1^A \;=\; \delta\lambda_2^A \;=\; \frac{2\lambda^2}{16\pi^2\epsilon}\,\frac{3-\dfrac{4\lambda}{16\pi^2\epsilon}}{1-\dfrac{6\lambda}{16\pi^2\epsilon}+\dfrac{8\lambda^2}{(16\pi^2\epsilon)^2}}$$

$$=\; -\,\lambda \;+\; \frac{(16\pi^2\epsilon)^2}{8\lambda} \;+\; O(\epsilon^3)\;,$$

$$\delta\lambda_1^B \;=\; \delta\lambda_2^B \;=\; \frac{2\lambda^2}{16\pi^2\epsilon}\,\frac{1}{1-\dfrac{2\lambda}{16\pi^2\epsilon}}$$

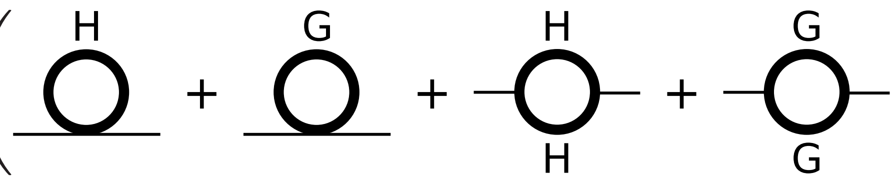
$$=\; -\,\lambda \;-\; \frac{16\pi^2\epsilon}{2} \;-\; \frac{(16\pi^2\epsilon)^2}{4\lambda} \;+\; O(\epsilon^3)\;,$$

$$\delta m_1^2 \;=\; \frac{4\lambda m^2}{16\pi^2\epsilon}\,\frac{1}{1-\dfrac{4\lambda}{16\pi^2\epsilon}} \;=\; -\,m^2 \;-\; m^2\frac{16\pi^2\epsilon}{4\lambda} \;+\; O(\epsilon^2)\;.$$

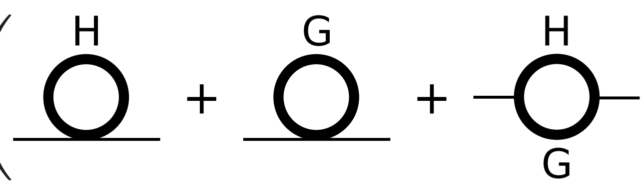
• Finite-Width Effects within Quantum Loops

Equations of Motion including Sunset Diagrams:

- $$\Delta_H^{-1}(p) \;=\; p^2 \;-\; (3\lambda + \delta\lambda_1^A + 2\delta\lambda_1^B)v^2 \;+\; m^2 \;+\; \delta m_1^2$$

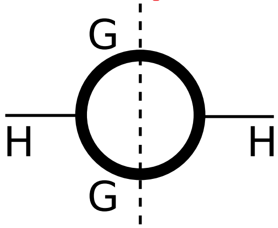
  $$-i \Bigg($$  $$\Bigg)\,,$$

- $$\Delta_G^{-1}(p) \;=\; p^2 \;-\; (\lambda + \delta\lambda_1^A)v^2 \;+\; m^2 \;+\; \delta m_1^2$$

  $$-i \Bigg($$  $$\Bigg)\,,$$

- $$v\,\Delta_G^{-1}(0) \;=\; 0\,.$$

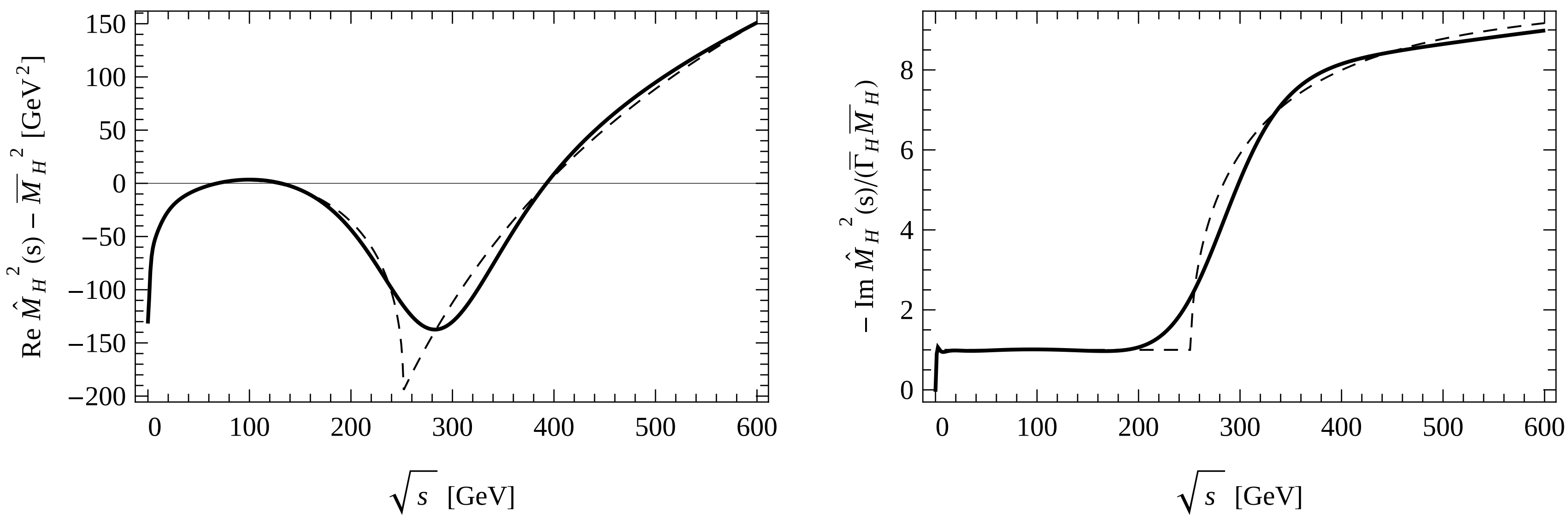
Absorptive Effects:

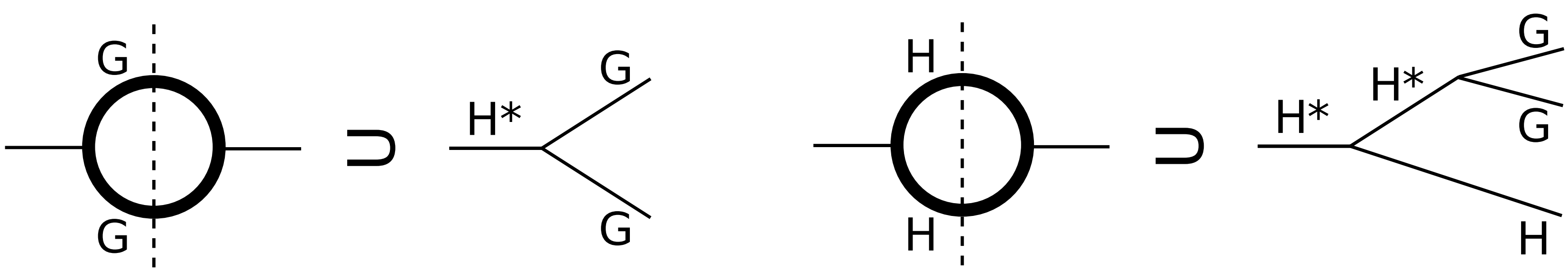


G consistently massless $\Longleftrightarrow$ threshold at $s \equiv p^2 = 0$

– Higgs Selfenergy in CJT

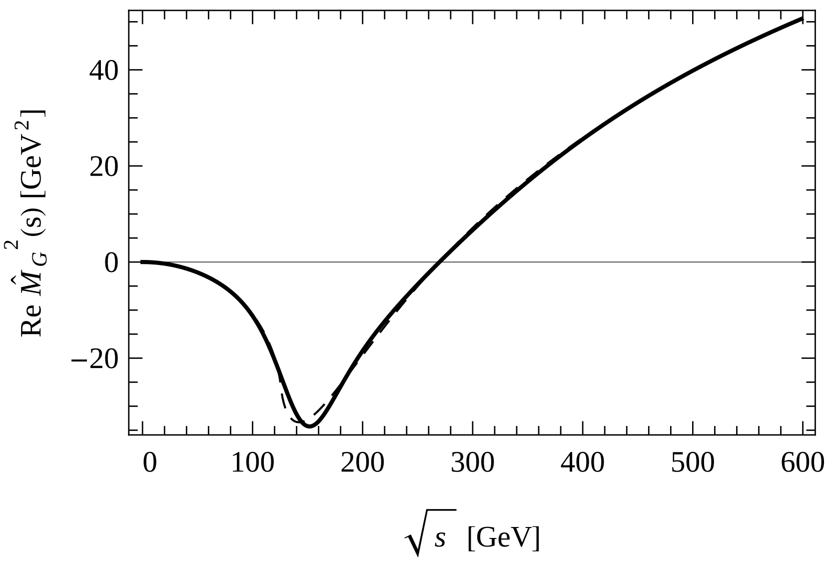
[AP, D. Teresi, NPB874 (2013) 594]

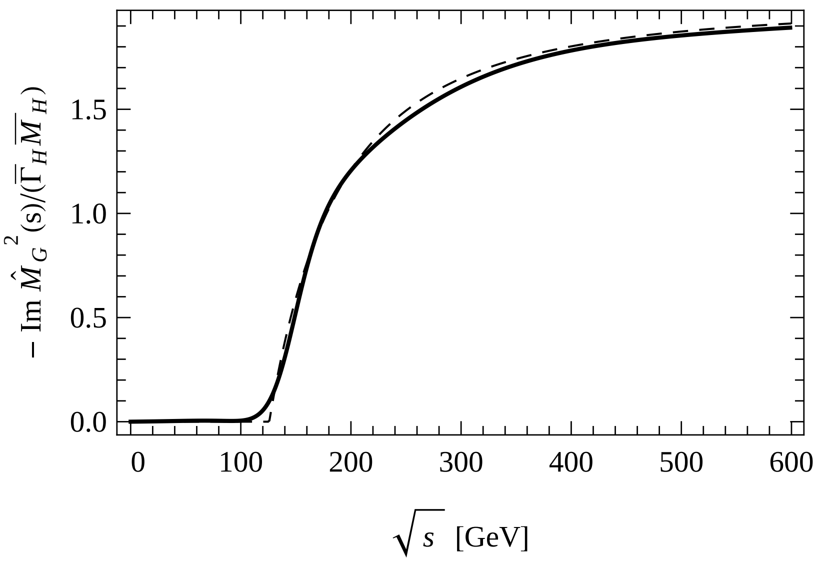


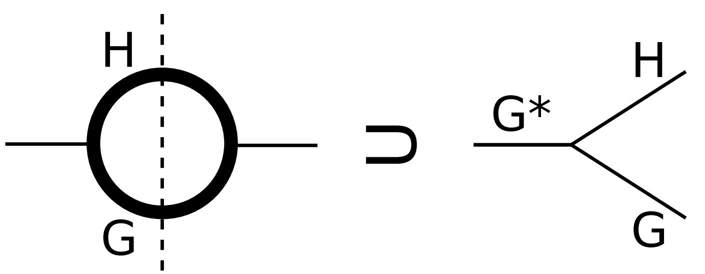


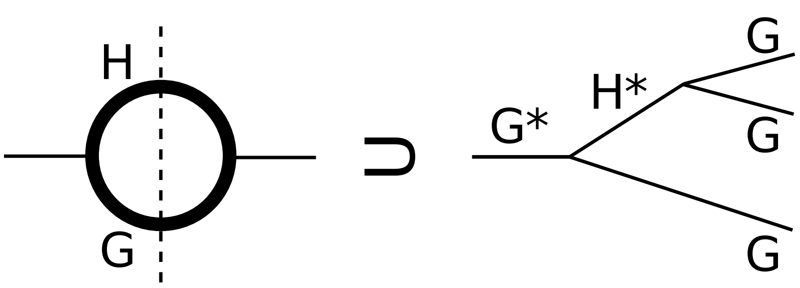
– Goldstone Selfenergy in CJT

[AP, D. Teresi, NPB874 (2013) 594]









• Symmetry-Improved Effective Higgs Potential in CJT

Symmetry-Improved Effective Potential $\widetilde{V}_{\rm eff}(\phi)$ from 1PI Ward Identity:

$$\phi\,\Delta_G^{-1}(k=0;\phi) \;=\; -\,\frac{d\widetilde{V}_{\rm eff}(\phi)}{d\phi}\;.$$

Solution:

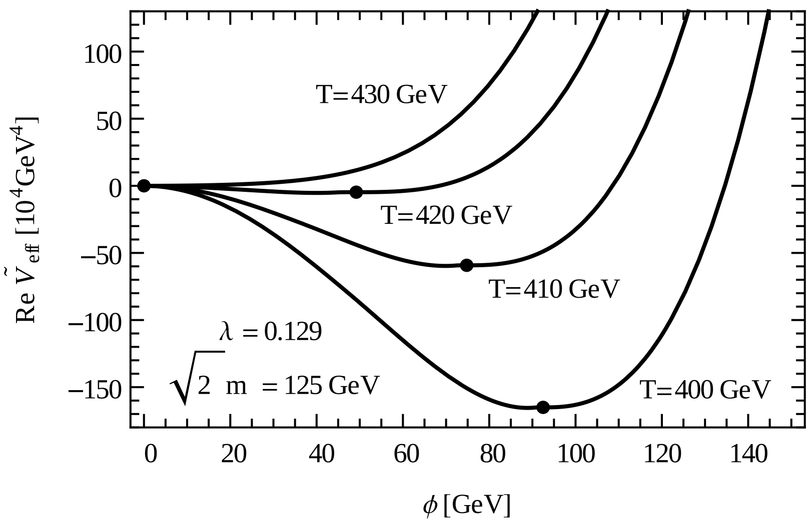
$$\begin{aligned}\widetilde{V}_{\rm eff}(\phi) &= -\int_0^{\phi} d\phi\,\phi\,\Delta_G^{-1}(k=0;\phi)\;+\;\widetilde{V}_{\rm eff}(\phi=0)\\ &= -\int_v^{\phi} d\phi\,\phi\,\Delta_G^{-1}(k=0;\phi)\;+\;P(T,\mu)\;,\end{aligned}$$

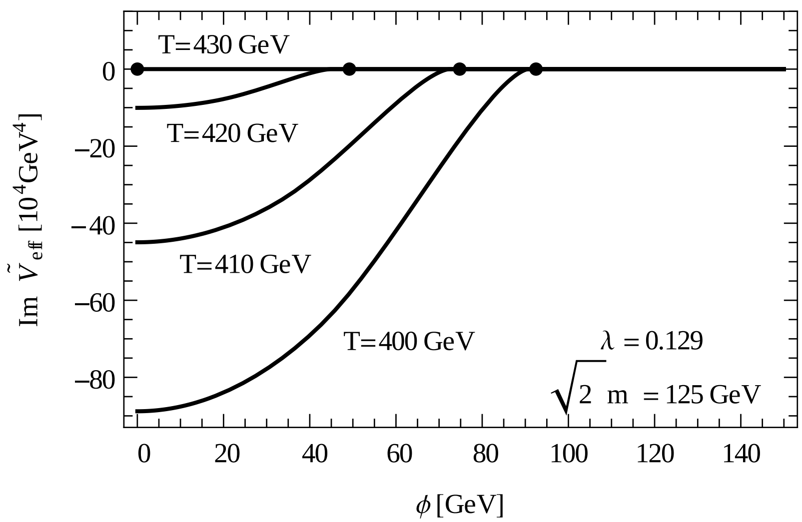
where $P(T,\mu)$ is the thermodynamic pressure $=$ hydrostatic pressure, i.e. it satisfies Baym's thermodynamic consistency.

[G. Baym, PR127 (1962) 1391]

– CJT Effective Higgs Potential

[AP, D. Teresi, NPB874 (2013) 594]





Note: $\mathrm{Im}\,\widetilde{V}_{\rm eff}(\phi) < 0$, for $0 < \phi < v$

$\Longrightarrow$ Vacuum instability for the concave part of the potential.

[E.J. Weinberg, A.-Q. Wu, PRD36 (1987) 2474]

• Conclusions

- **Maintaining symmetries in CJT loopwise is a long-standing problem**

- **Novel Approach to Global Symmetries:**

  $\Longrightarrow$ **Symmetry-Improved CJT Effective Action**

  $\longrightarrow$ Massless Goldstone Bosons
  $\longrightarrow$ 2nd-Order Phase Transition in the HF Approximation

- $\overline{\rm MS}$ **Renormalization with T-independent Resummed Counterterms**

- **Absorptive Effects properly described:**

  $\longrightarrow$ Smooth thresholds consistent with massless Goldstone bosons
  $\longrightarrow$ Consistent resummation within Quantum Loops.

- **Symmetry-Improved Effective Higgs Potential is unique, with proper thermodynamic properties**

• Future Directions

- **Extension: 2PI $\longrightarrow$ nPI Effective Actions**
- **Extension to Local Symmetries: $\mathbb{U}(1)$, $\mathbb{SU}(N)$**
- **Spontaneous Breaking of Local Gauge Symmetries**
- **Higher Precision Predictions in the 2PI Formalism**

$\vdots$

• Future Directions

- **Extension: 2PI $\longrightarrow$ nPI Effective Actions**
- **Extension to Local Symmetries: $\mathbb{U}(1)$, $\mathbb{SU}(N)$**
- **Spontaneous Breaking of Local Gauge Symmetries**
- **Higher Precision Predictions in the 2PI Formalism**

$\vdots$

New Era of Analytical Non-Perturbative QFT?