What does gravity do with axions?

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- the axion as Cold Dark Matter
- the right question: **How to distinguish axions from WIMPs?**
  Sikivie's scenario:
  1. "gravitational thermalisation" drives axions to a Bose Einstein condensate
  2. an axion BE condensate supports vortices $\Rightarrow$ caustics in galactic dark matter
- focus on 1.: what is a Bose Einstein condensate?
  - dunno
  - but do need $entropy$ generation

  ? estimate a $dissipative$ gravitational interactions for axions in early U?
  - veerry slow...
- anything worth remembering from this talk?

You all know the axion...

- strong CP problem of QCD: $-\frac{1}{4g^2}F^A_{\mu\nu}F^{\mu\nu A} - \frac{\theta}{4g^2}F^A_{\mu\nu}\widetilde{F}^{\mu\nu A}$
  θ is CPV — neutron edm $\Rightarrow \theta \lesssim 10^{-10}$

Pich deRafael
Pospelov, Ritz

- axial anomaly : rotate θ onto quark masses :(still CPV
- solution: add fields such that rotate θ to the phase of a complex SM-singlet scalar Φ, who gets a vev

Peccei Quinn

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... Srednicki NPB85

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light and feebly coupled $\Leftrightarrow$ stellar energy loss bounds: (upper bd on cplg)

$$m_a \lesssim 10^{-2} \text{ eV} \qquad\qquad (f_{PQ} \gtrsim 10^9 \text{ GeV})$$

Non-thermal axion production cosmology...it is *Cold* Dark Matter!

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2. then at PQPT $\Phi \to f_{PQ} e^{ia/f_{PQ}}$
   $*$ a random in $-\pi \to \pi$ from one horizon to next,
   but $\langle a^2 \rangle_{U\ today} \sim \pi^2 f_{PQ}^2/3$
   $*$ one string/horizon

3. QCD Phase Transition ($T \sim 200$ MeV):
   $*$ strings go away (radiate cold axion particles, $\vec{p} \sim H \ll m_a$) Hiramatsu etal 1012.5502
   $*$ $m_a(t) : 0 \to f_\pi m_\pi / f_a$
   ($\delta\rho \sim \rho$ on horizon-scale !)
   when $m_a > H$, axion field oscillates around the minimum...energy density redshifts as $1/R^3(t)$ (and density today higher for smaller mass)

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Summary:

⋆ osc. axion field + (cold) axion particles (from strings) redshift like CDM
⋆ not overclose U for $m_a \gtrsim 10^{-4}$ eV ($f_{PQ} \lesssim 10^{11}$ GeV) recall astrophys: $m_a \lesssim 10^{-2}$ eV
(PQ before inflation can have smaller m_a by tuning $a \ll f_{PQ}$)
⋆ axion field also grows linear density fluctuations like CDM

Ratra, Hwang+Noh

To cosmologically distinguish axions from WIMPs?

• axions = feebly-coupled, very light one-parameter BSM

• non-thermal production in cosmology $\Rightarrow \left\{ \begin{array}{l} \text{redshift like WIMPs} \\ \text{grow linear fluctuations like WIMPs} \end{array} \right.$

$\Rightarrow$ *might axions differ from WIMPs during non-linear structure formation?*

(**Umm...** non-linear/N-body is hard!)

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Sikivie's scenario:

Erken,Sikivie,Tam,Yang

1. at $T_\gamma \sim$ keV, "gravitational thermalisation" of axions drives them to a Bose-Einstein Condensate

Saikawa Yamaguchi

2. BEC can support vortices, which allow caustics in the galactic DM distribution $\Leftrightarrow$ axion DM signature?

BEC galactic halos:
Rindler-Daller+Shapiro

Rest of the talk is about step 1.

$\Rightarrow$ what is a Bose Einstein condensate?

$\Rightarrow$ what does gravity do with axions during linear structure formation?

$\Rightarrow$ how to find dissipation/thermalisation in gravitational interactions?

What is a Bose Einstein condensate? (I don't know. Please tell me if you do!)

1. in equilibrium stat mech: bosons pile into the $\vec{p} = 0$ mode

2. in equilibrium Finite Temp FT: a phase transition $\leftrightarrow$ form a vev
   store a density of conserved charge in a homogeneous $+$ isotropic classical field

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   store a density of conserved charge in a homogeneous + isotropic classical field
3. for alkali gases in atomic traps: coherent collective behaviour (all the same $\vec{p}$; but not necc $\vec{p} = 0$)
4. Sikivie says: lowest energy state (not necc homogeneous) pragmatically, it needs to support vortices?
5. for purposes of this talk: the inhomogeneous classical axion field of misalignment mechanism is *not* a BE condensate. Need to increase entropy to get there...
   $\Rightarrow$ find dissipation!

Is a BE condensate just a (non-relativistic) charge-carrying classical field?
Or as well as being "coherent", does it need to be homogeneous + isotropic, ie, the $\vec{p} = 0$ *mode?*

What does gravity do with axions?

consider early evolution of the Universe, until $\delta\rho \sim \rho$
Eqns of motion inside the horizon thermalisation is causal, so neglecting H^2/m_a^2,... for axion field in perturbed FRW (Newtonian gauge, ϕ = Newtonian potential comes from metric) can be obtained from

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1. get Eqns of motion for the axion field a:

$$\ddot{a} + 3H\dot{a} + k^2 a + m_a^2 a \sim \pm \sum_{q,p} \frac{8\pi G_N m^2}{q^2} aaa + \text{other } \delta\rho \text{ terms}$$

non-linear... can calculate rate for axions to emit a graviton of any wavelength.

Interpretation: the interaction rate from 1 is a thermalisation rate. $\Gamma > H$ after $T_\gamma \sim$ keV, so hugely occupied low-$\vec{k}$ modes equilibrate = form BE condensate.

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   non-linear... can calculate rate for axions to emit a graviton of any wavelength.

2. Or get Eqns of motion for a fluid with scalar perturbations
   $$\ddot{\delta} + 2H\dot{\delta} - 4\pi\rho\delta + \frac{c_s^2}{R^2(t)}\nabla^2\delta = 0$$
   can solve (in fourier space); gives evolution of axion density fluctuations.

Interpretation: the interaction rate from 1 is a thermalisation rate. $\Gamma > H$ after $T_\gamma \sim$ keV, so hugely occupied low-$\vec{k}$ modes equilibrate = form BE condensate.
But... 2 is the same eqns with different variables, and says (at least some of) those gravitons are growing fluctuations?
Why is that a thermalisation rate? (bath? fluctuations to sum??)

Looking for dissipation in the gravitational interactions of axions

1. Assume BE condensation requires dissipation

2. Assume no dissipation/thermalisation at leading order of classical equations of motion usual non-equilibrium field theory — must sum a bath of fluctuations to dissipate with t-reversal invariant eqns

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3. Look for gravitational interactions of axions that are neglected in obtaining expanding U with inhomogeneity growth:
   T_{ij}, $i \neq j$, is gauge invariant, of $\mathcal{O}(|\vec{p}|^2/m_a^2)$, and neglected in equations for density fluctuations.

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4. match axion in perturbed U onto imperfect fluid in FRW:

$$T^i{}_j(\vec{x},t) = -\frac{(1+2\phi)}{R^2(t)}\partial_i\phi\partial_j\phi \quad = \quad -\eta(t)(\partial_j U^i(\vec{x},t) + \partial^i U_j(\vec{x},t))$$

   $\eta =$ viscosity, $U = 4-$velocity. An imperfect fluid can grow density fluctuations, but contains dissipation...

5. estimate a dissipation scale:
   $<$ the Jeans length $1/\sqrt{m_a H}$, distance below which fluctuations oscillate due to axion pressure

Summary (review of well-known things)

Maybe New Physics is *light!* Maybe the axion, feebly coupled solution to the θ-problem, is the Dark Matter. It should satisfy 10^{-4} eV $\lesssim m_a \lesssim 10^{-2}$ eV (upper bd from astrophys).

Non-thermally produced axions can be the *Cold* Dark Matter of the Universe. There are axion field oscillations, and if PQPT after inflation, cold axion particles from strings.

To confirm/distinguish axions from WIMPs?

$1-$ direct detection: find WIMPs or axions in terrestrial searches (CAST, ADMX...)

$2-$ during structure formation: redshift, grow linear density growth same

$\Rightarrow$ *are axions different from WIMPs during non-linear structure formation ?*

Speculations...on distinguishing axions from WIMPs during non-linear structure formation

? Maybe a classical field forms a galaxy differently from WIMPs?
$\Rightarrow$ ask a friendly N-body person to write an axion code?
$\Rightarrow$ (if there are axions from strings, do they condense to a field??)

Sikivie answer :
1-gravity drives Bose Einstein condensation of axions
2-the BE condensate supports vortices, and the vortices give observable caustics

caveat about 1: remains to be shown that the gravitational interaction rate of axions is a thermalisation rate, or how it changes the axion distribution

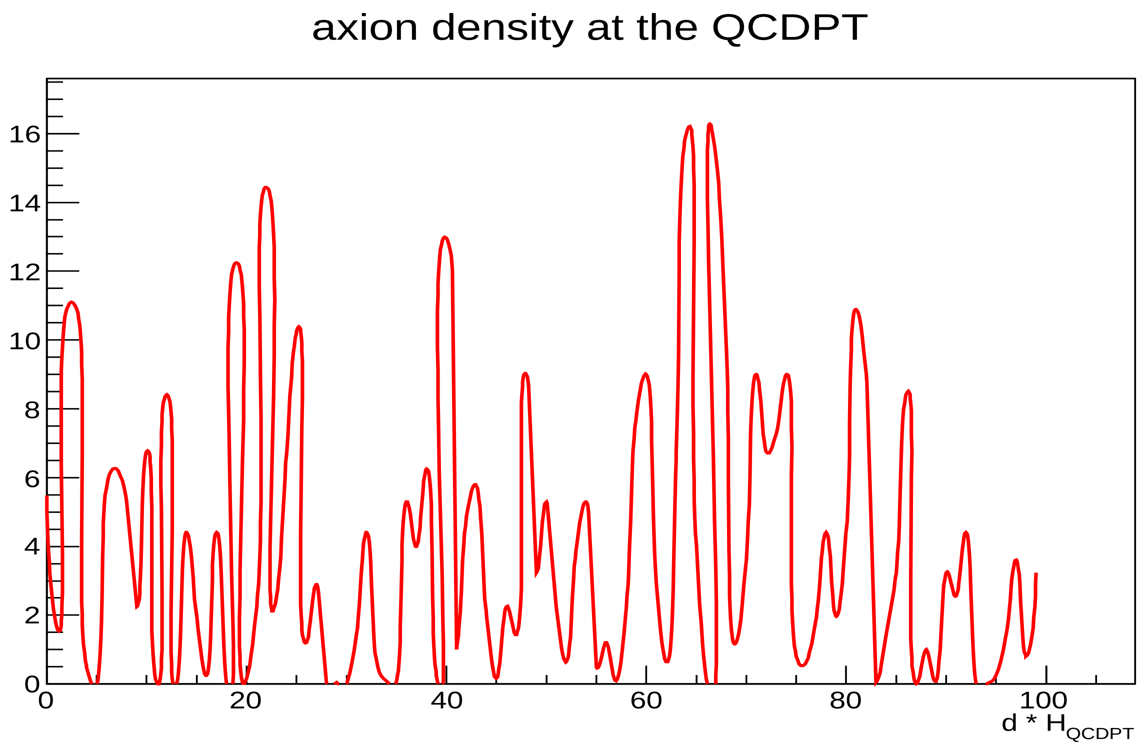
$*$ leading order classical equations (no entropy generation?) for axions in perturbed FRW reproduce the gravitational interaction rate Sikivie identifies as a thermalisation rate. Maybe its the gravitational interactions growing the density fluctuations?

$*$ We found some dissipative gravitational interactions, but suppressed by p_a^2/m_a^2...

Backup

Inhomogeneities are $\mathcal{O}(1)$ on the QCD horizon scale

$a(\vec{x},t)$ random from one horizon(~ 5km) to next; $\rho_a(\vec{x},t) \simeq m_a^2 a^2(\vec{x},t)$



$\Rightarrow$ its *not* a spatially homogeneous distribution of particles various momenta

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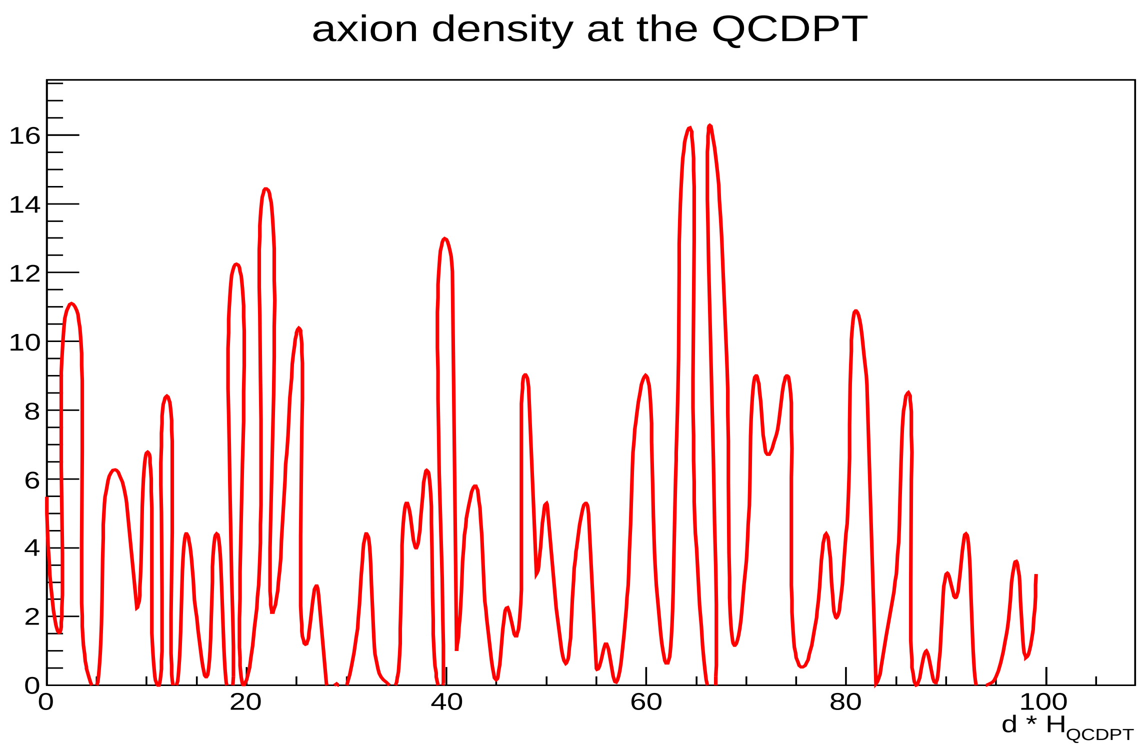

But how can axions form a *homogeneous-on-QCD-horizon-scale* bose-einstein condensate = zero mode of field? ??

$v = H_{QCDPT}/m_a \lesssim 10^{-6}c$...not "free-stream" QCD-horizon distance before t_{eq}:

$$d(t) = \int^t \frac{H_{QCDPT}}{m_a R(t')} dt' \sim \frac{H_{QCDPT}}{m_a} \frac{1}{H(t)R(t)} = \frac{R(t)}{m_a} \ll \frac{R(t)}{H_{QCDPT}}$$

(RD U, $R(t)$ =1@QCDPT)

The (beautiful) calculation of Saikawa and Yamaguchi

Suppose PQ PT after inflation. The classical axion field can be represented as a coherent state of axion particles (of momentum $\lesssim H_{QCDPT}$).
QFT rate for axions (momentum $\vec{k}$) to emit gravitons:

$$i\frac{\partial \hat{n}_k}{\partial t} = \left[\hat{H}_{int}, \hat{n}_k\right] \simeq \frac{G_E}{H(t)^2}\rho_a^2 \gg H(t)n_k$$

Saikawa+Yamaguchi

(evaluated in coherent state ⇔ classical field caln.)

Sikivie interprets as gravitational thermalisation rate: hugely occupied low-$\vec{p}$ modes, equilibrium after $T_\gamma \lesssim$ keV, → BE condensate.

But are some of those gravitons expanding the U, and some growing fluctuations?

Why is that a thermalisation rate??

thermalisation in closed unitary systems?

$$\text{entropy} = \sum_{states\ s} P_s \ln P_s \quad \text{increases}$$

- unitary evolution creates no entropy $\Leftrightarrow$ NO entropy generation in closed systems ... BUT... can calculate "effective" thermalisation: a subset of observables evolve towards equilibrium expectations
  $\Rightarrow$ the "rest" of the system is the bath??

- ex: couple two SHOs. Solve one, substitute into Eqns of second, and find dissipation.

- ...$K - \bar{K}$ evolution is non-unitatry, because not also follow 2π 3π states...

? $\Rightarrow$ divide axions+gravity into

1. U expansion + structure growth
2. other fluctuations which are the bath?

gravity and the second law

1. undergraduate memories say that gravitational collapse of a gas cloud to a star respects the second law...

2. story of $\Omega_{baryon} = 1$ U
   (a) quasi-homogeneous dust clouds collapse
   (b) ...generations of stars, supernovae, black holes...
   (c) ... ... proton decays...
   (d) venerable homogeneous and isotropic U full of photons and gravitons

3. so gravitational thermalisation of axions will happen.
   But does it happen before the U a year old?

Particles vs fields

Develop field operator

$$\hat{a}(t,\vec{x}) = \frac{1}{[R(t)L]^{3/2}}\int \frac{d^3k}{(2\pi)^3}\Big\{\hat{b}_{\vec{k}}\frac{\chi(t)}{\sqrt{2\omega}}e^{i\vec{k}\cdot\vec{x}} + \hat{b}^{\dagger}_{\vec{k}}\frac{\chi^*(t)}{\sqrt{2\omega}}e^{-i\vec{k}\cdot\vec{x}}\Big\}$$

then write the coherent state:

$$|a(\vec{x},t)\rangle \quad \propto \quad \exp\left\{\int \frac{d^3p}{(2\pi)^3}a(\vec{p},t)b^{\dagger}_{\vec{p}}\right\}|0\rangle$$

which satisfies $\hat{b}_{\vec{q}}|a(\vec{x},t)\rangle = a(\vec{q},t)|a(\vec{x},t)\rangle$ (can check $\hat{b}_{\vec{q}}\{1+\int \frac{d^3p}{(2\pi)^3}a(\vec{p},t)b^{\dagger}_{\vec{p}}\}|0\rangle = a(\vec{q},t)|0\rangle$)
where the classical field is

$$a(t,\vec{x}) = \frac{1}{[R(t)L]^{3/2}}\int \frac{d^3k}{(2\pi)^3}\Big\{a(\vec{k},t)\frac{\chi(t)}{\sqrt{2\omega}}e^{i\vec{k}\cdot\vec{x}} + a^*(\vec{q},t)\frac{\chi^*(t)}{\sqrt{2\omega}}e^{-i\vec{k}\cdot\vec{x}}\Big\}$$