Galileon Scalars
& Massive Gravity

Mark Trodden
Center for Particle Cosmology
University of Pennsylvania

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Overview

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- Conclusions.

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- We'll see - too early to know if these will be useful or not - but it is turning out to be great fun trying.

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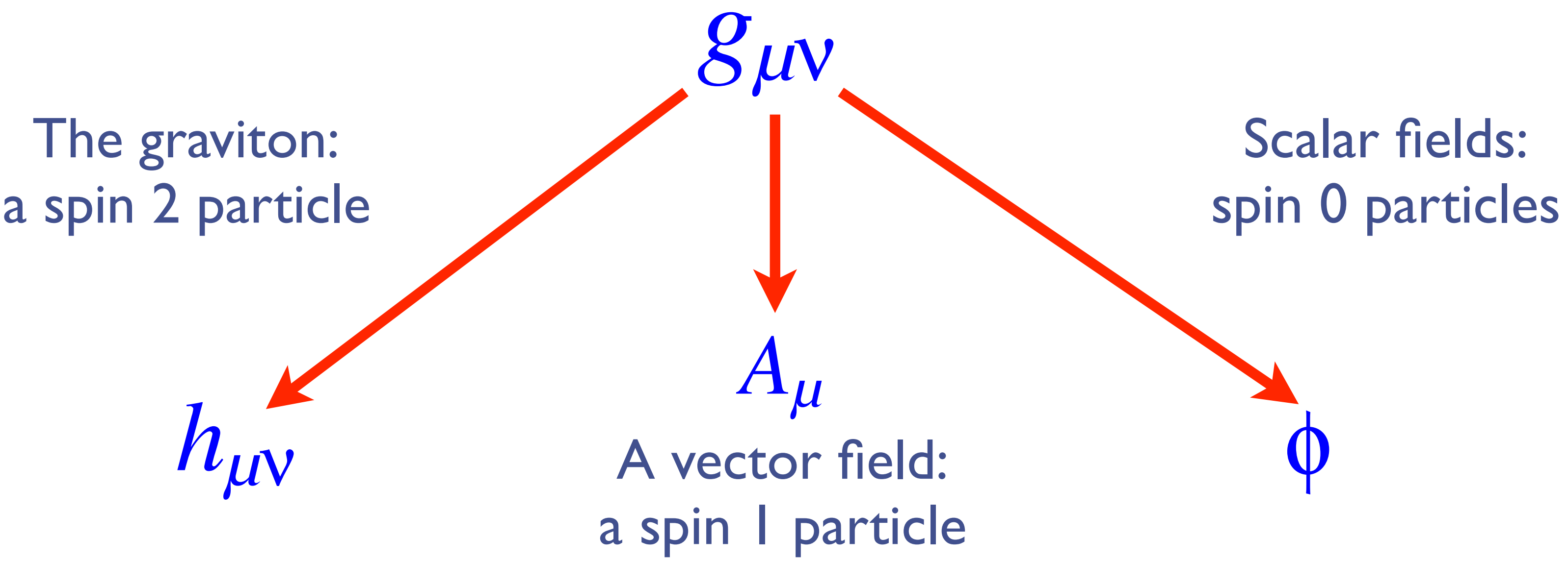

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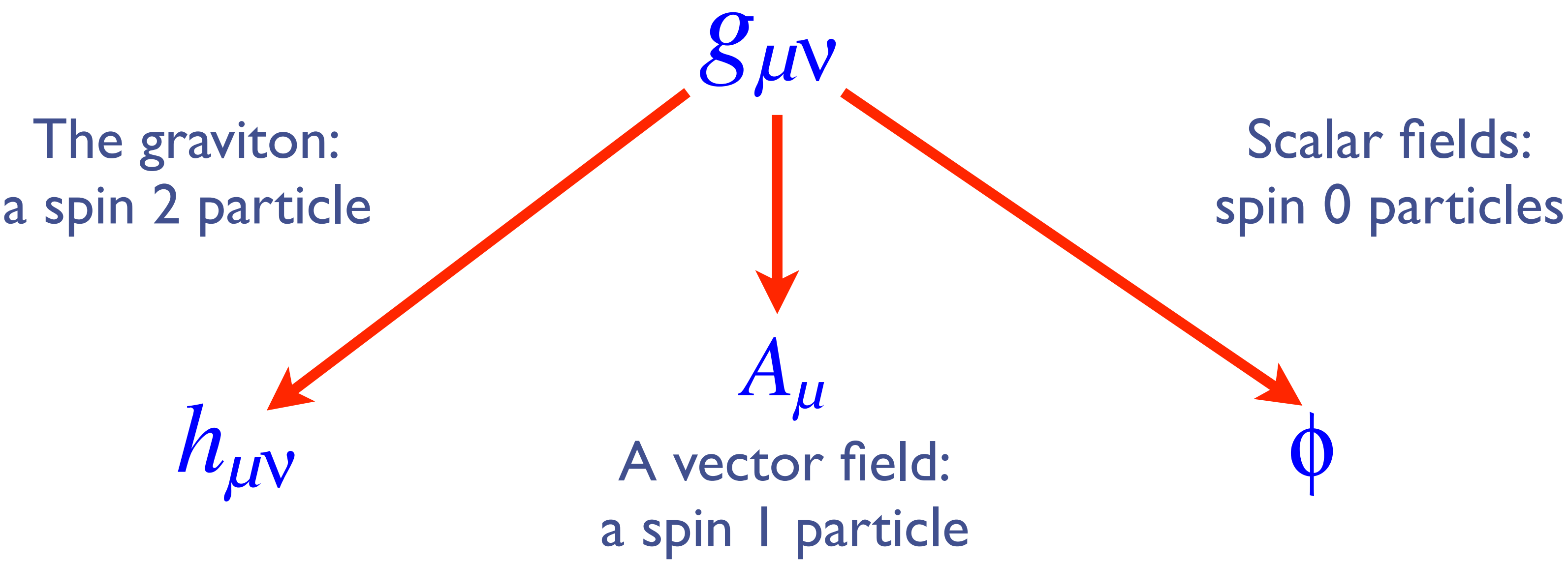

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Which degrees of freedom propagate depends on the action.

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- In each case should "resum" theory about the relevant background, and EFT of excitations around a nontrivial background is not the naive one.
- Around the new background, theory is safe from local tests of gravity.

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$$\pi(x) \to \pi(x) + c + b_\mu x^\mu$$

The *Galilean* symmetry!

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Now proven to be ghost free, and investigations of the resulting cosmology - acceleration, degravitation, ... are underway. but in a limit this also yields ...

[Hassan & Rosen(2011)]

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  - Are non-renormalized! (More soon).

Luty, Porrati, Ratazzi (2003); Nicolis, Rattazzi (2004)

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So forces much smaller than gravitational strength within the Vainshtein radius - hence safe from 5th force tests.

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$$\mathcal{L} = -3(\partial\varphi)^2 + \frac{2}{\Lambda^3}\underline{\left(\partial_\mu\partial_\nu\pi_0 - \eta_{\mu\nu}\Box\pi_0\right)}\,\partial^\mu\varphi\partial^\nu\varphi - \frac{1}{\Lambda^3}(\partial\varphi)^2\Box\varphi + \frac{1}{M_4}\varphi\delta T$$

$$\sim \left(\frac{R_v}{r}\right)^{3/2}$$

The Vainshtein Effect

Suppose we want to know the the field that a source generates within the Vainshtein radius of some large body (like the sun, or earth)

Perturbing the field and the source

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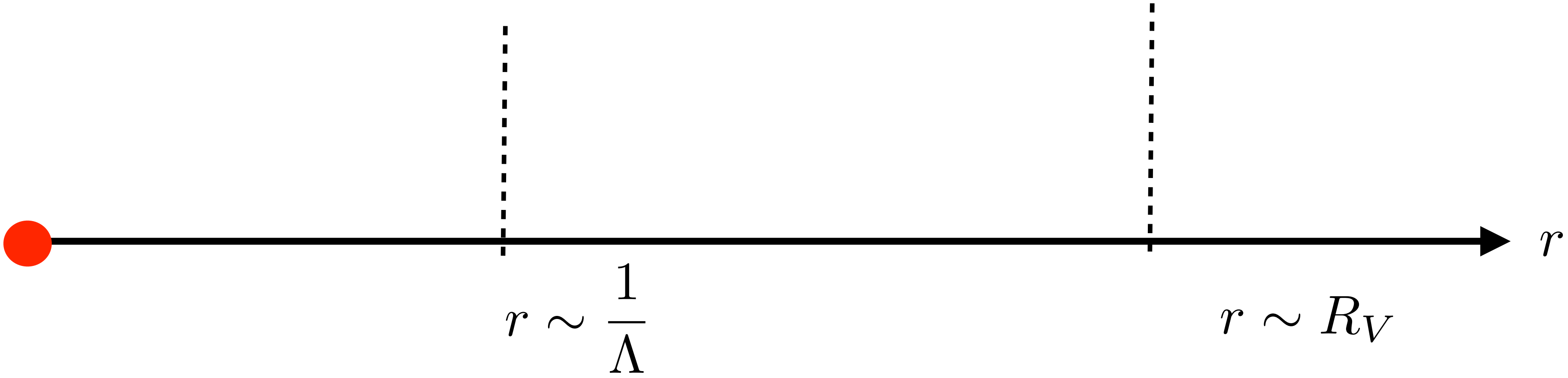
Thus, if we canonically normalize the kinetic term of the perturbations, we raise the effective strong coupling scale, and, more importantly, heavily suppress the coupling to matter!

Regimes of Validity



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$$r \ll \frac{1}{\Lambda}$$

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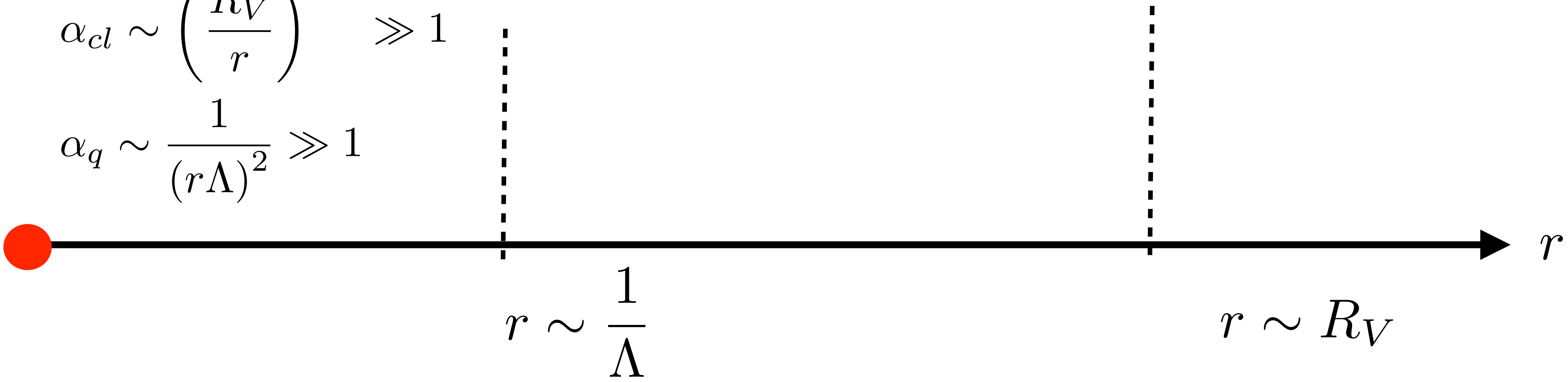

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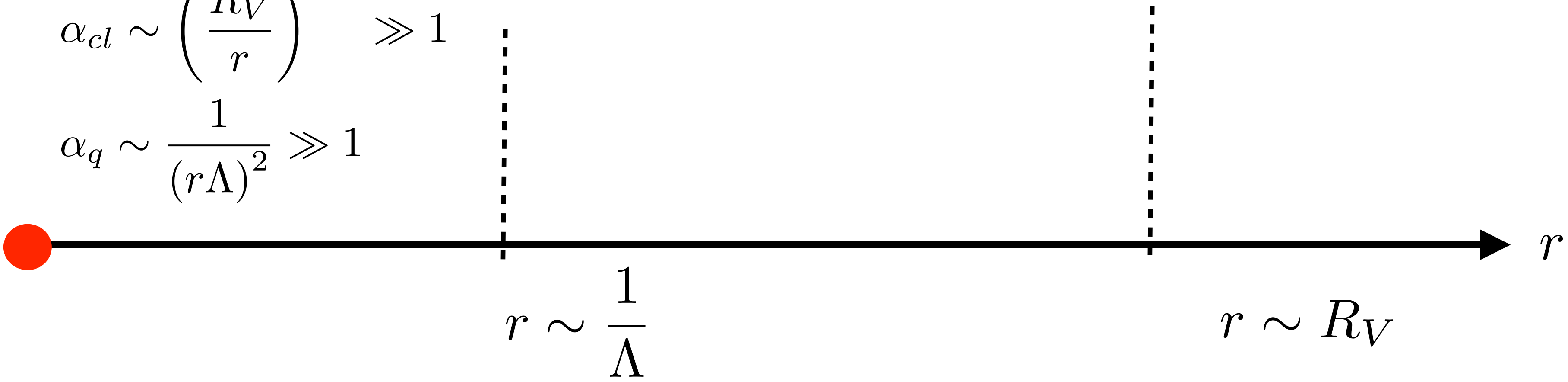

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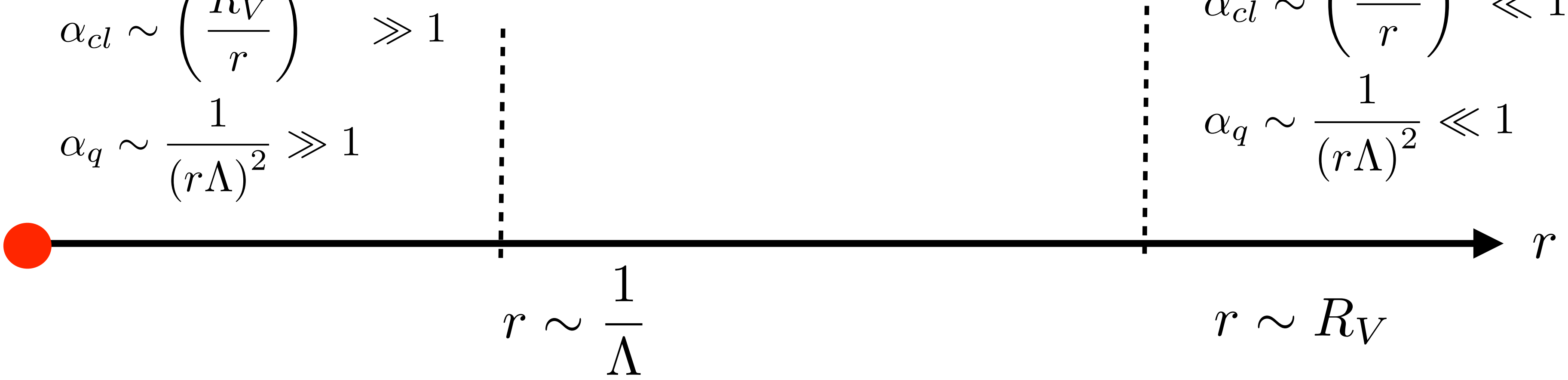

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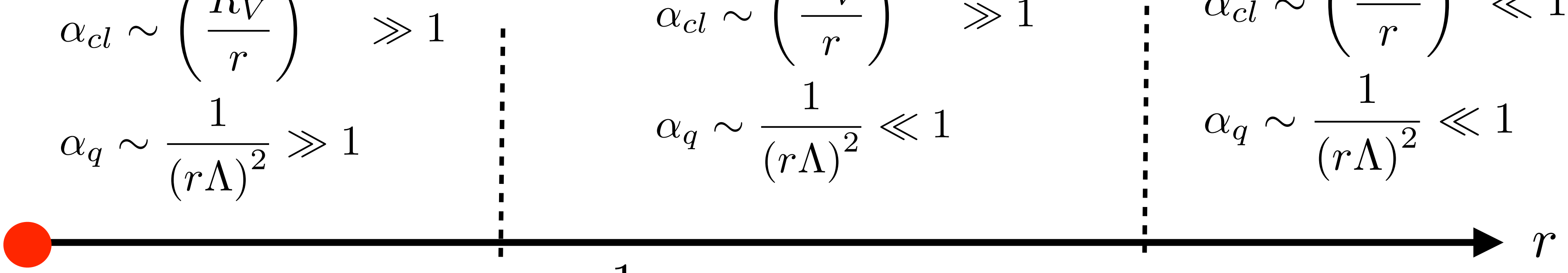

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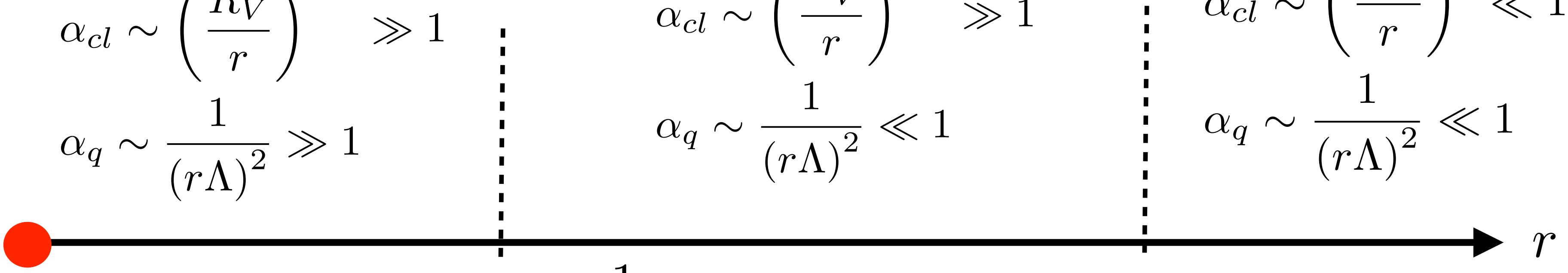

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A new classical regime, with order one nonlinearities

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Instead of extending Poincare symmetry by galilean one, might seek to extend to other useful symmetries. Making relativistic:

$$\delta\pi = c + b_\mu x^\mu - b^\mu \pi \partial_\mu \pi$$

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makes full symmetry group P(4,1), spontaneously broken to P(3,1).

Again get n terms in n-dimensions, and the galileons in the small field limit

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If we instead extend to the conformal group

$$\delta\pi = c - c x^\mu \partial_\mu \pi$$

$$\delta\pi = b_\mu x^\mu + \partial_\mu \pi \left(\frac{1}{2} b^\mu x^2 - (b \cdot x) x^\mu \right)$$

CONFORMAL GALILEONS

makes full symmetry group SO(4,2), spontaneously broken to P(3,1).
Again get n terms in n-dimensions. e.g.

$$\mathcal{L}_2 = -\frac{1}{2} e^{-2\hat{\pi}} (\partial\hat{\pi})^2$$

$$\mathcal{L}_3 = \frac{1}{2} (\partial\hat{\pi})^2 \Box\hat{\pi} - \frac{1}{4} (\partial\hat{\pi})^4$$

Constructing Galileons: Probe Branes

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[de Rham & Tolley]

Embed a flat 3-brane in a 5d flat bulk

Symmetries are:

$$\begin{aligned}\delta_P X^A &= \omega^A{}_B X^B + \epsilon^A \\ \delta_g X^A &= \xi^\mu \partial_\mu X^A\end{aligned}$$

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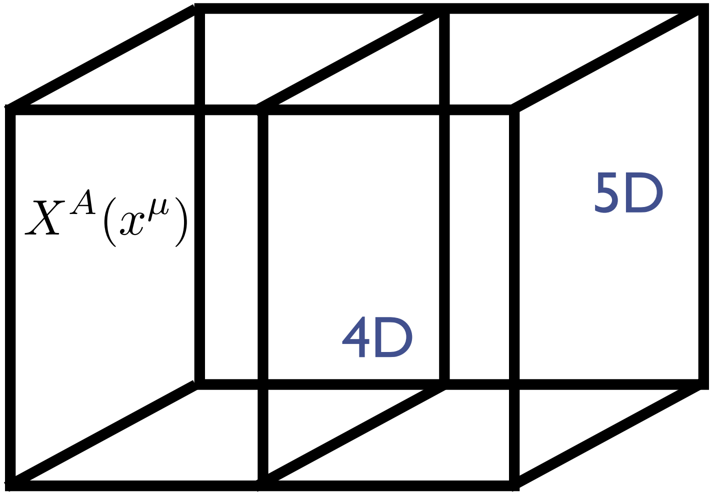

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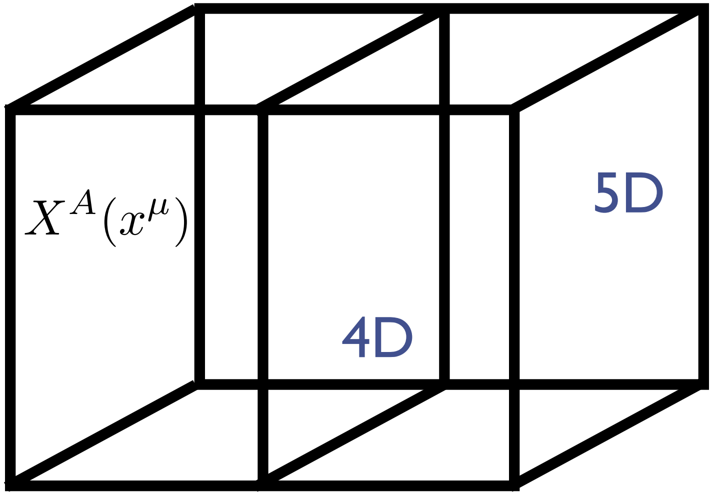

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A Poincare transformation ruins this choice, **but**: a simultaneous brane reparametrization restores it, so that the combination

$$\delta_{P'}\pi = \delta_P \pi + \delta_g \pi = -\omega^\mu{}_\nu x^\nu \partial_\mu \pi - \epsilon^\mu \partial_\mu \pi + \omega^5{}_\mu x^\mu - \omega^\mu{}_5 \pi \partial_\mu \pi + \epsilon^5$$

is still a symmetry

What remains is to construct actions

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$$S=\int d^4x\,\sqrt{-g}F\left(g_{\mu\nu},\nabla_\mu,R^\rho{}_{\sigma\mu\nu},K_{\mu\nu}\right)\Big|_{g_{\mu\nu}=\eta_{\mu\nu}+\partial_\mu\pi\partial_\nu\pi}$$

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This approach is extremely powerful - allows generalizing the theory in a large number of ways to theories that would be hard to find another way.

Higher co-Dimension Probe Branes

[K. Hinterbichler, M.T., D. Wesley, *Phys. Rev.* D82 (2010) 124018.]

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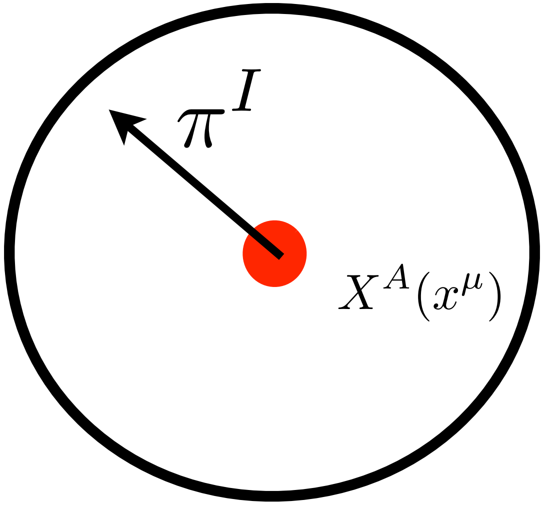

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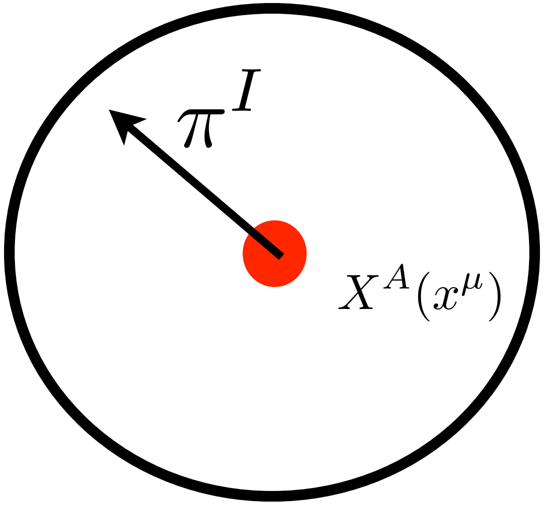

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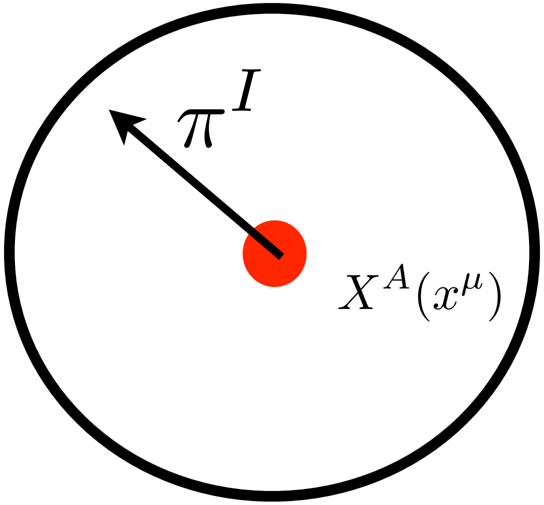

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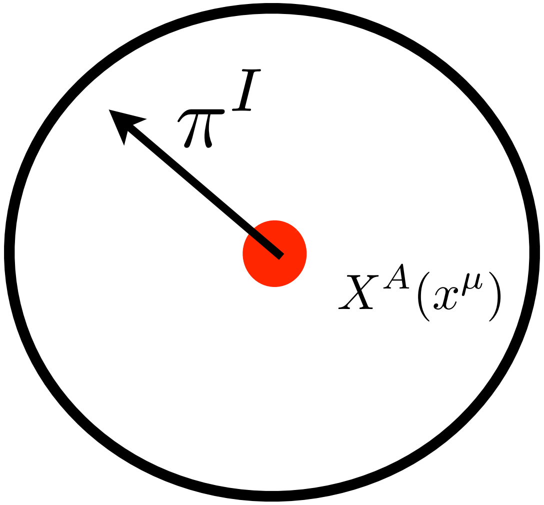

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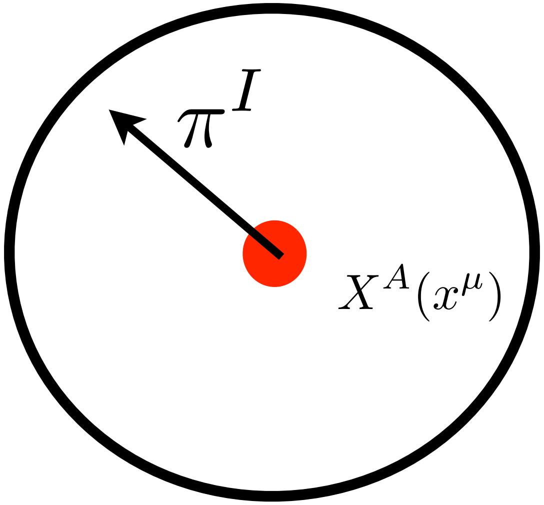

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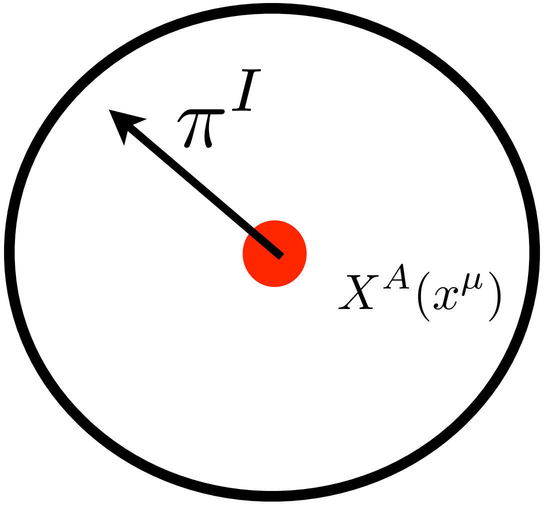

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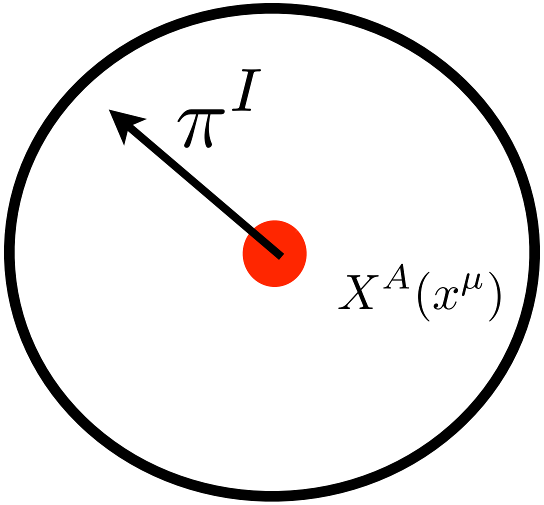

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$$\nabla_\rho K^i_{\mu\nu} = \partial_\rho K^i_{\mu\nu} - \Gamma^\sigma_{\rho\mu} K^i_{\sigma\nu} - \Gamma^\sigma_{\rho\nu} K^i_{\mu\sigma} + \beta^i_{\rho j} K^j_{\mu\nu}$$

The Multi-Galileon Limit

[K. Hinterbichler, M.T., D.Wesley, *Phys. Rev.* D82 (2010) 124018;
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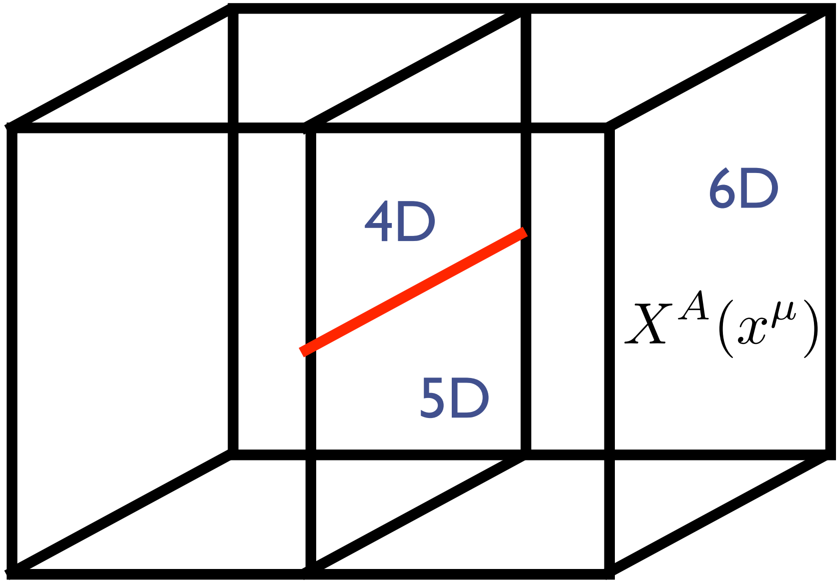

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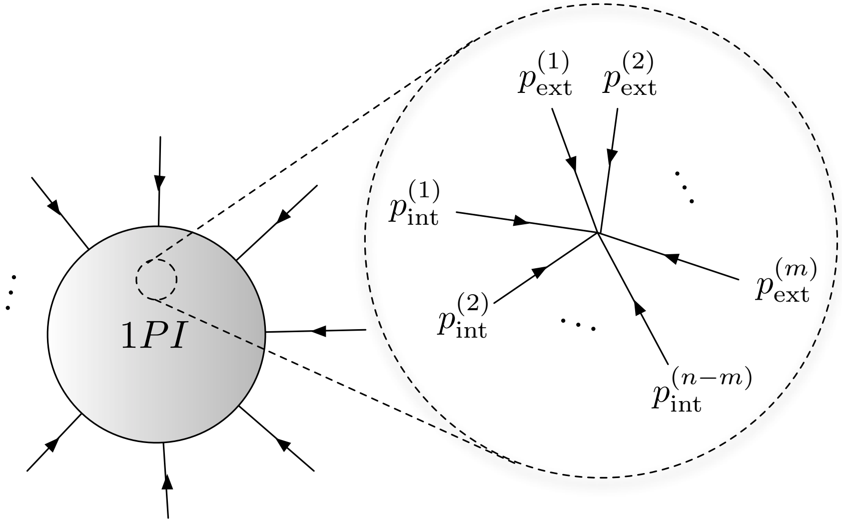

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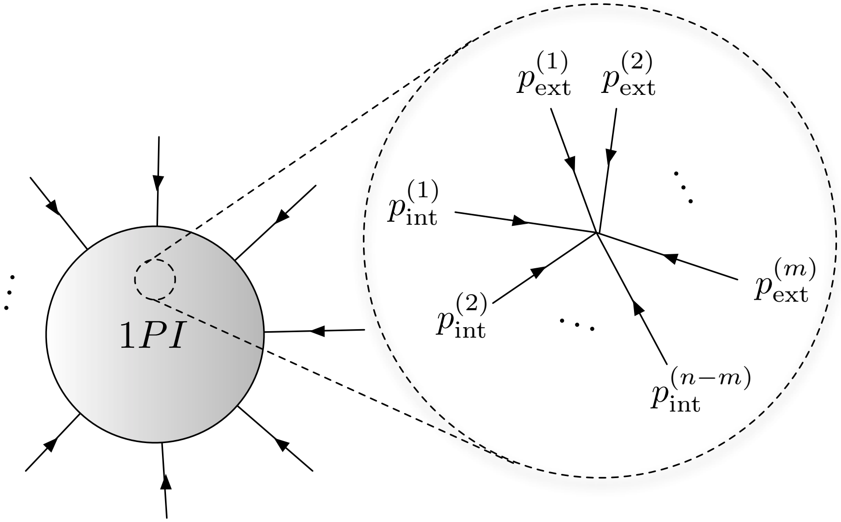

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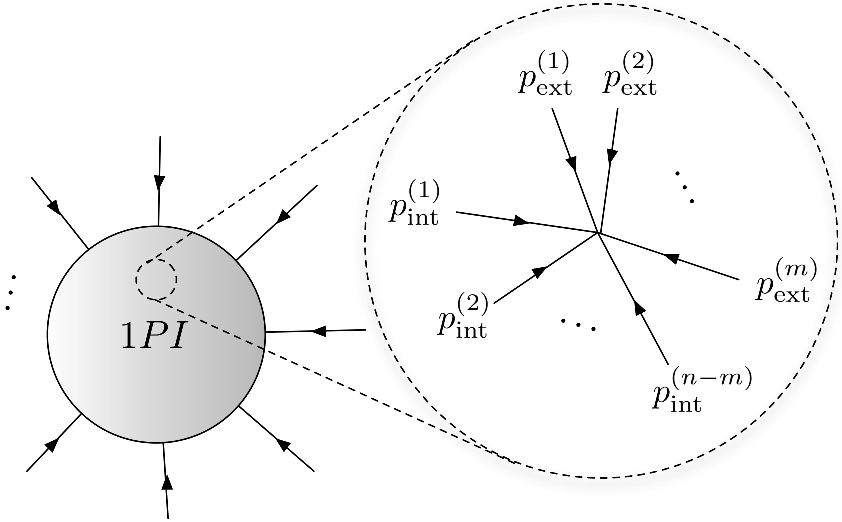

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Can even add a mass term and remains technically natural

Galileons on General Backgrounds

[Goon, Hinterbichler, M.T., *Phys. Rev.Lett.* 106, 231102 (2011).
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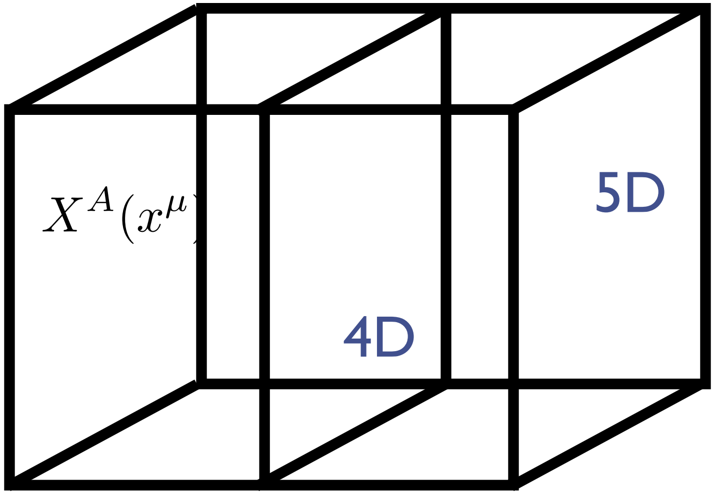

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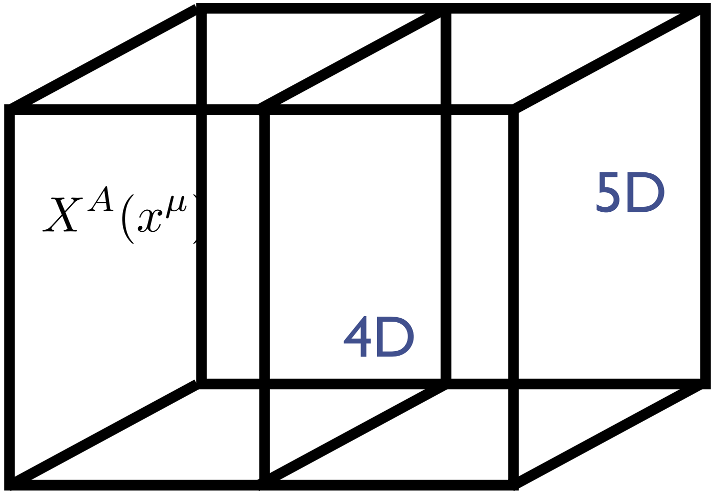

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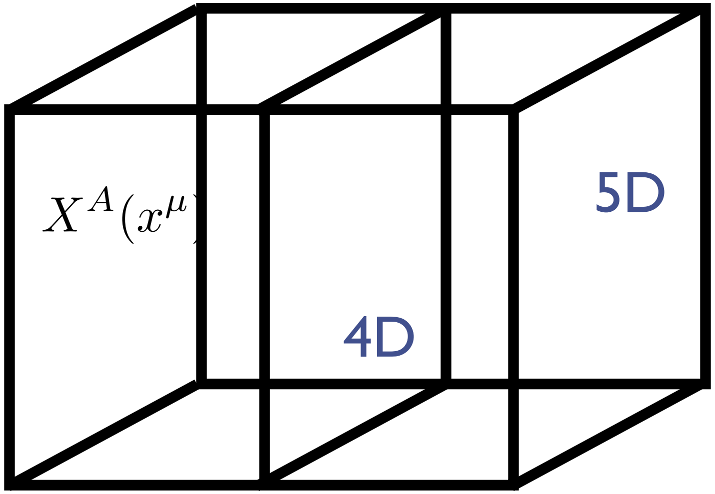

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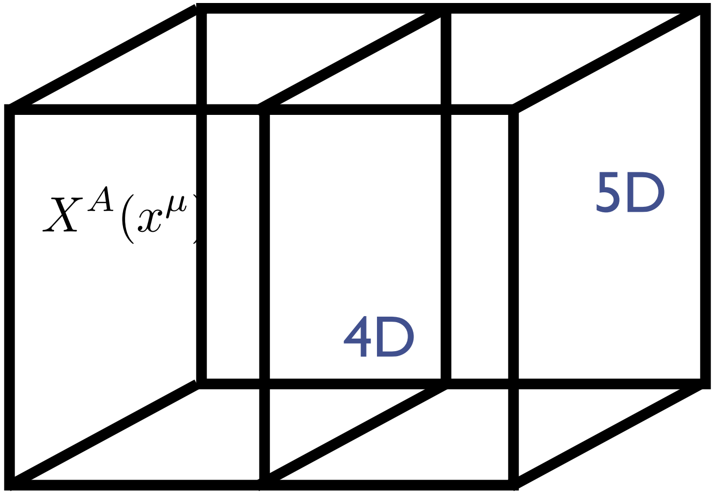

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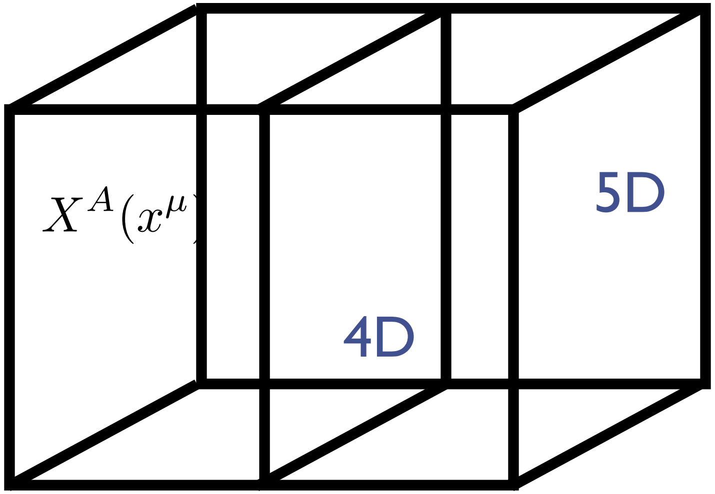

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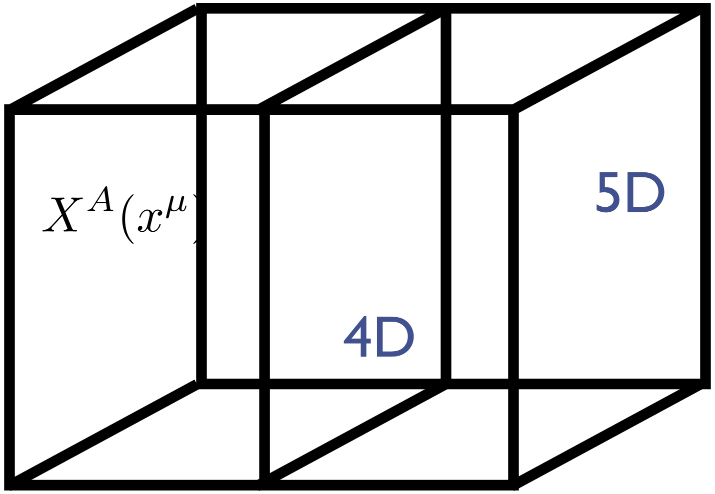

Galileons with symmetry

$$(\delta_K + \delta_{g,\mathrm{comp}})\pi = -a^i k_i^\mu(x)\partial_\mu \pi + a^I K_I^5(x,\pi) - a^I K_I^\mu(x,\pi)\partial_\mu \pi$$

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Brane metric

Ambient metric	AdS_4	M_4	dS_4
AdS_5	AdS DBI galileons $so(4,2) \to so(3,2)$ $f(\pi) = \mathcal{R}\cosh^2(\rho/\mathcal{R})$	Conformal DBI galileons $so(4,2) \to p(3,1)$ $f(\pi) = e^{-\pi/\mathcal{R}}$	type III dS DBI galileons $so(4,2) \to so(4,1)$ $f(\pi) = \mathcal{R}\sinh^2(\rho/\mathcal{R})$
M_5	×	DBI galileons $p(4,1) \to p(3,1)$ $f(\pi) = 1$	type II dS DBI galileons $p(4,1) \to so(4,1)$ $f(\pi) = \pi$
dS_5	×	×	type I dS DBI galileons $so(5,1) \to so(4,1)$ $f(\pi) = \mathcal{R}\sin^2(\rho/\mathcal{R})$

Small field limit ↓

AdS galileons	normal galileons	dS galileons

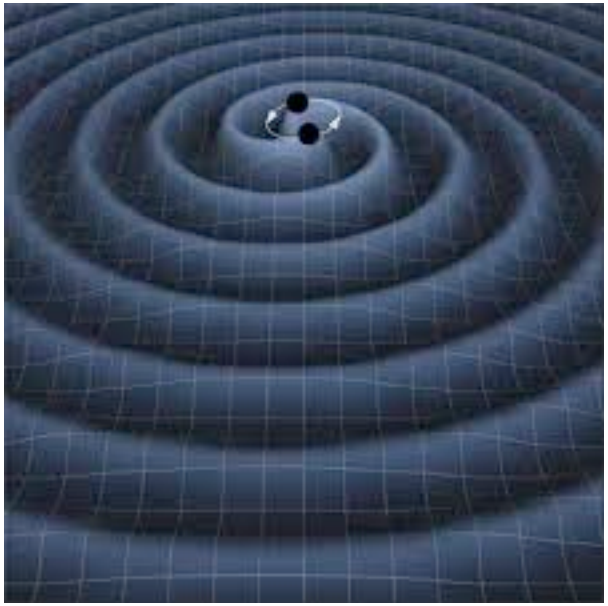
Some interest in these is motivated from cosmology

But might they predict other new effects, and how might we understand them better?

Example - Galileon Waves

[Chu & M.T., arxiv:1210.6651; (also de Rham & Tolley; Matas, de Rham & Tolley)]

- Recently have constructed Galileon retarded Green's function about background field of a central mass (& exact static Green's function)



- Have applied to radiation spectrum due to motion of n point masses gravitationally bound to central mass M.
- Have focused on the non-relativistic limit

Interesting surprise - direct consequence of structure of Galileon radial Green's function.

- In the high frequency limit ($\omega r_{\rm V} \gg l$), get anticipated Vainshtein screening of Galileon radiation at low multipole orders. BUT!
- At high enough multipoles, high frequency Galileon radiation enhanced!

Galileon Waves

- If Galileon waves exist, in principle detectable by GW detectors.
- Ordinary matter experiences an effective weakly curved metric
- Tidal forces experienced by the arms of the interferometers of GW detectors would now be due to both the transverse-traceless graviton and the Galileon waves.

$$g_{\mu\nu} = \eta_{\mu\nu} + h^{(\mathrm{eff})}_{\mu\nu}$$

Now trying to do the relevant equal-mass binary calculation

Interesting directions for future work:

Introduce quartic and quintic Galileon terms and carry out an analogous analysis.

Develop understanding of backreaction of power loss on the motion of n point masses.

Other Work & the Future

- Galileons are Wess-Zumino terms! In d dimensions are d-form potentials for (d+1)-forms which are non-trivial co-cycles in Lie algebra cohomology of full symmetry group relative to unbroken one. Slightly different stories for DBI and conformal Galileons.

[Goon, Hinterbichler, Joyce & M.T., arxiv:1203.319]

- Our models tell you what Galileons do propagating on cosmological spaces. What about driving cosmology? Need dynamical (massive) gravity; and we now know how to do this.

[Gabadadze, Hinterbichler, Khoury, Pirtshkalava & M.T., arxiv:1208.5773]

- And that theory is ghost-free!

[Andrews, Goon, Hinterbichler, Stokes & M.T., arxiv:1303.1177]

- We've begun investigating cosmology.

[Hinterbichler, Stokes & M.T., arxiv:1301.4993; Andrews, Hinterbichler, Stokes & M.T., arxiv:1306.5743]

- Systematic Tests of Gravity Analysis

[Chu & M.T., arxiv:1210.6651;
Andrews, Chu Hinterbichler & M.T., arxiv:1305.2194]

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