Q-balls, oscillons and total screening from ultramassive scalar fields

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Harmonically coupled, inverted pendulums

The signum-Gordon model

Real scalar field $\varphi(x,t)$ in 1+1 dimensions with

$$L = \frac{1}{2}(\partial_t\varphi\partial_t\varphi - \partial_x\varphi\partial_x\varphi) - g\,|\varphi|.$$

The signum-Gordon equation

$$\partial_t^2\varphi - \partial_x^2\varphi + g\,\mathrm{sign}\,\varphi = 0,$$

$\mathrm{sign}\,0 = 0$, $g > 0$, all variables dimensionless.

Comparing with the φ^4 scalar field:

- Lack of linear regime around the vacuum field $\varphi = 0$
- the V-shaped field potential $\rightarrow$ '$m_0^2 = \infty$', the ultramassive field

General features in any dimensions $d \geq 2$:

- Scale invariance of the 'on shell' type with unusual exponent
- Parabolic approach to the vacuum field (compactness)
- Relatively easy to solve analytically

The scale invariance

Take a solution $\varphi(x,t)$ of s-G equation; then

$$\varphi_\lambda(x,t) = \lambda^2\ \varphi(\frac{x}{\lambda}, \frac{t}{\lambda}),$$

$\lambda > 0$, also obeys that equation. The action is not invariant.
There exists a rich variety of self-similar solutions.

The total energy

$$E[\varphi] = \frac{1}{2}\int dx\ [(\partial_t\varphi)^2 + (\partial_x\varphi)^2] + g\ \int dx\ |\varphi|$$

scales as

$$E[\varphi_\lambda] = \lambda^3 E[\varphi].$$

In the limit $\lambda \to 0$: high frequencies, short waves, small amplitude

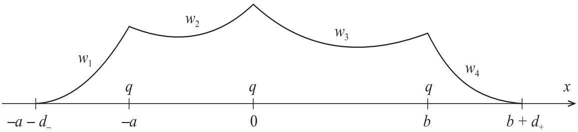
and small energy .

For the massless φ^4: $\lambda^2 \to \lambda^{-1}$, small energy and amplitude appear in the limit $\lambda \to \infty$. Then frequencies are low and waves long.

The total screening of charges (example)

Signum-Gordon equation with point-like external sources on the r.h.s.

Three identical point charges totally screened by the scalar field φ:



$$w_1(x) = \frac{g}{2}\,(x+a+d_-)^2\,, \quad w_3(x) = \frac{g}{2}\,(x-b-d_+)^2 + q(x-b), \quad \ldots$$

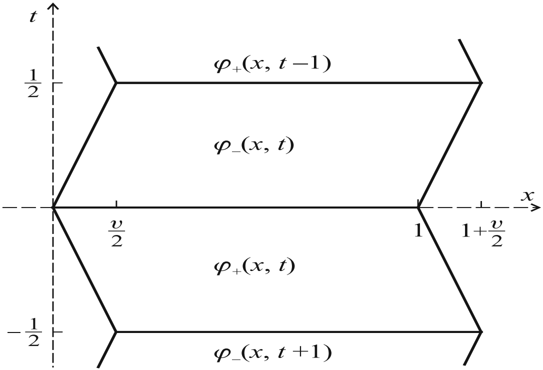
$$d_- = \frac{3a_*}{2} - \frac{2a+b}{3}, \quad d_+ = \frac{3a_*}{2} - \frac{a+2b}{3}, \quad a_* = \frac{q}{g}.$$

The total force on q at $x=-a$: $\quad F(-a) = q^2\left(1 - \frac{2a+b}{3a_*}\right).$

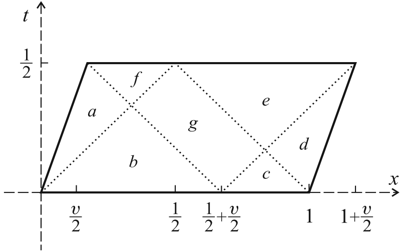
3-body component is present: $\quad F_{3body}(-a) = q^2\,\frac{2a+b}{6a_*}.$

World-sheet of the swaying oscillon

$gt \to t,\ gx \to x$



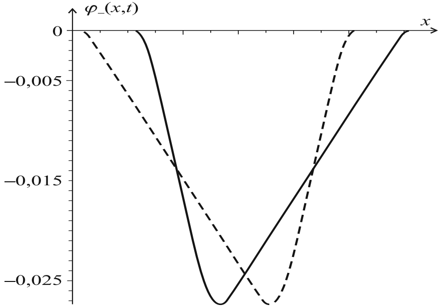
The structure of $\varphi_-(x, t)$



$$\varphi_a = -\frac{(x - vt)^2}{2(1 - v^2)}, \quad \varphi_b = \frac{t^2}{2} - \frac{tx}{1 + v}, \quad \varphi_c = \frac{t^2}{2} + \frac{t(x - 1)}{1 - v}, \ \ldots$$

Plot of $\varphi_{-}(x, t)$

$v = 1/2$



the dashed line: $t = 1/8$, the continuous line: $t = 3/8$

Q-balls in the complex signum-Gordon model

The complex signum-Gordon model: $L = \partial_\mu \psi^* \partial^\mu \psi - g\,|\psi|$.

The U(1) charge: $Q = \frac{1}{2i} \int d^d x\, (\psi^* \partial_0 \psi - \partial_0 \psi^* \psi)$.

Time-periodic solutions of the form $\psi(\vec{x}, t) = F(r) \exp(i\omega t)$,
with finite total energy (nontopological solitons), real F, $\omega > 0$, $Q > 0$.

$$F'' + \frac{d-1}{r} F' + \omega^2 F = \frac{g}{2} \,\mathrm{sign}\, F,$$

$$Q = c_1 \frac{\lambda^2}{\omega^{d+3}}, \quad E = c_2\, g^{\frac{2}{d+3}} \left(\frac{Q}{c_1}\right)^{\frac{d+2}{d+3}},$$

where the numerical constants c_1, c_2 dependent only on d.
Explicit solutions in $d = 1, 2, 3$ with compact support and arbitrary $\omega > 0$ are known.

The model can be extended by including the electromagnetic field. Interesting new feature: a transition from spherical Q-balls to Q-shells at large values of the charge Q.

Odd features of the ultramassive fields

- The lack of linear regime in a vicinity of the vacuum field
- Numerical methods do not work well here
- Few applications as yet
  - the pendulums and the like
  - boson stars
- How to construct a quantum counterpart?

References:

arXiv: ... [hep-th], Phys. Rev. D and E, Acta Phys. Polon. B

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THANK YOU!