HIGGS MASS AND STABILITY OF THE EW VACUUM

Vincenzo Branchina

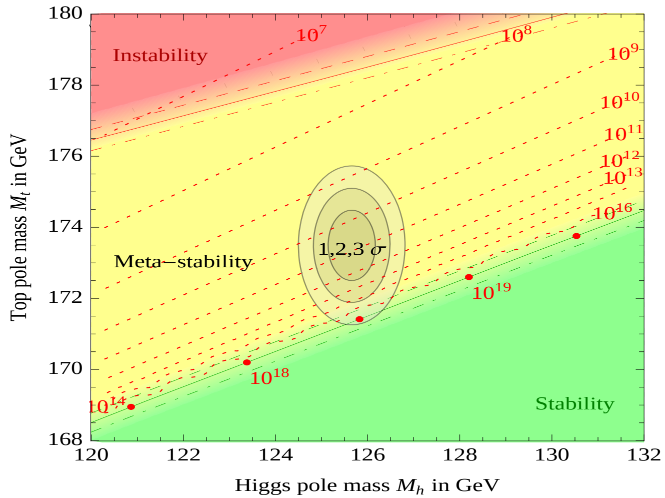
Department of Physics, University of Catania, Italy
and
INFN, Sezione di Catania

work done in collaboration with Emanuele Messina

Scalars - Warsaw 15 September 2013

Introduction and Summary

Stability / Metastability : very delicate issue...

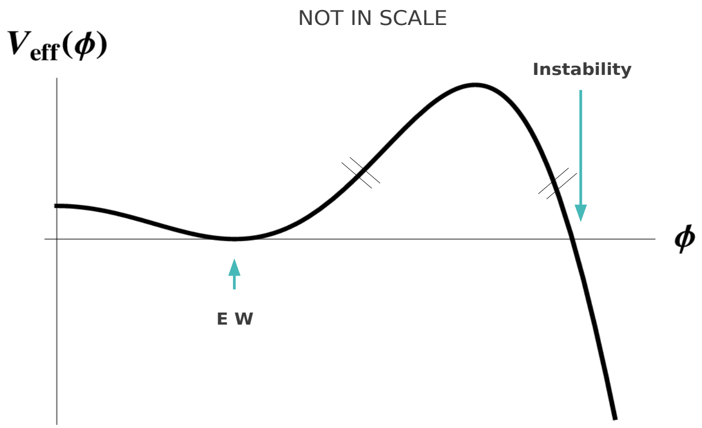


Degrassi et al., JHEP 2012 ; Buttazzo et al., arXiv:1307.3536 [hep-ph]

A complete analysis does not seem to lead to this phase diagram (Why?)

... We learn a quite different lesson ... Interesting consequences ?

One-Loop Effective Potential $V^{1l}(\phi)$

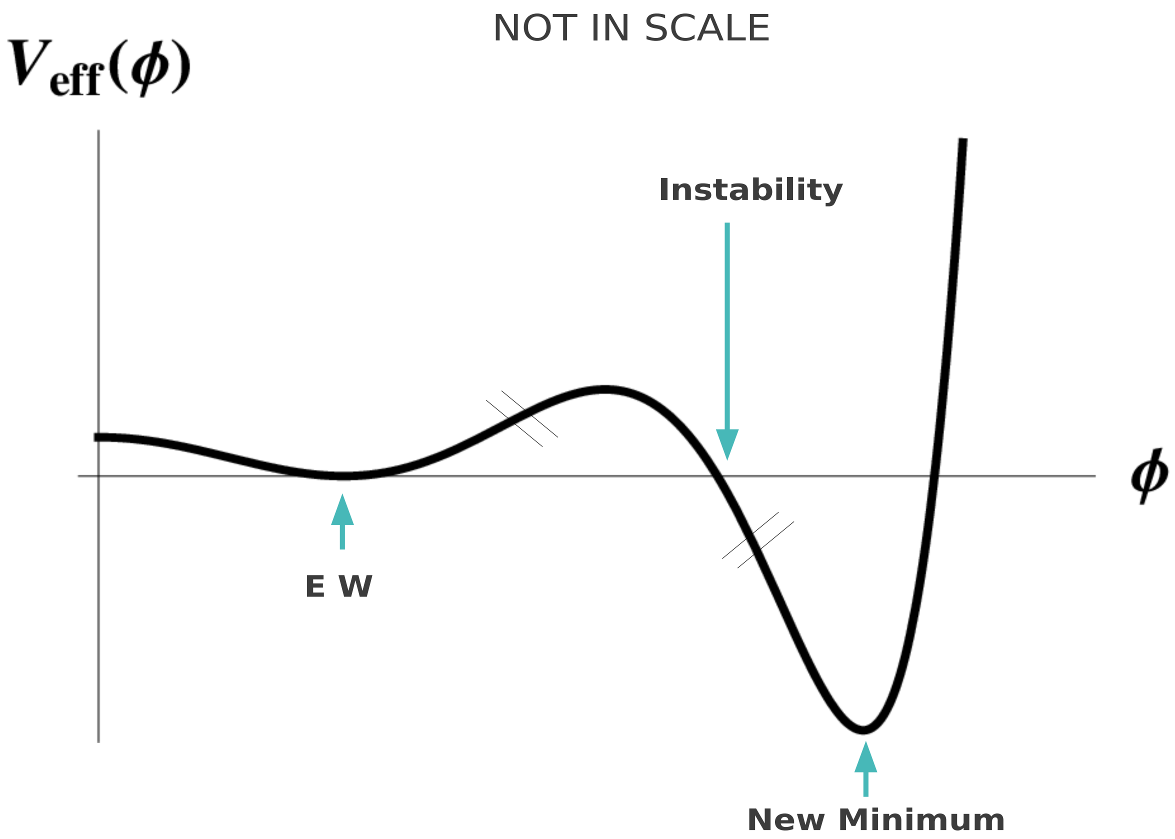


$$V^{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{24}\phi^4 + \frac{1}{64\pi^2}\left[\left(m^2 + \frac{\lambda}{2}\phi^2\right)^2\left(\ln\left(\frac{m^2+\frac{\lambda}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right)\right.$$

$$+3\left(m^2 + \frac{\lambda}{6}\phi^2\right)^2\left(\ln\left(\frac{m^2+\frac{\lambda}{6}\phi^2}{\mu^2}\right) - \frac{3}{2}\right) + 6\,\frac{{g_1}^4}{16}\phi^4\left(\ln\left(\frac{\frac{1}{4}{g_1}^2\phi^2}{\mu^2}\right) - \frac{5}{6}\right)$$

$$\left.+3\,\frac{\left({g_1}^2+{g_2}^2\right)^2}{16}\phi^4\left(\ln\left(\frac{\frac{1}{4}\left({g_1}^2+{g_2}^2\right)\phi^2}{\mu^2}\right) - \frac{5}{6}\right) - 12\,h_t^4\phi^4\left(\ln\frac{g^2\phi^2}{\mu^2} - \frac{3}{2}\right)\right]$$

RG Improved Effective Potential $V_{eff}(\phi)$



The instability occours for large values of the field

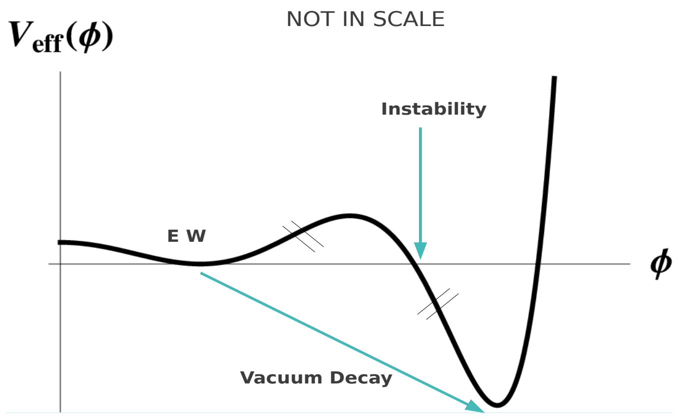
$\Rightarrow$ $V_{eff}(\phi)$ **well approximated by keeping only the quartic term :**

$$V_{eff}(\phi) \sim \frac{\lambda_{eff}(\phi)}{24}\phi^4$$

$\lambda_{eff}(\phi)$ **depends on** ϕ **essentially as** $\lambda(\mu)$ **depends on** μ

$\Rightarrow$ **Read the Effective Potential from the** $\lambda(\mu)$ **flow**

Metastability Scenario



Tunnelling between the Metastable EW Vacuum and the True Vacuum.

As long as EW vacuum lifetime larger than the age of the Universe ....

.... we may well live in the Meta-Stable (EW) Vacuum ....

How is the EW vacuum lifetime (tunneling time) computed?

Tunnelling time τ

$$\frac{1}{\tau} = T_U^3 \, \frac{S[\phi_b]^2}{4\pi^2} \left| \frac{\det' \left[-\partial^2 + V''(\phi_b)\right]}{\det \left[-\partial^2 + V''(v)\right]} \right|^{-1/2} e^{-S[\phi_b]}$$

$\phi_b(r)$ **: Bounce Solution to the Euclidean Equation of Motion**

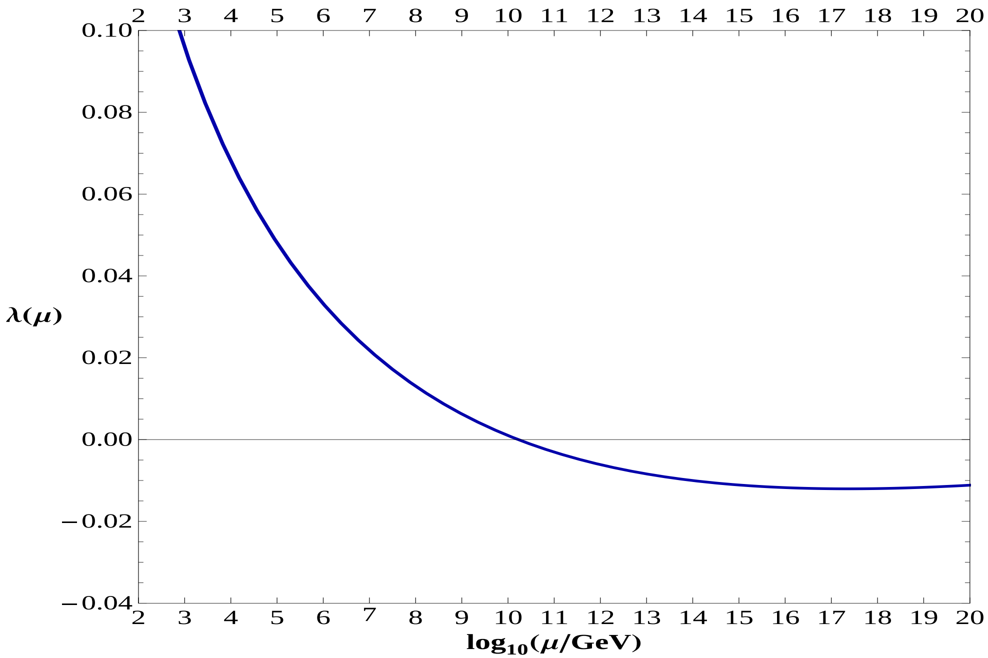
$$r^2 = x_\mu x_\mu$$

S. Coleman, Phys. Rev. D 15 (1977) 2929

C.G.Callan, S.Coleman, Phys. Rev. D 16 (1977) 1762

Tunnelling and Bounces

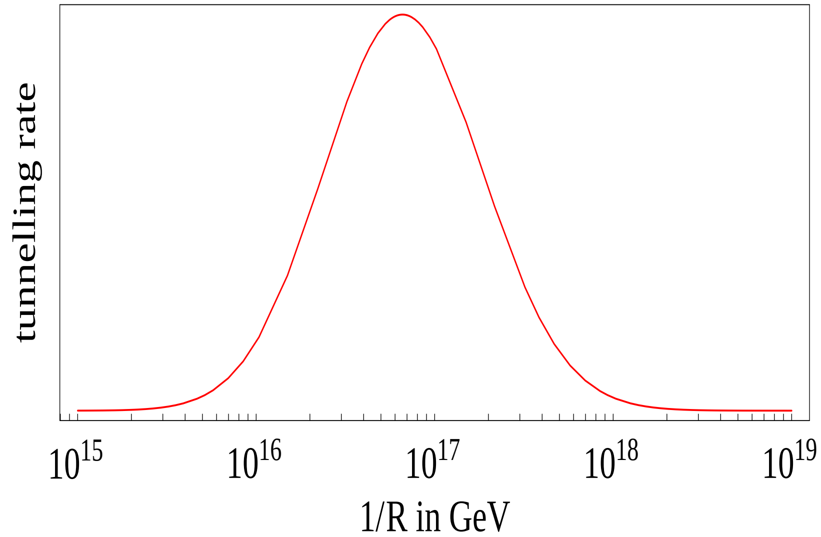
Bounce solutions to the euclidean equation of motion for $V(\phi) = \frac{\lambda}{4}\phi^4$ with constant negative λ, a good approximation in this range!



Bounces : $\phi_b(r) = \sqrt{\frac{2}{|\lambda|}} \frac{2R}{r^2+R^2}$, $S[\phi_b] = \frac{8\pi^2}{3|\lambda|}$

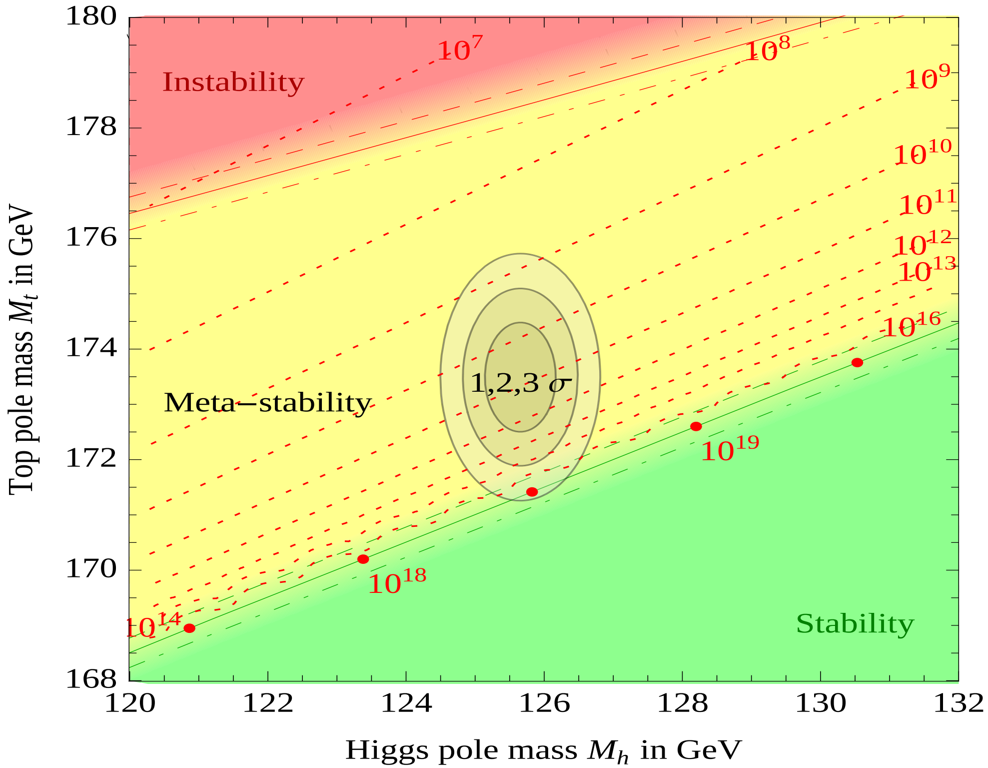
R **: size of the bounce - Degeneracy at the Classical Level**

Degeneracy removed at the Quantum Level



G. Isidori, G. Ridolfi, A. Strumia, Nucl.Phys.B 609 (2001) 387

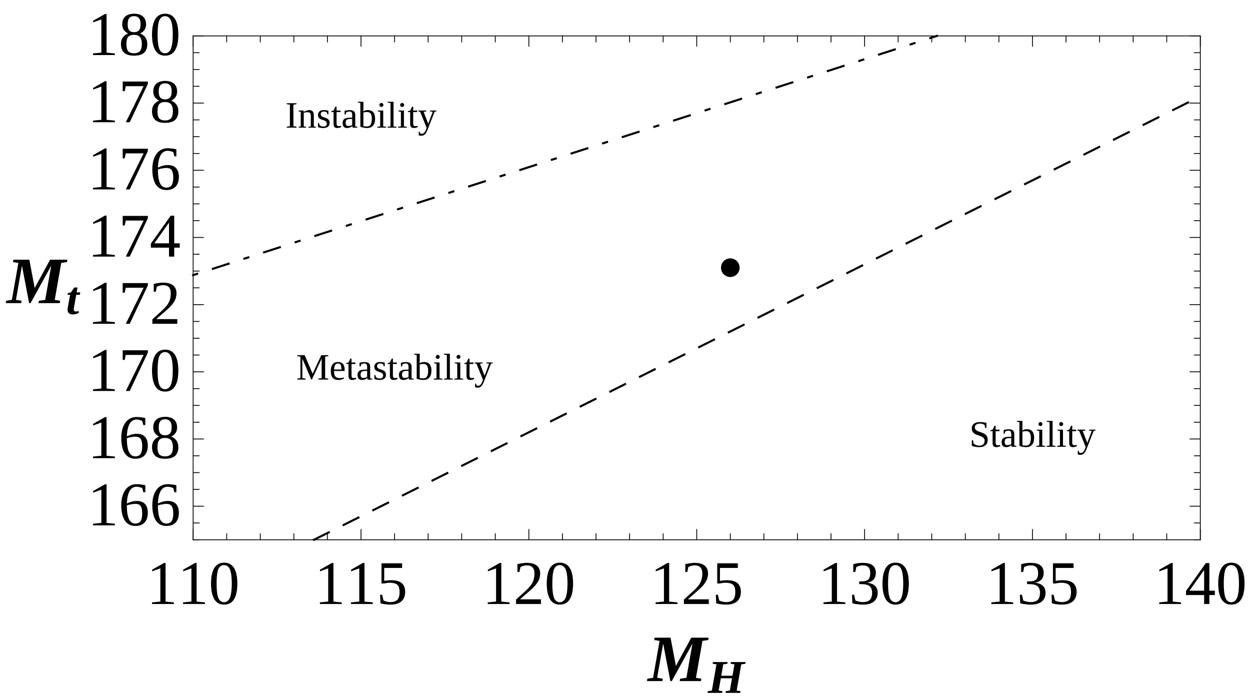
Phase Diagram in the $M_H - M_t$ plane



Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia arXiv:1307.3536 [hep-ph]

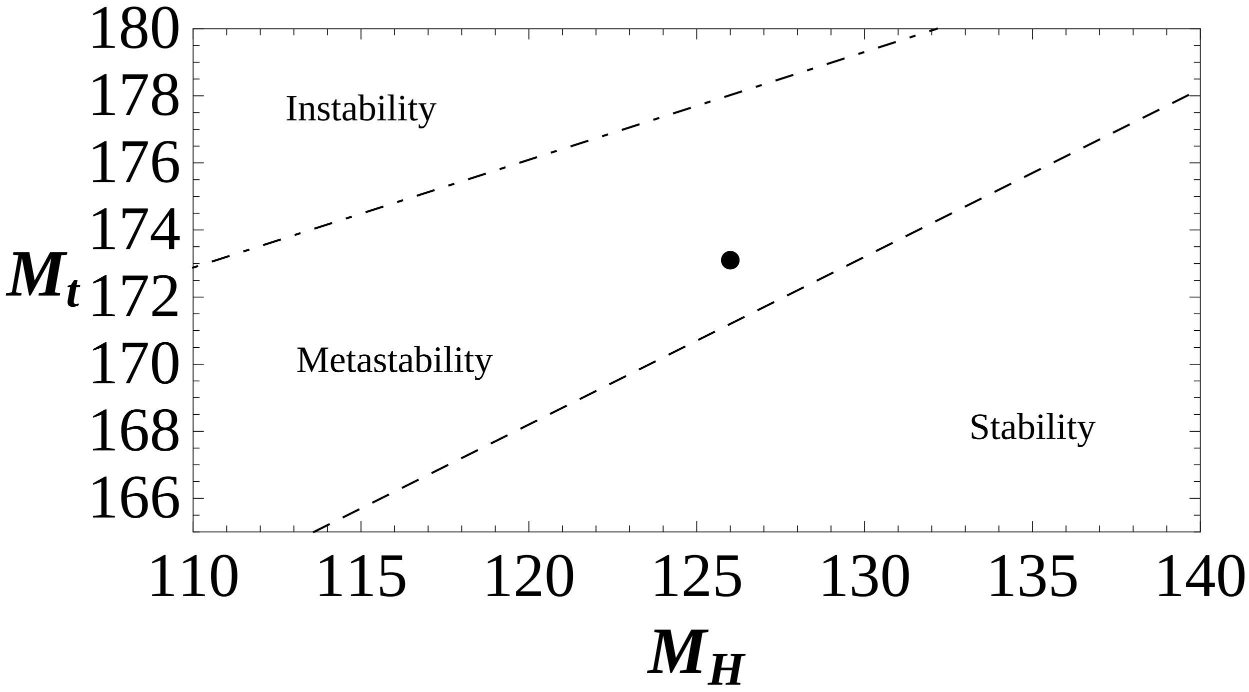
Phase Diagram in the $M_H - M_t$ plane

V.B. , E. Messina, arXiv:1307.5193 [hep-ph]



Stability region : $V_{eff}(v) < V_{eff}(\phi^{(2)}_{min})$. *Meta-stability* region : $\tau > T_U$.
Instability region : $\tau < T_U$. Dashed line : $V_{eff}(v) = V_{eff}(\phi^{(2)}_{min})$.
Dashed - dotted line : M_H and M_t such that $\tau = T_U$.

For $M_t \sim 173.1$ **GeV** , $M_H \sim 126$ **GeV : SM within Metastability Region**



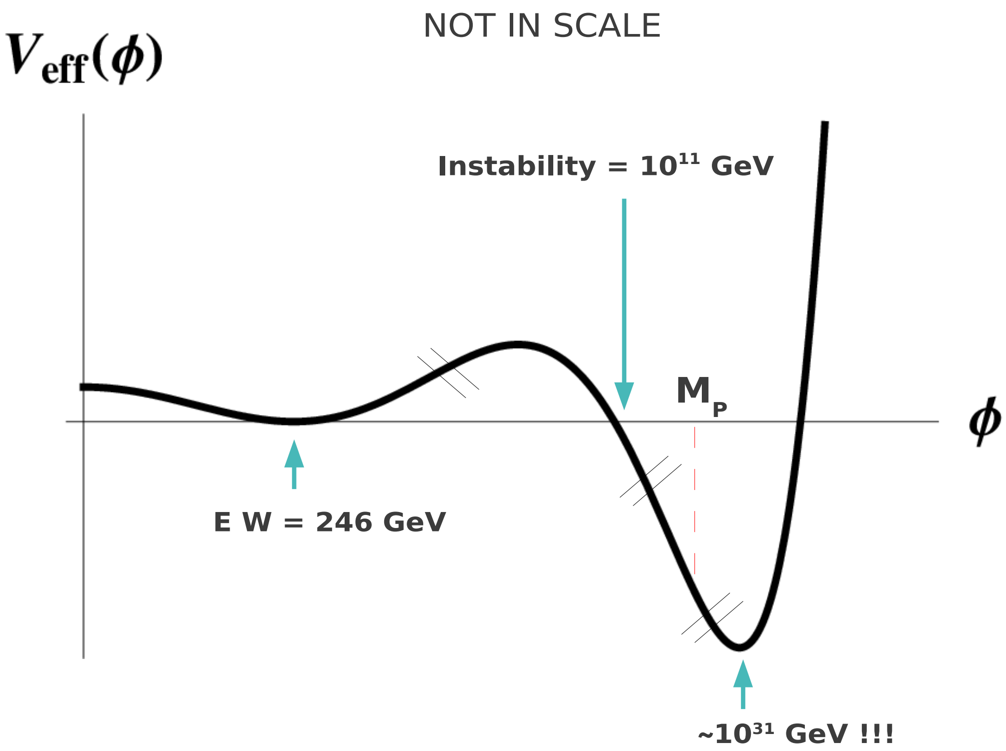
Conclusion(s) : This phase-diagram shows that for present values of M_H and M_t :

- Standard Model valid all the way up to the Planck scale ...

- We are close to criticality ...

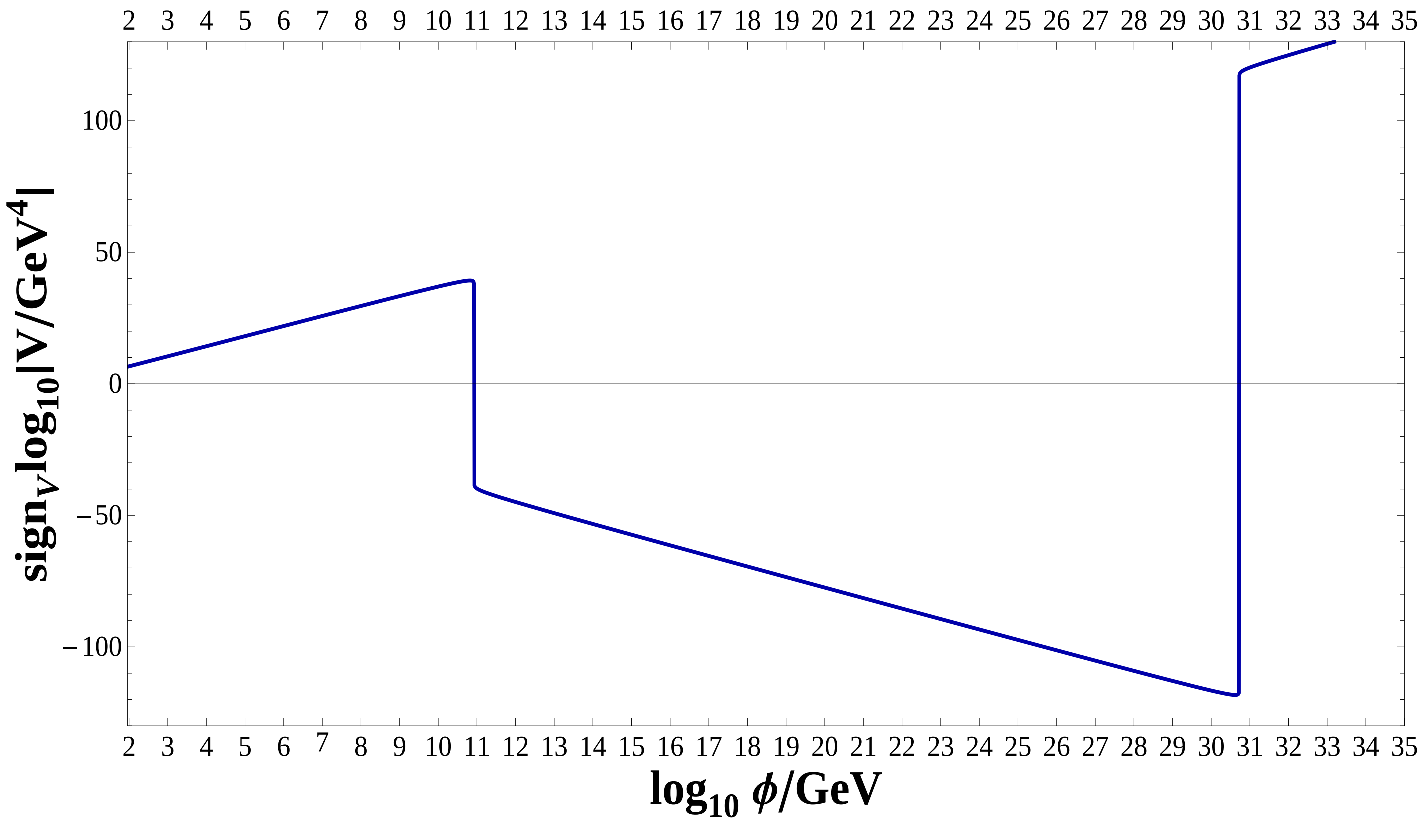
..... **However** .....

$M_H = 126$ **GeV** $\qquad M_t = 173.1$ **GeV**



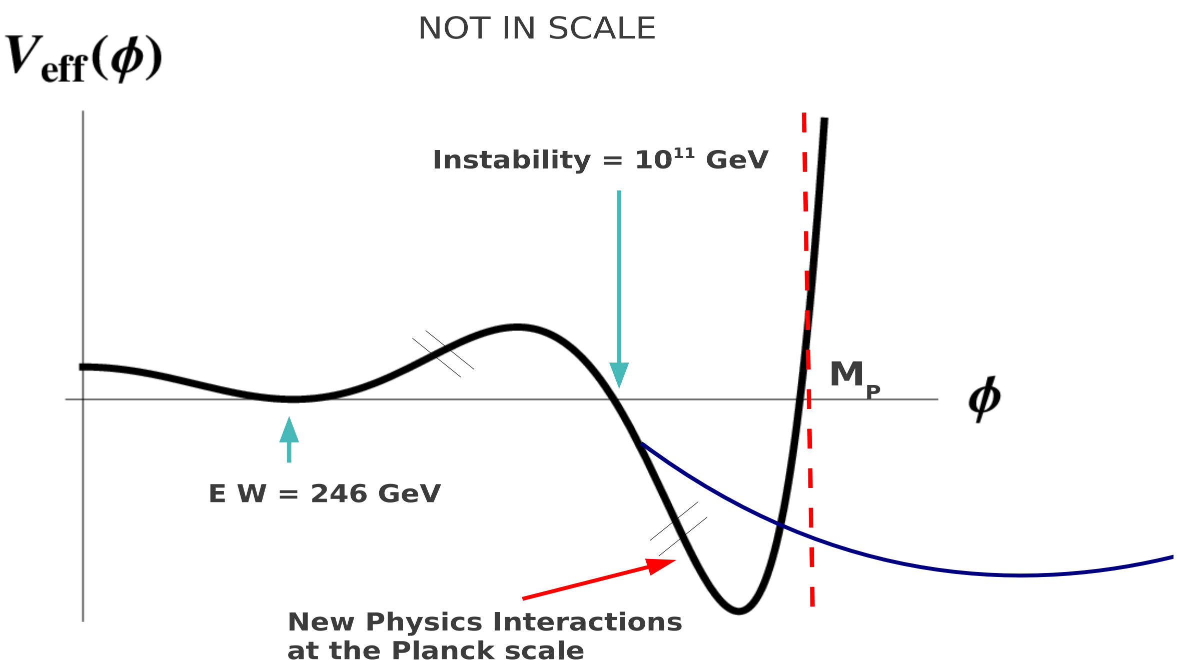
New minimum at $\phi \sim 10^{31}\, Gev$ **!!!! .... Now picture in scale ....**

Effective Potential ($M_H = 126$ GeV, $M_t = 173.1$ GeV)



Log-Log

New Physics Interactions
stabilize the potential around the Planck scale



Are New Physics Operators harmless for computing τ?

... It was thought they are ... But ...

... New Physics Interactions at the Planck scale ...

Add ϕ^6 and ϕ^8 to the SM Higgs potential:

$$V(\phi) \;=\; \frac{\lambda}{4}\phi^4 + \frac{\lambda_6}{6}\frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_P^4}$$

The Effective Potential $V_{eff}^{new}(\phi)$ is modified as :

$$V_{eff}^{new}(\phi) = V_{eff}(\phi) + \frac{\lambda_6(\phi)}{6M_P^2}\xi(\phi)^6\phi^6 + \frac{\lambda_8(\phi)}{8M_P^4}\xi(\phi)^8\phi^8$$

For $\phi < M_P$ **:** $V_{eff}^{new}(\phi)$ **practically coincides with** $V_{eff}(\phi)$

For $\phi \sim M_P$ **:** $V_{eff}^{new}(\phi)$ **depends on** λ_6 **and** λ_8**...**

Two different representative cases

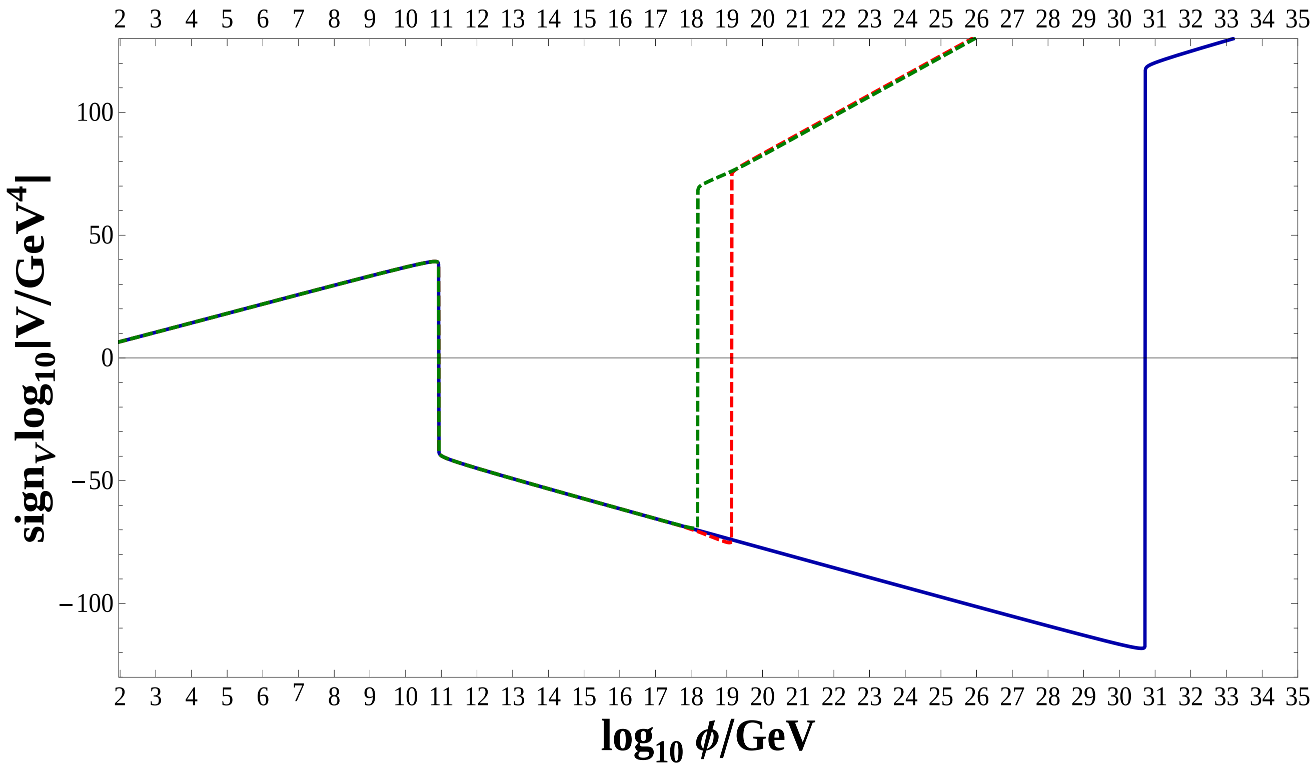
1. λ_6 **negative and** λ_8 **positive**

$$\textbf{Example :}\ \lambda_6(M_P) = -2 \qquad \lambda_8(M_P) = 2.1$$

2. both λ_6 **and** λ_8 **positive**

$$\textbf{Example :}\ \lambda_6(M_P) = 1 \qquad \lambda_8(M_P) = 0.5$$

Effective Potential $M_H = 126$ $M_t = 173.1$ Log-Log Plot

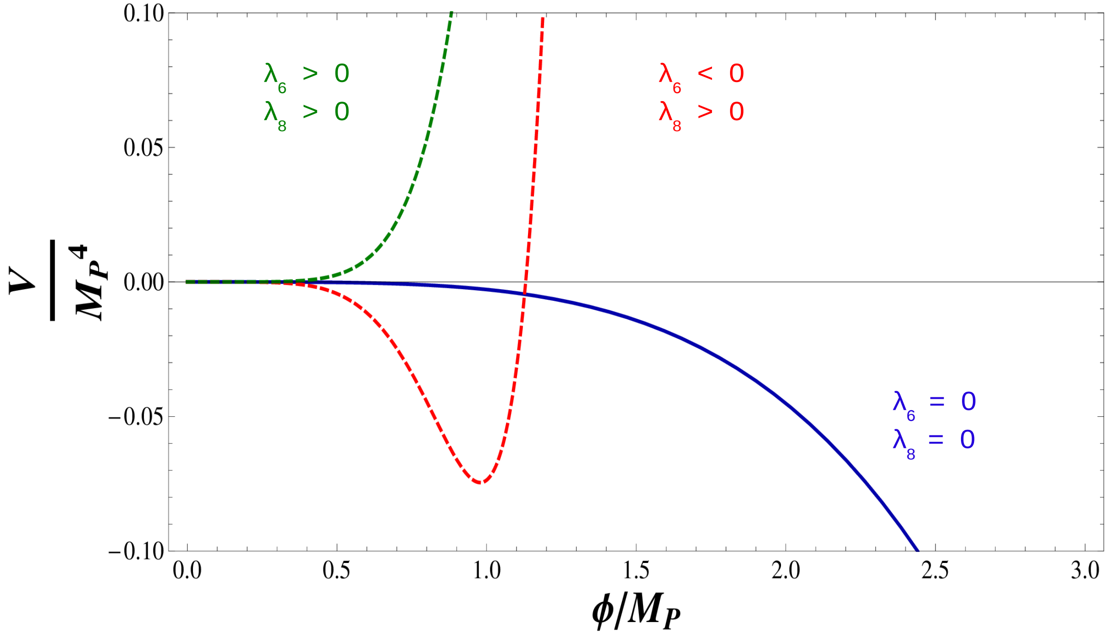


Blue line : $V_{eff}(\phi)$ **no higher order terms**

Red line : $V_{eff}^{new}(\phi)$ **with** $\lambda_6(M_P) = -2$ $\lambda_8(M_P) = 2.1$

Green line : $V_{eff}^{new}(\phi)$ **with** $\lambda_6(M_P) = 1$ $\lambda_8(M_P) = 0.5$

Zoom around the Planck scale



Blue line : $V_{eff}(\phi)$ **no higher order terms**

Red line : $V_{eff}^{new}(\phi)$ **with** $\lambda_6(M_P) = -2$ $\lambda_8(M_P) = 2.1$

Green line : $V_{eff}^{new}(\phi)$ **with** $\lambda_6(M_P) = 1$ $\lambda_8(M_P) = 0.5$

1. The case $\lambda_6 < 0$ $(\lambda_6 = -2)$ $\lambda_8 > 0$ $(\lambda_8 = 2.1)$

a. Up to $\eta \simeq 0.78 M_P$, V_{eff}^{new} very well approximated by :

$$V_{eff}^{new}(\phi) = \frac{\lambda_{eff}}{4}\phi^4$$

$$\lambda_{eff} = \lambda + \frac{2}{3}\lambda_6\frac{\eta^2}{M_P^2} + \frac{1}{2}\lambda_8\frac{\eta^4}{M_P^4} \simeq -0.437.$$

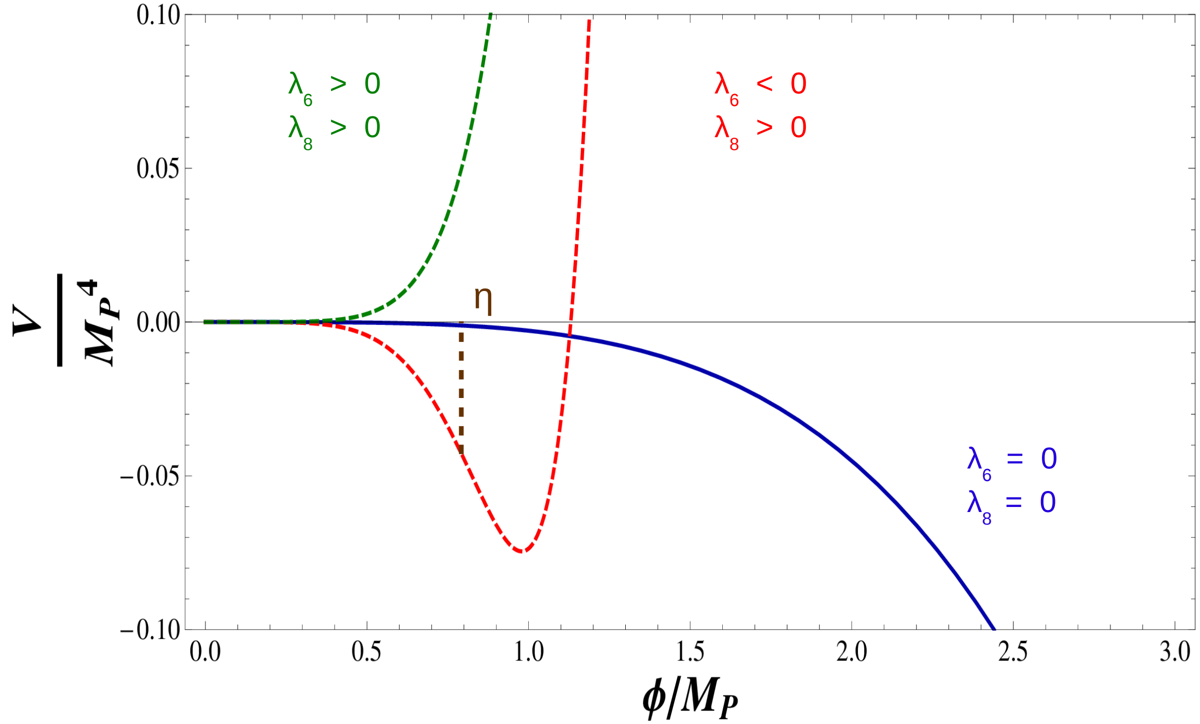
b. For $\phi \gtrsim \eta$, $V_{eff}^{new}(\phi)$ can be linearized :

$$V(\phi) = \frac{\lambda_{eff}}{4}\eta^4 - \frac{\lambda_{eff}\,\eta^3}{\gamma}(\phi - \eta)$$

$$\gamma = -\,\lambda_{eff}\;\eta^3\left(\lambda\eta^3 + \lambda_6\frac{\eta^5}{M_P^2} + \lambda_8\frac{\eta^7}{M_P^4}\right)^{-1}$$

This is all what we need to know to compute τ !!!

K. Lee, E.J. Weinberg, Nucl. Phys. B 267 (1986) 181



0.10
0.05
0.00
−0.05
−0.10
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\frac{V}{M_P^4}$
ϕ/M_P
$\lambda_6 > 0$
$\lambda_8 > 0$
$\lambda_6 < 0$
$\lambda_8 > 0$
$\lambda_6 = 0$
$\lambda_8 = 0$
η

Equation of motion admits Two Types (!!) of bounce solutions:

Type 1 (Old Friend !!!) : $\phi_b^{(1)}(r) = \sqrt{\frac{2}{|\lambda_{eff}|}}\frac{2\,R}{r^2+R^2}$

R **: size of these bounces**

Action degenerate with R **:** $S[\phi_b^{(1)}] = \frac{8\,\pi^2}{3|\lambda_{eff}|}$

Type 2 : (**New Comer** !!!) $\phi_b^{(2)}(r) = \begin{cases} 2\eta - \eta^2\sqrt{\frac{|\lambda_{eff}|}{8}}\frac{r^2+\overline{R}^2}{\overline{R}} & 0 < r < \overline{r} \\ \sqrt{\frac{8}{|\lambda_{eff}|}}\frac{\overline{R}}{r^2+\overline{R}^2} & r > \overline{r} \end{cases}$

$$\overline{r}^2 = \frac{8\gamma}{\lambda_{eff}\eta^2}(1+\gamma) \quad , \quad \overline{R}^2 = \frac{8}{|\lambda_{eff}|}\frac{\gamma^2}{\eta^2}\,.$$

Only for $-1 < \gamma < 0$

$\overline{R}$ **: size of the bounce**

Action : $S[\phi_b^{(2)}] = (1-(\gamma+1)^4)\ \frac{8\pi^2}{3|\lambda_{eff}|}$

If New Physics Interactions not explicitly included

Only Type 1 Solution !!!

... Tunnelling time computed accordingly (literature !!!) ...

$$\frac{1}{\tau} = T_U^3 \, \frac{S[\phi_b]^2}{4\pi^2} \left| \frac{\det' \left[-\partial^2 + V''(\phi_b)\right]}{\det \left[-\partial^2 + \mathrm{V}''(\mathrm{v})\right]} \right|^{-1/2} e^{-S[\phi_b]}$$

Even considering tree-level only (neglecting determinants)

As $S[\phi_b^{(1)}] = \frac{8\,\pi^2}{3|\lambda_{eff}|}$ **and** $S[\phi_b^{(2)}] = (1-(\gamma+1)^4)\ \frac{8\pi^2}{3|\lambda_{eff}|}$

When Type 2 solution exists $(-1<\gamma<0)$: contribution from Type 1 bounces exponentially suppressed (!!!) with respect to contribution from Type 2 bounce

... Some numbers ...

Our case : $\lambda_6(M_P) = -2 \qquad \lambda_8(M_P) = 2.1 \qquad (\gamma \simeq -0.963)$

$$\frac{1}{\tau} = T_U^3 \frac{S[\phi_b^{(2)}]^2}{4\pi^2} \left| \frac{\det' \left[-\partial^2 + V''(\phi_b)\right]}{\det \left[-\partial^2 + \mathrm{V}''(\mathrm{v})\right]} \right|^{-1/2} e^{-S[\phi_b^{(2)}]}$$

$$\tau \sim 10^{-219}\, T_U <<< T_U \quad !!!!!$$

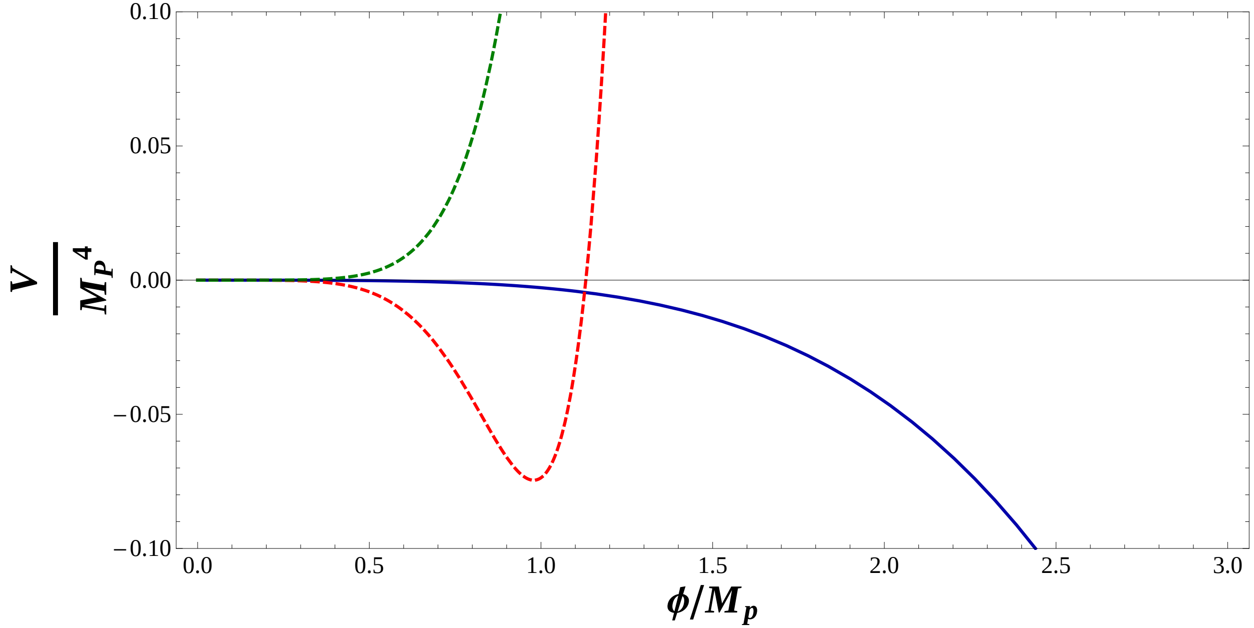
Literature case : $\lambda_6(M_P) = 0 \qquad \lambda_8(M_P) = 0$

$$\frac{1}{\tau} = T_U^3 \frac{S[\phi_b^{(1)}]^2}{4\pi^2} \left| \frac{\det' \left[-\partial^2 + V''(\phi_b)\right]}{\det \left[-\partial^2 + \mathrm{V}''(\mathrm{v})\right]} \right|^{-1/2} e^{-S[\phi_b^{(1)}]}$$

$$\tau \sim 10^{544}\, T_U >>> T_U \quad !!!!!$$

New Physics Interactions at the Planck scale do matter !!!

2. Case $\lambda_6 > 0$ and $\lambda_8 > 0$ $(\gamma < -1)$ ... only Type 1 bounces !!!



$$S[\phi_b^{(1)}] = \frac{8\,\pi^2}{3|\lambda_{eff}|} \qquad\qquad S[\phi_b^{(2)}] = (1 - (\gamma + 1)^4)\ \frac{8\pi^2}{3|\lambda_{eff}|}$$

γ **is the crucial parameter ! Two solutions only for :**

$$-1 < \gamma < 0$$

Otherwise only Type 1 : $\quad \tau = \tau_{SM}$ **!!!**

Determinant contributions do not change these results !!!

Logarithm of the fluctuation determinant (Higgs sector only) :

$$\log\left(\frac{\det'(-\partial^2+\mathrm{V}''(\phi_\mathrm{b}))}{\det(-\partial^2)}\right)^{1/2} = \frac{1}{2}\sum_{l=0}^{\infty}(l+1)^2\ln\rho_l$$

$$\rho_l = \lim_{r\to\infty}\rho_l(r)$$

Each $\rho_l(r)$ solution of the differential equation:

$$\rho_l''(r)+\frac{(2l+d-1)}{r}\rho_l'(r)-V''(\phi_b(r))\rho_l(r)=0 \qquad (\rho_l(0)=1\ ;\ \rho_l'(0)=0)$$

$\overline{MS}$ renormalized sum (excluding negative and zero modes):

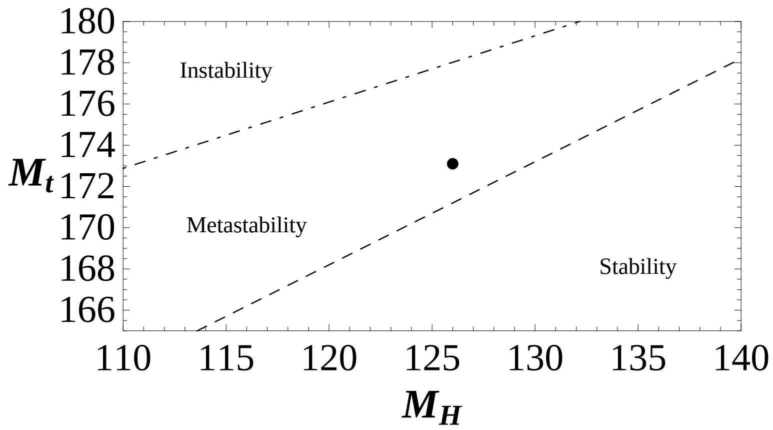
$$\left[\frac{1}{2}\sum_{l>1}^{\infty}(l+1)^2\ln\rho_l\right]_{Ren} = \frac{1}{2}\sum_{l>1}^{\infty}(l+1)^2\ln\rho_l-\frac{1}{2}\sum_{l=0}^{\infty}(l+1)^2\left[\frac{\int_0^\infty drrV''}{2(l+1)}-\frac{\int_0^\infty drr^3(V'')^2}{8(l+1)^3}\right]$$
$$-\frac{1}{8}\int_0^\infty drr^3(V'')^2\left[\ln\left(\frac{\mu r}{2}\right)+\gamma_E+1\right]$$

For our example ($\lambda_6(M_P)=-2 \qquad \lambda_8(M_P)=2.1$), adding determinant contribution (Higgs sector only), including negative and zero modes :

$$\tau\sim 10^{-212}\,T_U <<< T_U \quad !!!!!$$

Top , Gauge , Goldstone contributions do not change this result!

Phase Diagram in the $M_H - M_t$ plane (Literature)



For $M_H \sim 126$ GeV and $M_t \sim 173.1$ GeV

Neglecting New Interactions at the Planck scale

EW vacuum well inside Metastability Region, close to Stability line

Extremely long-lived Metastable State !

This is why often stated :

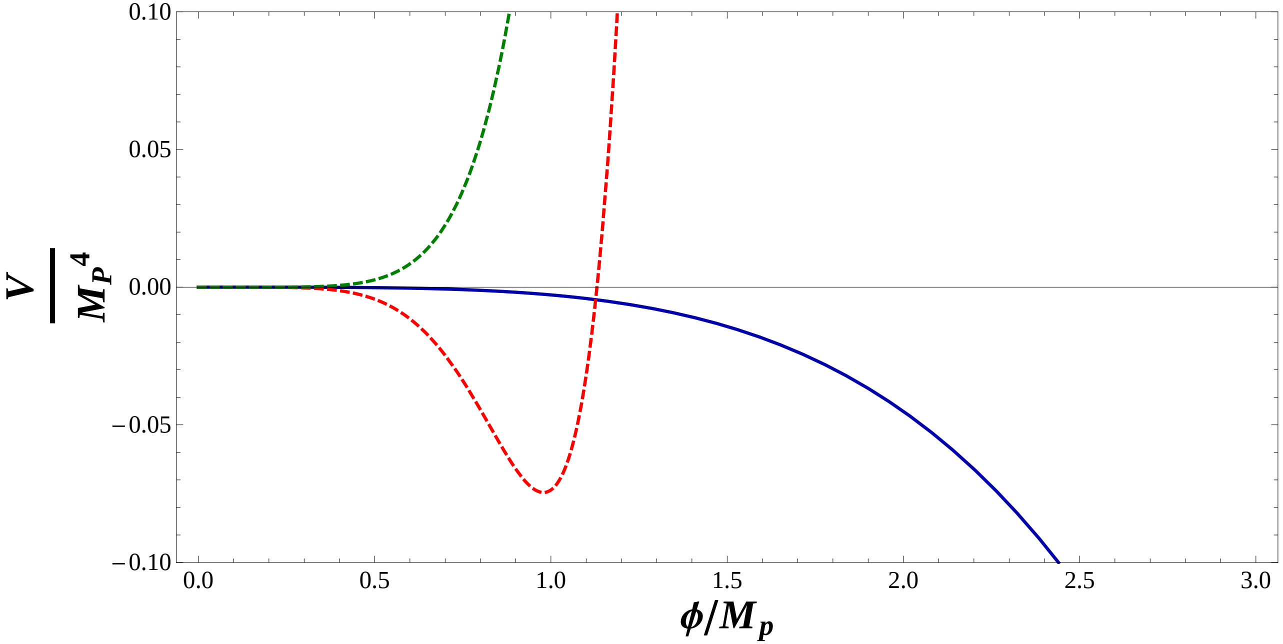
The SM is an effective theory that may be valid all the way up to M_P !

SM : effective theory valid up to M_P ???

Lesson : This is not generically true !!!

New physics interactions at the Planck scale may turn the EW vacuum from a very long-lived metastable state to a highly unstable state.

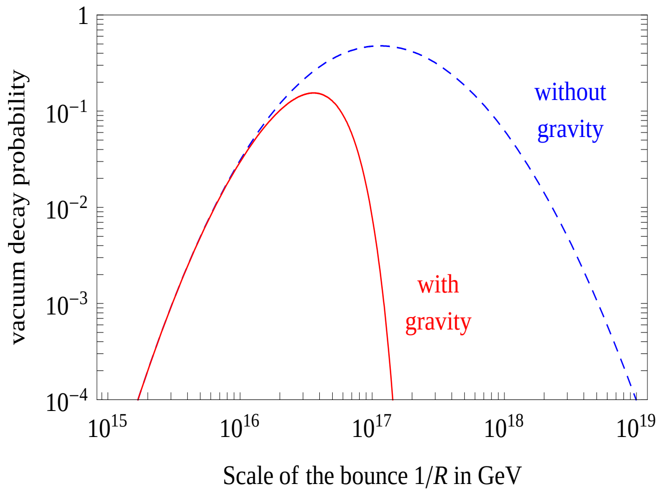
When $V_{eff}^{new}(\phi)$ lies above $V_{eff}(\phi)$, τ is not affected by new physics. On the contrary, when $V_{eff}^{new}(\phi)$ lies below $V_{eff}(\phi)$, New Physics Interaction at the Planck scale have strong impact on τ, turning $\tau >> T_U$ to $\tau << T_U$!



Conclusions

V.B. , E. Messina, arXiv:1307.5193 [hep-ph]

- **Lifetime τ of the EW vacuum strongly depends on New Physics Interactions at the Planck scale**
- **SM Phase Diagram strongly depends on New Physics Interactions at the Planck scale**
- **Metastability Scenario (based on the assumption that τ does not depend on New Physics Interactions at the Planck scale) has to be reconsidered**
- **These results provide constraints on New Physics beyond SM**
- **A similar analysis can be done also if the new physics scale lies below the Planck scale**
- **Aalysis also relevant for : Higgs potential with two degenerate minima, $\lambda(M_P) \sim 0$, $\beta(\lambda(M_P)) \sim 0$, Higgs driven inflation... In all of these cases, the relevant physical scale dangerously close to the Planck scale $\rightarrow$ high sensitivity to New Physics Interactions**



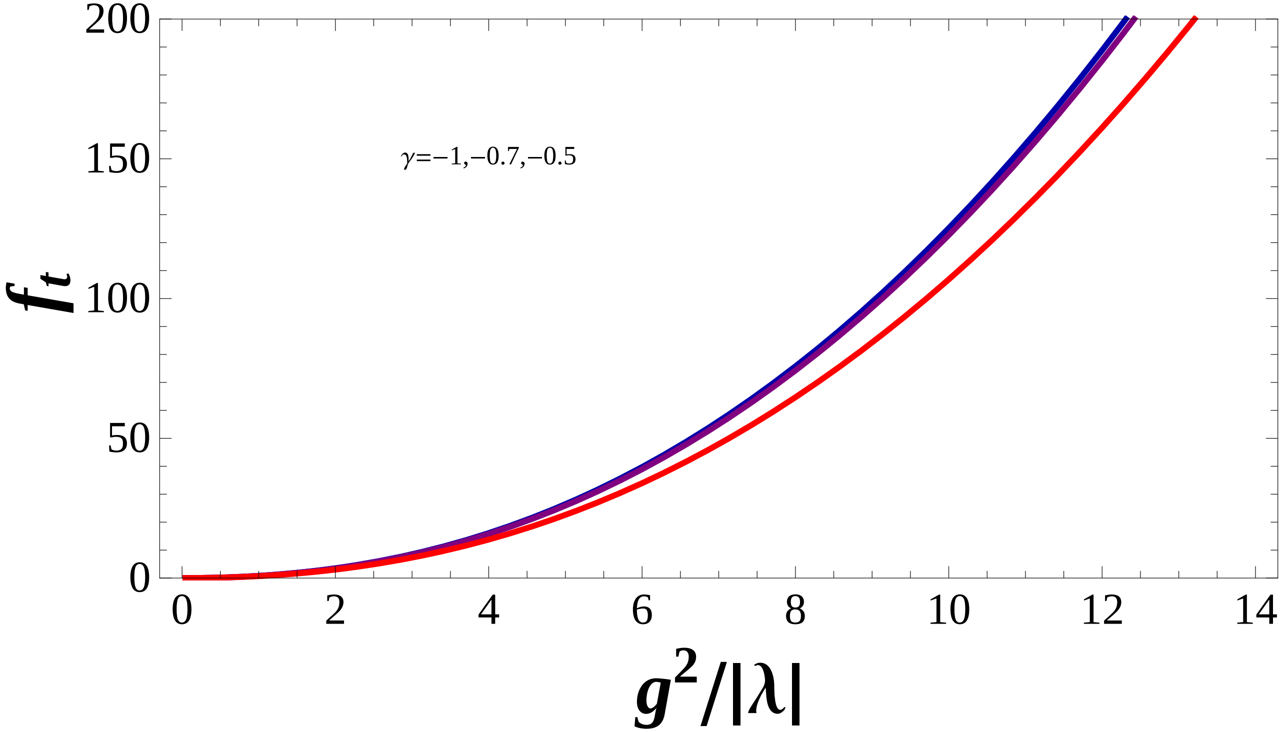
Isidori, Rychkov, Strumia, Tetradis, "Gravitational corrections to Standard Model vacuum decay" , Phys.Rev.D77:025034,2008

... Adding to the SM action possible dimension-6 non-renormalizable operators suppressed by the Planck scale ⇒ corrections to the bounce action. In particular, adding to the SM Lagrangian the operators

$$\Delta_6 = \frac{1}{M_{\rm Pl}^2}\left(-\xi M_{\rm Pl}^2 R|H|^2 + c_1\frac{|H|^6}{3!} + c_2|H|^2|D_\mu H|^2\right)$$

where ξ and $c_{1,2}$ are unknown dimensionless coefficients, gives the following correction

$$\Delta S'_{\rm gravity} = \frac{8\pi^2}{15(M_{\rm Pl}R\lambda)^2}\left(128\pi\xi + \frac{c_1}{|\lambda|} + 4c_2\right) \ldots$$



200
150
100
50
0
f_t
γ=−1,−0.7,−0.5
0
2
4
6
8
10
12
14
$g^2/|\lambda|$