Radiative corrections to the Higgs couplings in the triplet model

Mariko Kikuchi (Univ. of TOYAMA)

M. Aoki, S. Kanemura, M. K., K. Yagyu,
Phys.Lett. B714 (2012) 279-285
Phys.Rev. D87 (2013) 015012

Scalars2013 @ University of Warsaw , Sept. 12th -16th 2013

Contents

I. Introduction

Higgs coupling measurement is a probe of new physics

Future precision measurement ⇔ Radiative Corrections

II. Higgs triplet model

Motivation, Particle entries, Some characteristics of the model

Renormalization and loop calculations for Higgs couplings

III. Results

Deviation from SM in $h\gamma\gamma$, hWW, hZZ, hhh

IV. Summary

Higgs sector

m_h=126 GeV

Higgs boson was discovered last year !

Data indicate that it is a SM-like Higgs boson (h).

What is the shape of the Higgs sector?

No principle for minimal Higgs with one doublet

All extended Higgs sectors can predict the SM-like Higgs boson

Many new physics models predict specific extended Higgs sectors

Hierarchy, Neutrino Masses, Dark Matter,
Baryon Asymmetry, …

Higgs sector is a probe of new physics !

Test of Extended Higgs

Direct search

When there are additional Higgs bosons, Higgs sector can be determined by their direct discovery

H? A? H⁺? H⁺⁺?

Indirect search

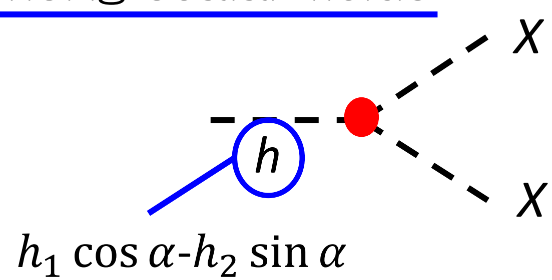
Physics of *h*

Deviation in coupling constants of *h* due to heavy Higgs bosons

There are **two possibilities** to change couplings of *h*

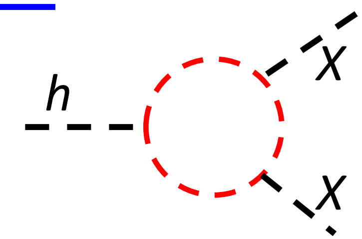
Mixing among scalar fields

EX) 2HDM



Radiative corrections

Deviation due to the loop contributions of additional new particles.



Feature of each new physics model appears in Higgs couplings.
We can discriminate an extended Higgs model by measuring the pattern of deviations in the *h* couplings.

Determination of *h* couplings

◆ LHC data for signal strength slightly deviate from SM predictions.

EX) $h \to \gamma\gamma$ 0.8(±0.3) (CMS) 1.6(±0.3) (ATLAS)

$h \to WW^*$, 0.75(±0.2) (CMS) 0.95(±0.2) (ATLAS)

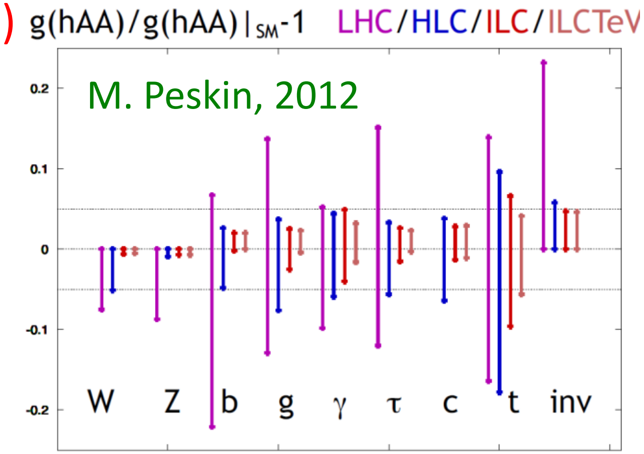
◆ Further deviations may be found at future precision measurements.

ILC

hZZ, *hWW* : O(1)% or better

hhh : O(10)%

Technical design report of ILC (2013)



They can be determined more precisely by combination with data from HL-LHC and ILC.

hγγ : about 5%(HL-LHC&LC500)

Markus, Remi, Tilman, Michael, and Dirk Zerwas(2013)

◆ With such future precision measurements, predictions on *h*-couplings at the one-loop level are important to determine the Higgs sector

Extended Higgs models

- Some extended Higgs models have the possibility to explain some problems
  - $\Phi + S$ (B-L Higgs, ...)
  - $\Phi + \Phi$ (SUSY, EW Baryogenesis, …)
  - $\Phi + \Delta$ (Type II seesaw, …)

S : singlet scalar field
Φ : doublet scalar field
Δ : triplet scalar field
…

- Important experimental constraints:
  - EW ρ parameter is almost unity

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{\sum_i \left[T_i(T_i+1) - Y_i^2\right] v_i^2}{\sum_i 2Y_i^2 v_i^2}$$

$$\rho_{exp} = 1.0008^{+0.0017}_{-0.0007}$$

PDG(2012)

T_i : Isospin ,
Y_i : hypercharge,
v_i : vacuum expectation value

  - FCNC must be suppressed

$$BR(K_L^0 \to \mu^+\mu^-)_{exp} = (6.84 \pm 0.11) \times 10^{-9}$$

PDG(2012)

Extended Higgs sector must satisfy them!

In this talk

- Basically we have to study all extended Higgs sectors in order to discriminate them by the future data.

- We here consider the Higgs triplet model ($\Phi+\Delta$), and study radiative corrections to the h-couplings in this model.

- Why the Higgs triplet model?
  - Neutrino mass can be explained
  - Theoretical interest for 1-loop calculation of such an exotic model with $\rho \neq 1$.

Higgs triplet model (HTM)

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad + \quad \Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & \frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

$$\phi^0 = \frac{1}{\sqrt{2}}(\phi + v_\phi + i\,\chi)$$

$$\Delta^0 = \frac{1}{\sqrt{2}}(\delta + v_\Delta + i\,\eta)$$

	$SU(2)_L$	$U(1)_Y$
Φ	2	1/2
Δ	3	1

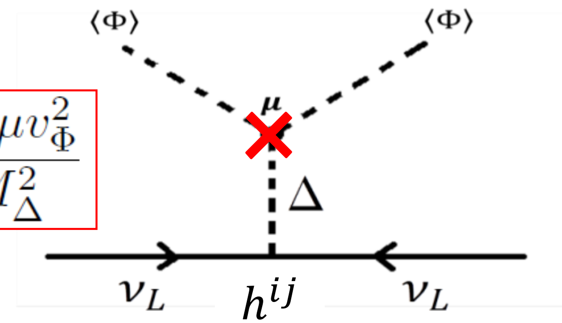
◆ Neutrino mass **Type-II seesaw**

Cheng, Li (1980), Mohapatra, Senjanovic(1981)

$$\mathcal{L}_\nu = h_{ij}\overline{L_L^i}^c i\tau_2 \Delta L_L^j + hc$$

Majorana neutrino masses are generated via the LNV parameter μ .

$$M_\nu^{ij} = \frac{h^{ij}\mu v_\Phi^2}{M_\Delta^2}$$



v_ϕ : doublet VEV
v_Δ : triplet VEV

$v^2 = v_\varphi^2 + v_\Delta^2 \simeq 246\text{GeV}$

◆ The rho parameter $\rho_{exp} \simeq 1.0008^{+0.0017}_{-0.0007}$

ρ is not equal to unity at tree.

We need to set $v_\Delta / v_\phi \ll 1$

$$\rho \equiv \frac{m_W{}^2}{\cos^2\theta_W m_Z{}^2} = \frac{v_\phi{}^2 + 2v_\Delta{}^2}{v_\phi{}^2 + 4v_\Delta{}^2} \neq 1$$

Higgs potential

$$V(\Phi,\Delta) = m^2\Phi^\dagger\Phi + M^2\mathrm{Tr}(\Delta^\dagger\Delta) + \left[\mu\Phi^T i\tau_2\Delta^\dagger\Phi + \mathrm{h.c.}\right]$$
$$+ \lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 + \lambda_3\mathrm{Tr}[(\Delta^\dagger\Delta)^2] + \lambda_4(\Phi^\dagger\Phi)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_5\Phi^\dagger\Delta\Delta^\dagger\Phi.$$

◆ Mass eigenstates

$$\begin{pmatrix}\phi\\ \delta\end{pmatrix} = \begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}h\\ H\end{pmatrix}, \quad \begin{pmatrix}\phi^\pm\\ \Delta^\pm\end{pmatrix} = \begin{pmatrix}\cos\beta & -\sin\beta\\ \sin\beta & \cos\beta\end{pmatrix}\begin{pmatrix}G^\pm\\ H^\pm\end{pmatrix}, \quad \begin{pmatrix}\chi\\ \eta\end{pmatrix} = \begin{pmatrix}\cos\beta' & -\sin\beta'\\ \sin\beta' & \cos\beta'\end{pmatrix}\begin{pmatrix}G^0\\ A\end{pmatrix},$$

Mass eigenstates: h, $H^{\pm\pm}, H^\pm, A, H$

SM-like Higgs boson — triplet-like Higgs bosons

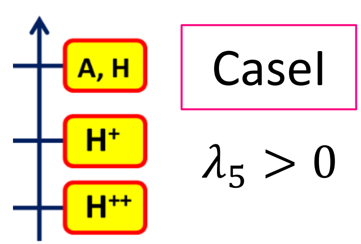
◆ Mass hierarchy

$v_\Delta{}^2 \ll v_\phi{}^2$ ⇒ Relation among masses

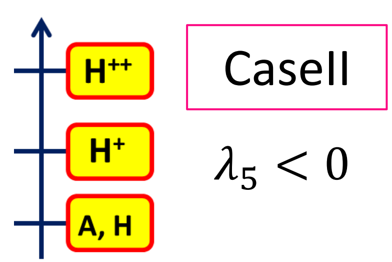
$$m_{H^{++}}{}^2 - m_{H^+}{}^2 \simeq m_{H^+}{}^2 - m_A{}^2 \simeq -\frac{\lambda_5}{4}v_\Phi{}^2$$

Three patterns for mass spectrum for the triplet-like Higgs bosons.

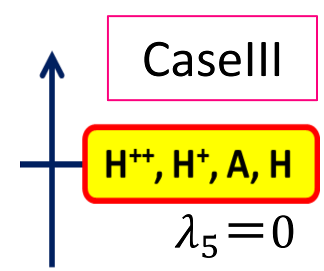
$\Delta m = m_{H^\pm} - m_{\text{lightest}}$



$\Delta m = m_{H^\pm} - m_{H^{\pm\pm}}$



$\Delta m = m_{H^\pm} - m_A$



$\Delta m = 0$

Current Bound from LHC

Decay of H++ depends on v_Δ

- v_Δ<1 MeV

  Mainly decay into dilepton $H^{++} \to l^+ l^+$

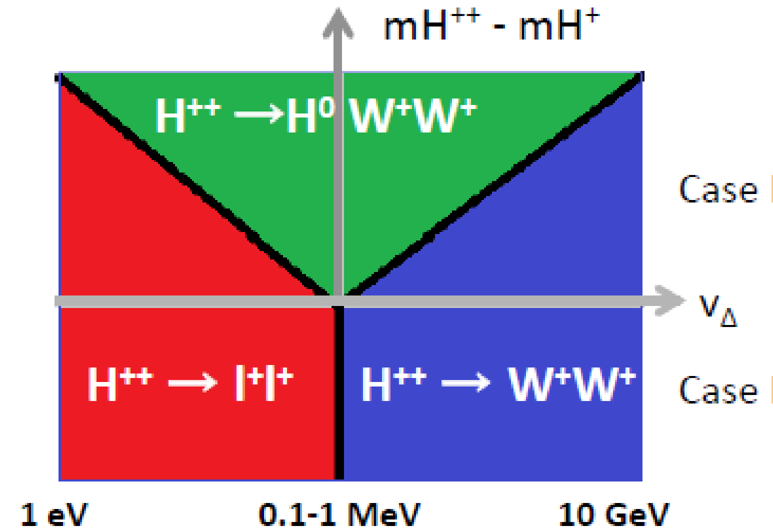
  LHC bound: 400 GeV < $m_{H^{++}}$

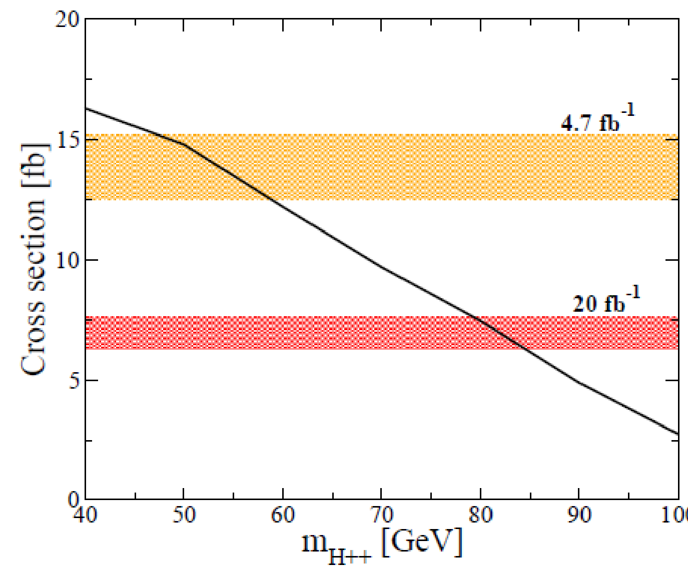
- v_Δ>1 MeV

  Mainly decay into diboson $H^{++} \to W^+ W^+$

  LHC bound: 80 GeV < $m_{H^{++}}$

  In this case, we can consider the light H^{++} which can be tested at ILC etc.





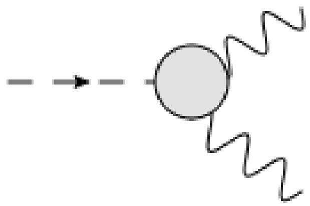
Kanemura, Yagyu, Yokoya (2013)

What do we calculate?

h-couplings

- ***hγγ*** *(1-loop induced, new loop contributions)*

  10% at LHC   5% LHC3000+ILC500



- ***hWW, hZZ*** *(Nature of Higgs mechanism)*

  10% at LHC   1% or better at ILC500

- ***hhh*** *(Structure of the Higgs potential)*

  rather difficult at LHC   O(10)% at ILC1000

➢ How do we calculate these coupling constants in the HTM?

- The renormalization scheme is different from the one in the SM because of $\rho \neq 1$.
- So we must construct the renormalization scheme in the HTM.
- Then we calculate these coupling constants

Renormalization

T. Blank, W. Hollik (1998), S. Kanemura, K. Yagyu (2012), P. H. Chankowski, S. Pokorski, J. Wagner, (2007); M. -C. Chen, S. Dawson, C. B. Jackson (2008).

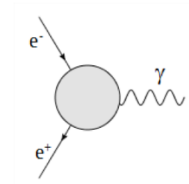
➢ The model with SM $\rho_{tree}=1$

- Lagrangian parameters ・・・ g, g', v
- Physical parameters ・・・ $m_W, m_Z, sin\theta_W, G_F, \alpha_{em}$.
- Renormalization conditions

On-shell condition

$$Re\Pi_{ZZ}(p^2)|_{p^2=m_Z^2} = 0, \quad \delta m_Z^2 = Re\Pi_{ZZ}^{1PI}(m_Z^2),$$

$$Re\Pi_{WW}(p^2)|_{p^2=m_W^2} = 0, \quad \delta m_W^2 = Re\Pi_{WW}^{1PI}(m_W^2),$$



$$\Big|_{p^2=0} = -ie\gamma^\mu$$

$$\frac{\delta\alpha_{em}}{\alpha_{em}} = \frac{d}{dp^2}\Pi_{\gamma\gamma}^{1PI}(p^2)|_{p^2=0} - \frac{2s_W}{c_W}\frac{\Pi_{\gamma Z}^{1PI}(0)}{m_Z^2}$$

➢ The model with HTM $\rho_{tree}\neq 1$

$$\rho = \frac{m_W{}^2}{m_Z{}^2 cos^2\theta_W} \neq 1$$

- Lagrangian parameters ・・・ g, g', v, v_Δ
- Physical parameters ・・・ $m_W, m_Z, sin\theta_W, G_F, \alpha_{em}$.
- Renormalization conditions

$$sin^2\theta_W \neq 1 - \frac{m_W{}^2}{m_Z{}^2} \Rightarrow \frac{\delta s_W^2}{s_W^2} = \frac{c_W^2}{s_W^2}\left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2}\right)$$

$$cos\theta_W{}^2 = \frac{m_W{}^2}{m_Z{}^2}\frac{2}{(1+cos\beta'^2)} \Rightarrow$$

$$\begin{aligned}\delta\bar{s}_W^2 &= -\delta\bar{c}_W^2 \\ &= \frac{2m_W^2}{m_Z^2(1+c_{\beta'}^2)}\left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2} - \frac{2s_{\beta'}c_{\beta'}}{(1+c_{\beta'}^2)}\delta\beta'\right)\end{aligned}$$

β' is a mixing angle among CP-odd scalar field.
δβ' is determined by

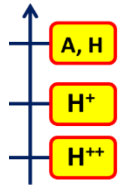
$$\Pi_{AG^0}(p^2)\Big|_{p^2=m_A{}^2,0} = 0$$

$h\gamma\gamma$

- Ratio of the event rate for $h \to \gamma\gamma$

A. Arhrib, R. Benbrik, M. Chabab, G. Moultaka (2012);
A. G. Akeroyd, S. Moretti (2012);

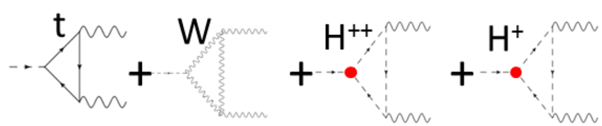
$$R_{\gamma\gamma} \equiv \frac{\sigma(gg \to h)_{\rm HTM} \times {\rm BR}(h \to \gamma\gamma)_{\rm HTM}}{\sigma(gg \to h)_{\rm SM} \times {\rm BR}(h \to \gamma\gamma)_{\rm SM}}$$



Results at LHC

$R_{\gamma\gamma}^{\rm exp}$ = 0.5 – 1.1 (CMS)

$R_{\gamma\gamma}^{\rm exp}$ = 1.3 – 1.9 (ATLAS)

$\Gamma(h \to \gamma\gamma)_{HTM} = \Big|$  $\Big|^2$

$R_{\gamma\gamma}$ depends on λ_4.

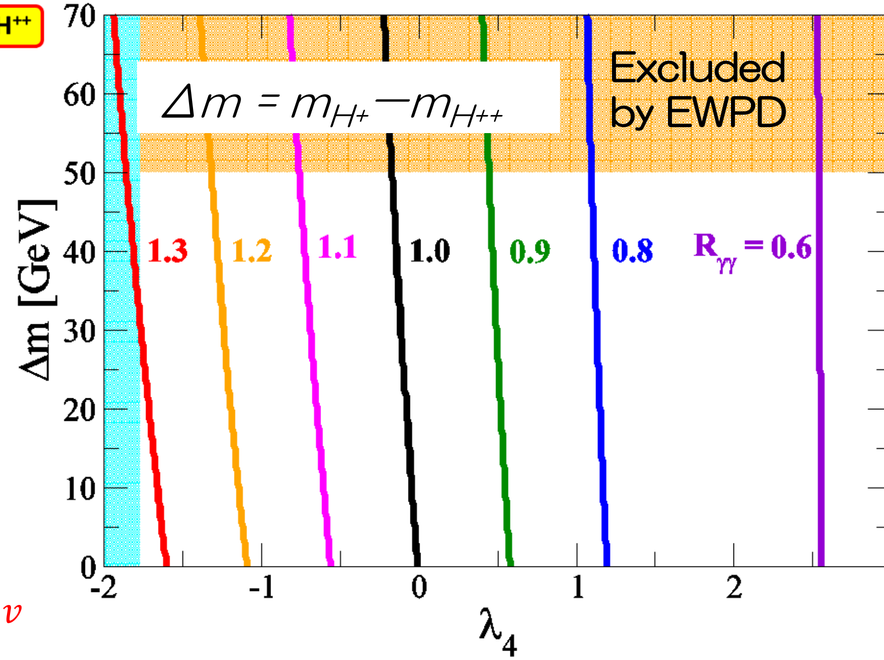
$\lambda_{hH^{++}H^{--}} \approx -\lambda_4 v$

$\lambda_{hH^{+}H^{-}} \approx -(\lambda_4 + \frac{\lambda_5}{2})v$

$\lambda_5 \geq 0$

Case I

$m_{H^{++}}$=300GeV, v_Δ=1MeV



M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

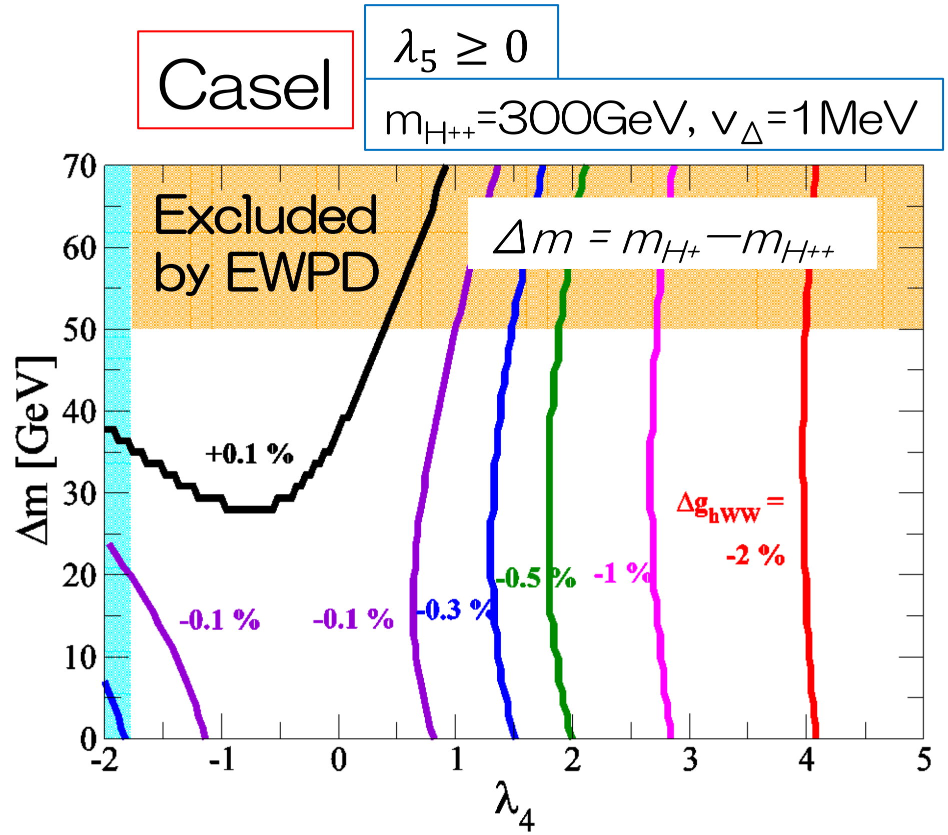
$R_{\gamma\gamma}$ can be enhanced or reduced depending on the sign of $\lambda_{hH^{++}H^{--}}$.

hWW

- Deviations for *hWW* from the SM predictions

$$\Delta g_{hVV} \equiv \frac{\mathrm{Re}M_1^{hVV} - \mathrm{Re}M_1^{hVV}(\mathrm{SM})}{\mathrm{Re}M_1^{hVV}(\mathrm{SM})}$$

$$\Delta g_{hWW} \simeq -\frac{v^2}{32\pi^2}\left[\frac{\lambda_4{}^2}{6m_{H++}{}^2} + \frac{\left(\lambda_4+\frac{\lambda_5}{2}\right)^2}{6m_{H+}{}^2} + \frac{(\lambda_4+\lambda_5)^2}{6m_A{}^2} + \frac{(\lambda_4+\lambda_5)^2}{6m_H{}^2}\right] + \frac{1}{4\pi^2}\frac{2(c_W{}^2-s_W{}^2)}{3s_W{}^2}\left[\frac{(m_{H++}-m_{H+})^2}{v^2} + \frac{(m_{H+}-m_A)^2}{v^2}\right]$$



M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

Deviations for *hWW* from the SM predictions can be several %.

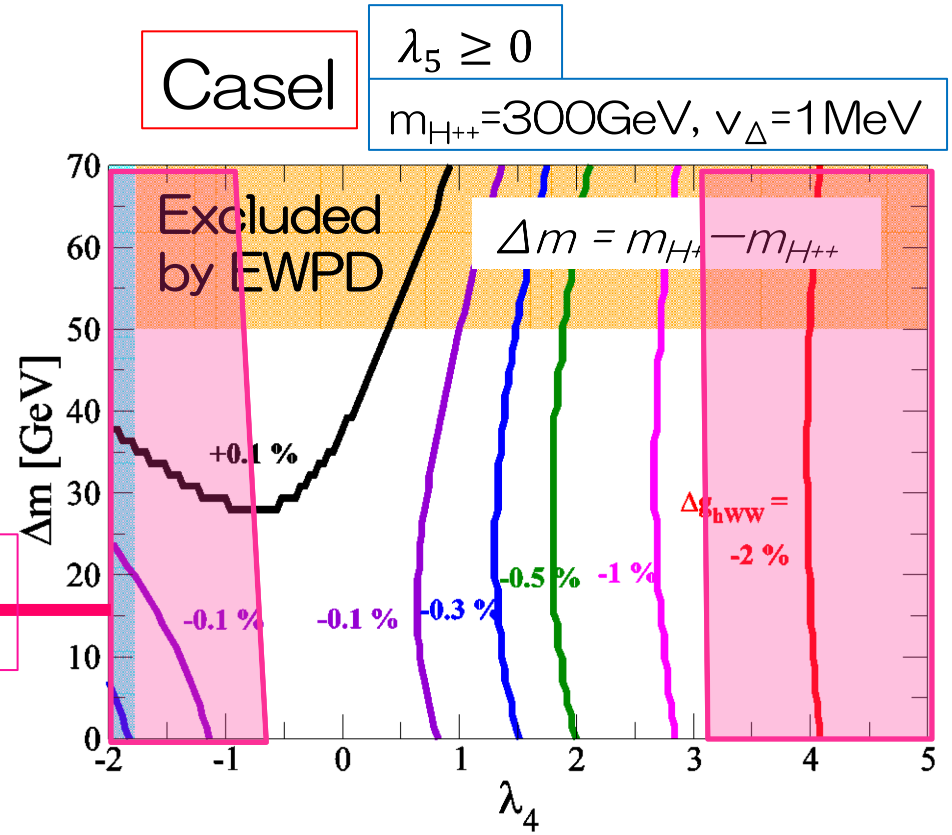
hWW

- Deviations for *hWW* from the SM predictions

$$\Delta g_{hVV} \equiv \frac{\mathrm{Re}M_1^{hVV} - \mathrm{Re}M_1^{hVV}(\mathrm{SM})}{\mathrm{Re}M_1^{hVV}(\mathrm{SM})}$$

$$\Delta g_{hWW} \simeq -\frac{v^2}{32\pi^2}\left[\frac{{\lambda_4}^2}{6{m_{H++}}^2} + \frac{\left(\lambda_4+\frac{\lambda_5}{2}\right)^2}{6{m_{H+}}^2} + \frac{(\lambda_4+\lambda_5)^2}{6{m_A}^2} + \frac{(\lambda_4+\lambda_5)^2}{6{m_H}^2}\right] + \frac{1}{4\pi^2}\frac{2({c_W}^2-{s_W}^2)}{3{s_W}^2}\left[\frac{(m_{H++}-m_{H+})^2}{v^2} + \frac{(m_{H+}-m_A)^2}{v^2}\right]$$

If we take into account the CMS data for $R_{\gamma\gamma}$, pink regions are excluded.



M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

Δg_{hVV} can be 1%.

Deviation is detectable at ILC !

hZZ

- Deviations for *hZZ* from the SM predictions

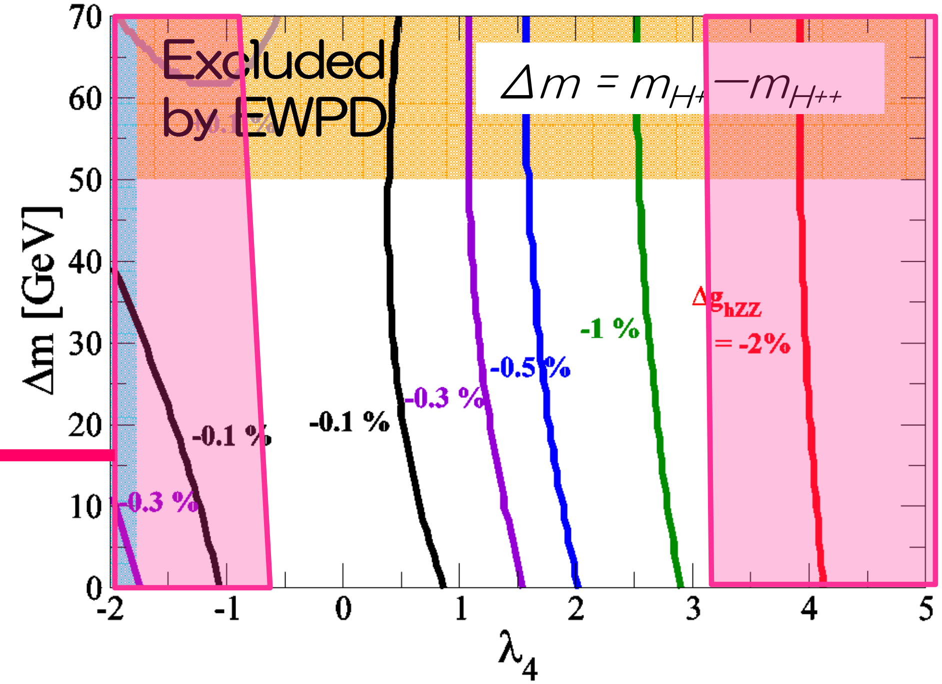
$$\Delta g_{hVV} \equiv \frac{\mathrm{Re}M_1^{hVV} - \mathrm{Re}M_1^{hVV}(\mathrm{SM})}{\mathrm{Re}M_1^{hVV}(\mathrm{SM})}$$

$$\Delta g_{hWW} \simeq -\frac{v^2}{32\pi^2}\left[\frac{{\lambda_4}^2}{6{m_{H++}}^2} + \frac{\left(\lambda_4+\frac{\lambda_5}{2}\right)^2}{6{m_{H+}}^2} + \frac{(\lambda_4+\lambda_5)^2}{6{m_A}^2} + \frac{(\lambda_4+\lambda_5)^2}{6{m_H}^2}\right] + \frac{1}{4\pi^2}\frac{2({c_W}^2-{s_W}^2)}{3{s_W}^2}\left[\frac{(m_{H++}-m_{H+})^2}{v^2} + \frac{(m_{H+}-m_A)^2}{v^2}\right]$$

CaseI

$\lambda_5 \geq 0$

m_{H++}=300GeV, v_Δ=1MeV



If we take into account results of $R_{\gamma\gamma}$ a, pink region is excluded.

M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

Δg_{hVV} can be 1%.

Deviations are detectable at ILC !

hhh

➤ Deviations for *hhh* from the SM predictions

$$\Delta\Gamma_{hhh} \equiv \frac{\mathrm{Re}\Gamma_{hhh} - \mathrm{Re}\Gamma^{\mathrm{SM}}_{hhh}}{\mathrm{Re}\Gamma^{\mathrm{SM}}_{hhh}}$$

$$\Delta\Gamma_{hhh} \simeq \frac{-v}{48\pi^2 m_h^2}\left(\frac{\lambda_{H^{++}H^{--}h}{}^3}{m_{H^{++}}{}^2} + \frac{\lambda_{H^+H^-h}{}^3}{m_{H^+}{}^2} + \frac{4\lambda_{hAA}{}^3}{m_A{}^2} + \frac{4\lambda_{hHH}{}^3}{m_H{}^2}\right)$$

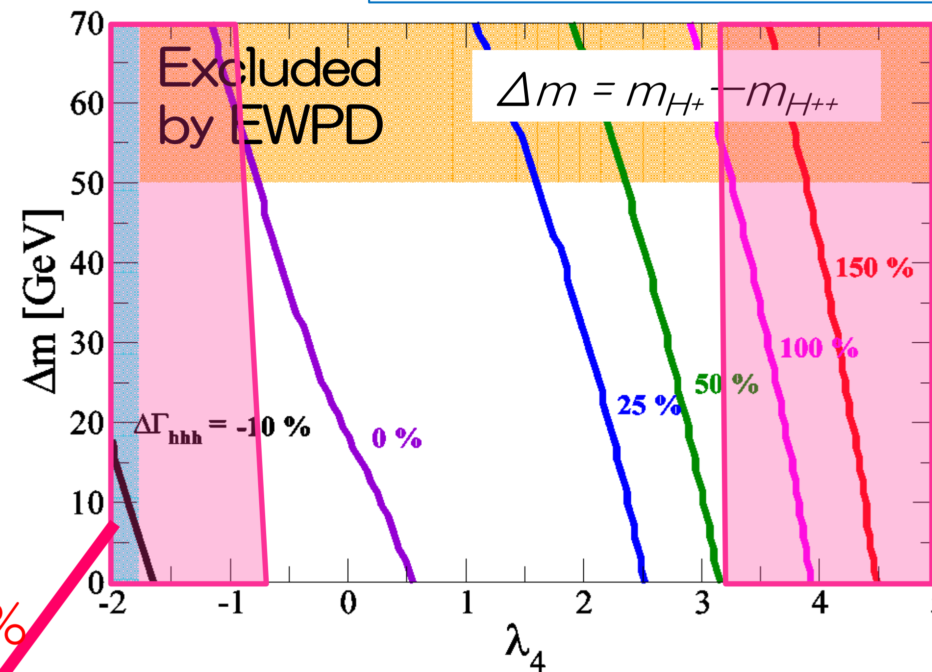
$$\simeq \frac{v^4}{48 m_h^2 \pi^2}\left[\frac{\lambda_4{}^3}{m_{H^{++}}{}^2} + \frac{\left(\lambda_4 + \frac{\lambda_5}{2}\right)^3}{m_{H^+}{}^2} + \frac{(\lambda_4+\lambda_5)^3}{2m_A{}^2} + \frac{(\lambda_4+\lambda_5)^3}{2m_H{}^2}\right]$$

Deviations in hhh coupling from the SM prediction can be -10 %∼ +150%

CaseI

$\lambda_5 \geq 0$

$m_{H^{++}}$=300GeV, v_Δ=1MeV



M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

If we take into account results of $R_{\gamma\gamma}$, pink region is excluded.

$\Delta\Gamma_{hhh}$ can be 50%. ➡ Deviations are detectable at ILC !

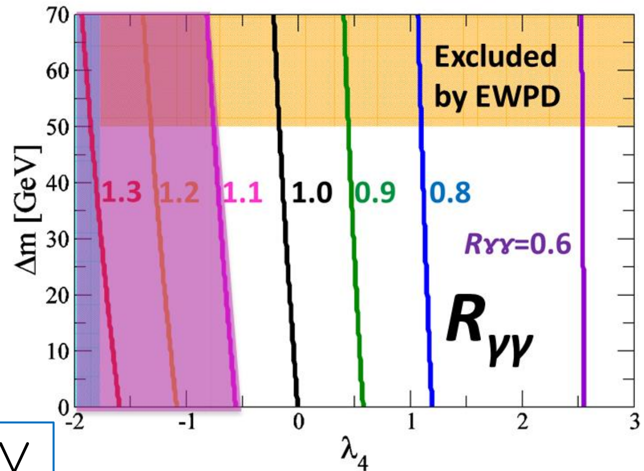
Correlations of the deviation

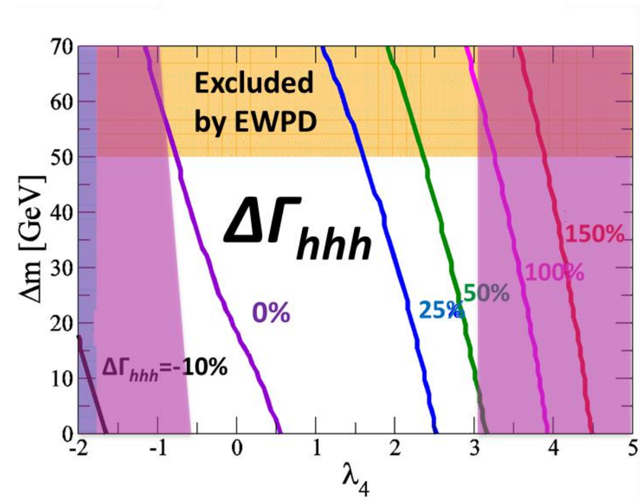
Contributions to $R_{\gamma\gamma}$ is opposite to one of $\Delta\Gamma_{hhh}$.

Case-I

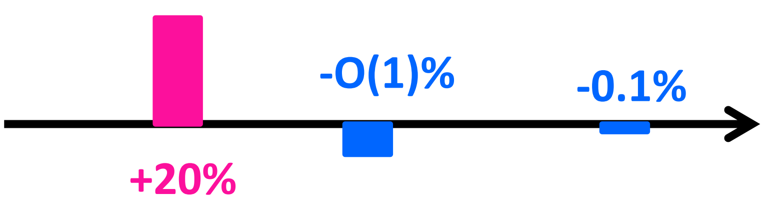
$\Delta m = m_{H+} - m_{H++}$

$m_{lightest}$=300GeV, v_{Δ}=1MeV

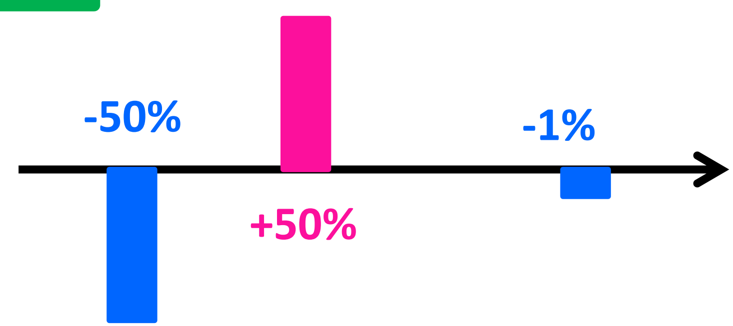




$\lambda_4 = -0.5$ $R_{\gamma\gamma}$ $\Delta\Gamma_{hhh}$ $\Delta g_{hWW}, \Delta g_{hZZ}$



$\lambda_4 = 3$ $R_{\gamma\gamma}$ $\Delta\Gamma_{hhh}$ $\Delta g_{hWW}, \Delta g_{hZZ}$



There is a correlation in deviations in $h\gamma\gamma$ and hhh.
By detecting this, we can discriminate the model from the others.

Summary

- We calculate radiative corrections to the h-coupling constants in the Higgs triplet model to compare future precision measurements.

- Results：

In the region where LHC (CMS) data($\Gamma(h\to\gamma\gamma)$) allows,

  - deviations for *hWW (hZZ)* can be about -0.1% to -1%.
  - deviations for *hhh* can be about -5% to +50 %.

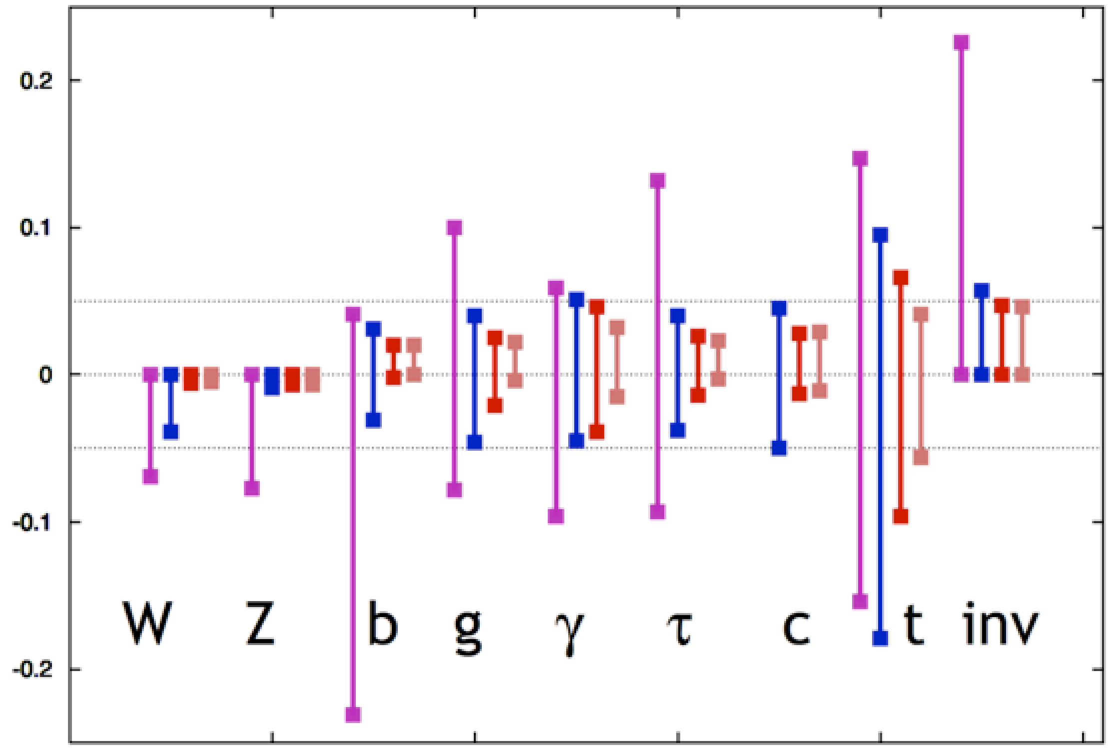
- Conclusion:

Deviations for these coupling constants can be large enough to be detected at ILC.

By detecting the pattern of deviations in coupling constants, we can discriminate the HTM from the other models.

Thank you for your attention !

$g(hAA)/g(hAA)|_{SM}-1$ LHC / HLC / ILC / ILCTeV



LHC : $\sqrt{s}$=14 TeV, L=300fb^{-1} in LHC
HLC : $\sqrt{s}$=250 GeV, L=250fb^{-1} in ILC
ILC : $\sqrt{s}$=500 GeV, L=500fb^{-1} in ILC
ILCTeV : $\sqrt{s}$=1000 GeV, L=1000fb^{-1} in ILC

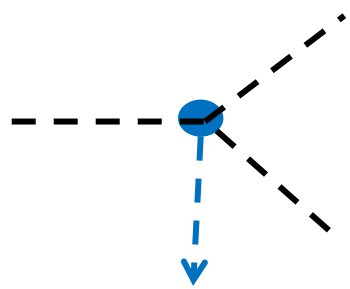
hhh

$$v_\Delta{}^2 \ll v_\phi{}^2$$
$$v^2 = v_\phi{}^2 + 2v_\Delta{}^2$$

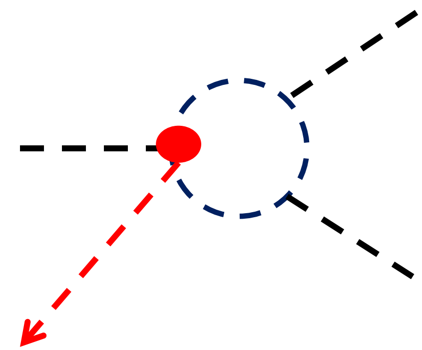
$$m_{H++}{}^2 \simeq M^2 + \frac{1}{2}\lambda_4 v^2$$
$$m_{H+}{}^2 \simeq M^2 + (\tfrac{1}{2}\lambda_4 + \tfrac{1}{4}\lambda_5)v^2$$
$$m_A{}^2 \simeq m_H{}^2 \simeq M^2 + \frac{1}{2}(\lambda_4 + \lambda_5)v^2$$

$$\Delta\Gamma_{hhh} \simeq \frac{v}{48\pi^2 m_h{}^2}\left(\frac{\lambda_{H++H--h}{}^3}{m_{H++}{}^2} + \frac{\lambda_{H+H-h}{}^3}{m_{H+}{}^2} + \frac{4\lambda_{hAA}{}^3}{m_A{}^2} + \frac{4\lambda_{hHH}{}^3}{m_H{}^2}\right)$$
$$\simeq \frac{v^4}{48 m_h{}^2 \pi^2}\left[\frac{\lambda_4{}^3}{m_{H++}{}^2} + \frac{\left(\lambda_4 + \frac{\lambda_5}{2}\right)^3}{m_{H+}{}^2} + \frac{(\lambda_4+\lambda_5)^3}{2m_A{}^2} + \frac{(\lambda_4+\lambda_5)^3}{2m_H{}^2}\right]$$

Coupling parameters of the loop diagrams are different from the one of the tree diagram.
So, deviations by loop correction can be large.



$$\lambda_{hhh} \simeq -\lambda_1 v$$



$$\lambda_{hH++H--} \simeq -\lambda_4 v$$
$$\lambda_{hH+H-} \simeq -\left(\lambda_4 + \frac{1}{2}\lambda_5\right)v$$
$$\lambda_{hAA} \simeq \lambda_{hH+H-} \simeq -\frac{1}{2}(\lambda_4 + \lambda_5)v$$

Global symmetries

This potential respects additional global symmetries in some limits.

$$V_{Higgs} = m^2\Phi^\dagger\Phi + M^2 Tr(\Delta^\dagger\Delta) + [\mu\Phi^T i\tau_2\Delta^\dagger\Phi + h.c.]$$
$$+ \lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2[Tr(\Delta^\dagger\Delta)]^2 + \lambda_3 Tr(\Delta^\dagger\Delta)^2$$
$$+ \lambda_4(\Phi^\dagger\Phi)Tr(\Delta^\dagger\Delta) + \lambda_5\Phi^\dagger\Delta\Delta^\dagger\Phi$$

- When the μ term is absent, there is the global U(1) symmetry in the potential. This symmetry conserves the lepton number.

Mass formula appear in this limit.

$$\xi \equiv m^2_{H^{++}} - m^2_{H^+} = m^2_{H^+} - m^2_A,$$
$$\Delta m \equiv m_{H^+} - m_{\text{lightest}}, \quad \text{with} \quad m_H = m_A$$

- When both the μ term and the λ5 term are zero, an additional global SU(2) symmetry appears.

This is the symmetry which rotate the doublet field and the triplet field with different angle.

In this case, all triplet-like Higgs bosons are degenerate in mass.

$$m_{H++}{}^2 = m_{H+}{}^2 = m_A{}^2 = m_H{}^2$$

Renormalization in EW

- Parameters ; $m_W, m_Z, sin\theta_W, G_F, \alpha_{em}$

- Relation ; $G_F = \frac{\pi\alpha_{em}}{\sqrt{2}m_W{}^2 sin\theta_W 2}$, $cos^2\theta_W = \frac{2m_W{}^2}{m_Z{}^2(1+cos^2\beta\prime)}$

- Input parameters ; $m_W, m_Z, \alpha_{em}, \underline{sin\theta_W}$
  $\Rightarrow$ β' (mixing angle among the CP-odd scalar field)

- Parameter shift ;

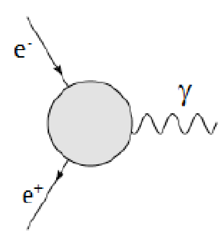
$$m_W{}^2 \rightarrow m_W{}^2 + \delta m_W{}^2,$$
$$m_Z{}^2 \rightarrow m_Z{}^2 + \delta m_Z{}^2,$$
$$\alpha_{em} \rightarrow \alpha_{em} + \delta\alpha_{em},$$
$$sin^2\theta_W \rightarrow sin^2\theta_W + \delta sin^2\theta_W$$

- Renormalization conditions ;

$$\mathrm{Re}\hat{\Pi}_{WW}[m_W^2] = 0, \quad \rightarrow \delta m_W{}^2 \qquad \mathrm{Re}\hat{\Pi}_{ZZ}[m_Z^2] = 0, \quad \rightarrow \delta m_Z{}^2,$$

$$\hat{\Gamma}_\mu^{\gamma ee}[q^2 = 0, \not{p}_1 = \not{p}_2 = m_e] = ie\gamma_\mu,$$

$$\rightarrow \delta\alpha_{em},$$



Renormalization in Higgs potential

Higgs potential

(α: Mixing angle among CP- even Higgs bosons,
β: Mixing angle among charged Higgs bosons)

- Parameters ･･･ $v, \alpha, \beta, \beta', m_{H\pm\pm}, m_{H\pm}, m_A, m_H, m$

Counter-terms

$\delta v, \delta\alpha, \delta\beta, \delta\beta', \delta m_{H++}{}^2, \delta m_{H+}{}^2, \delta m_A{}^2, \delta m_h{}^2, \delta m_H{}^2$

Tadpole : $\delta T_\varphi, \delta T_\Delta,$

Wave function renormalization : $\delta Z_h, \delta Z_H, \delta Z_A, \delta Z_{G0}, \delta Z_{H+}, \delta Z_{G+}, \delta Z_{H++}, \delta C_{hH}, \delta C_{AG0}, \delta C_{H+G+}$

Renormalization conditions

$\Pi_{\varphi\varphi}[p^2]$ =

$\delta m_\varphi{}^2$ ･･･ $\Pi_{\varphi\varphi}[m_\varphi{}^2] = 0,$

$v^2 = \frac{m_W{}^2 \sin^2\theta_W}{\pi^2 \alpha_{em}}$,

δv ･･･ Renormalizations in the gauge sector, ➡ $\frac{\delta v}{v} = \frac{1}{2}\left(\frac{\delta m_W^2}{m_W^2} - \frac{\delta\alpha_{\rm em}}{\alpha_{\rm em}} + \frac{\delta \bar{s}_W^2}{\bar{s}_W^2}\right)$

$\delta\alpha$ ･･･ $\Pi_{Hh}[m_h{}^2] = 0, \Pi_{Hh}[m_H{}^2] = 0,$ No mixing on-shell

$\delta\beta'$ ･･･ $\Pi_{AG}[m_A{}^2] = 0, \Pi_{AG}[m_G{}^2] = 0,$ No mixing on-shell

$\delta\beta$ ･･･ $\delta\beta = \frac{1+s_\beta^2}{\sqrt{2}}\delta\beta'$, ⬅ $\tan\beta = \frac{\sqrt{2}v_\Delta}{v_\phi}, \quad \tan\beta' = \frac{2v_\Delta}{v_\phi},$

$h\gamma\gamma$

A. Arhrib, R. Benbrik, M. Chabab, G. Moultaka (2012);
A. G. Akeroyd, S. Moretti (2012);

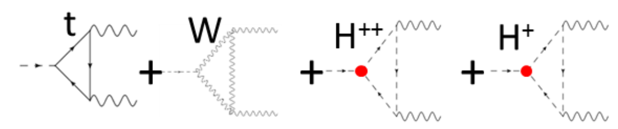
- Ratio of the event rate for $h \to \gamma\gamma$

$$R_{\gamma\gamma} \equiv \frac{\sigma(gg \to h)_{\rm HTM} \times {\rm BR}(h \to \gamma\gamma)_{\rm HTM}}{\sigma(gg \to h)_{\rm SM} \times {\rm BR}(h \to \gamma\gamma)_{\rm SM}}$$

R_{gg} depend on λ_4.

$$\lambda_{hH^{++}H^{--}} \approx -\lambda_4 v$$

$$\lambda_{hH^{+}H^{-}} \approx -(\lambda_4 + \frac{\lambda_5}{2})v$$

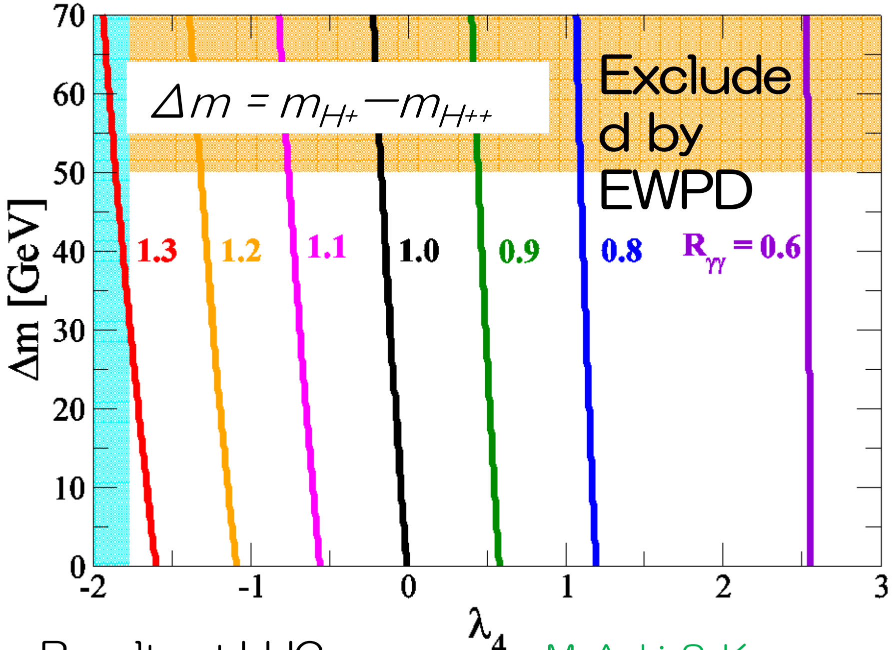
$\Gamma(h \to \gamma\gamma)_{HTM} = \Big|$  $\Big|^2$

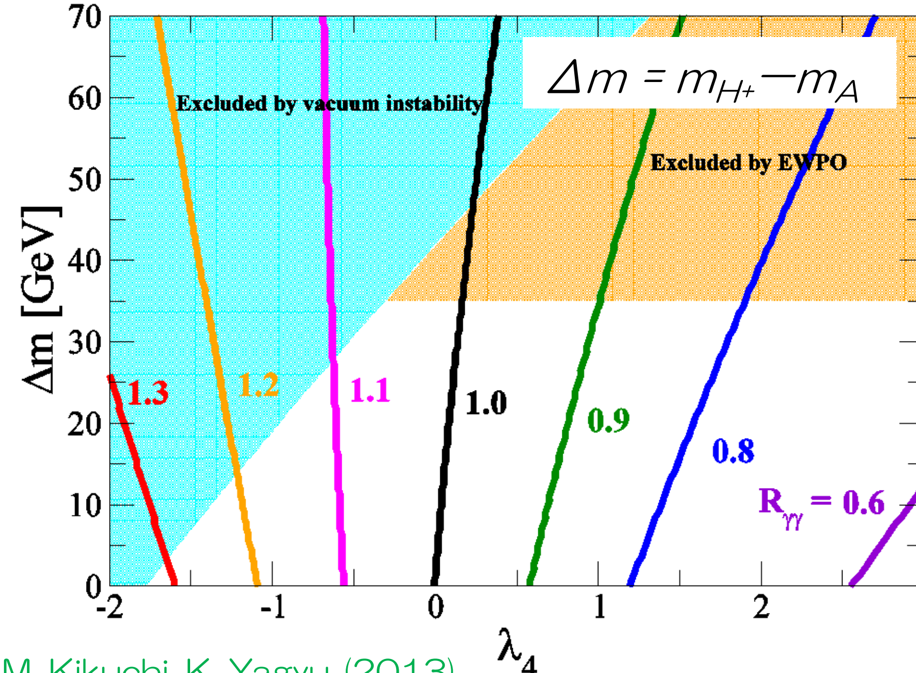
CaseI $\lambda_5 \geq 0$

$m_{H^{++}}$=300GeV, v_Δ=1MeV

CaseII $\lambda_5 \leq 0$

m_A=300GeV, v_Δ=1MeV





Results at LHC

M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

hWW

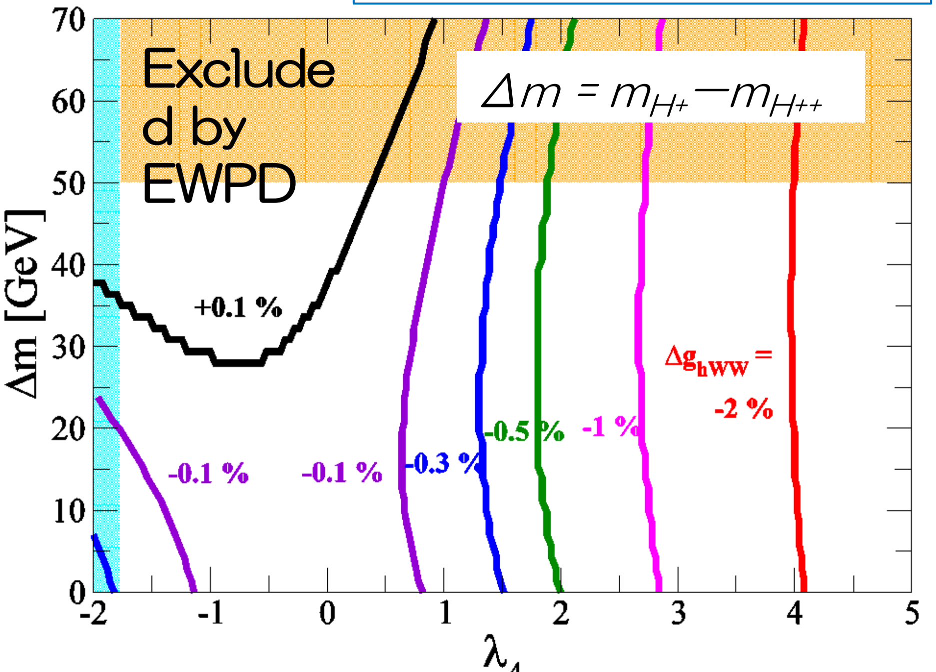
- Deviations for hWW from the SM predictions

$$\Delta g_{hVV} \equiv \frac{\mathrm{Re}M_1^{hVV} - \mathrm{Re}M_1^{hVV}(\mathrm{SM})}{\mathrm{Re}M_1^{hVV}(\mathrm{SM})}$$

$$\Delta g_{hWW} \simeq -\frac{v^2}{32\pi^2}\left[\frac{\lambda_4{}^2}{6m_{H^{++}}{}^2} + \frac{\left(\lambda_4+\frac{\lambda_5}{2}\right)^2}{6m_{H^+}{}^2} + \frac{(\lambda_4+\lambda_5)^2}{6m_A{}^2} + \frac{(\lambda_4+\lambda_5)^2}{6m_H{}^2}\right] + \frac{1}{4\pi^2}\frac{2(c_W{}^2-s_W{}^2)}{3s_W{}^2}\left[\frac{(m_{H^{++}}-m_{H^+})^2}{v^2} + \frac{(m_{H^+}-m_A)^2}{v^2}\right]$$

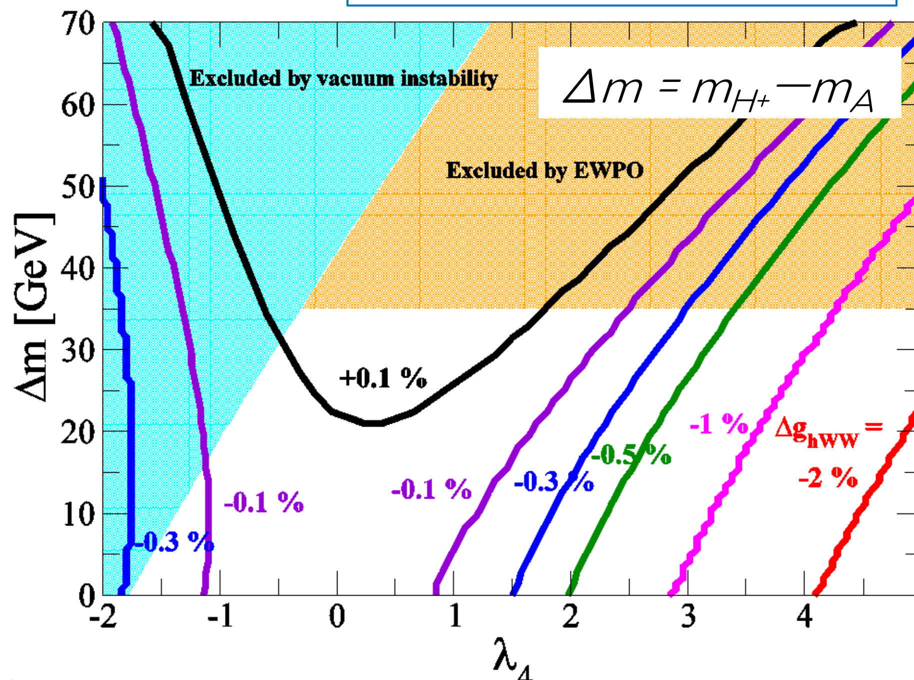
CaseI $\lambda_5 \geq 0$

$m_{H^{++}}$=300GeV, v_Δ=1MeV



CaseII $\lambda_5 \leq 0$

m_A=300GeV, v_Δ=1MeV



M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

Deviations for hWW from the SM predictions can be several %.

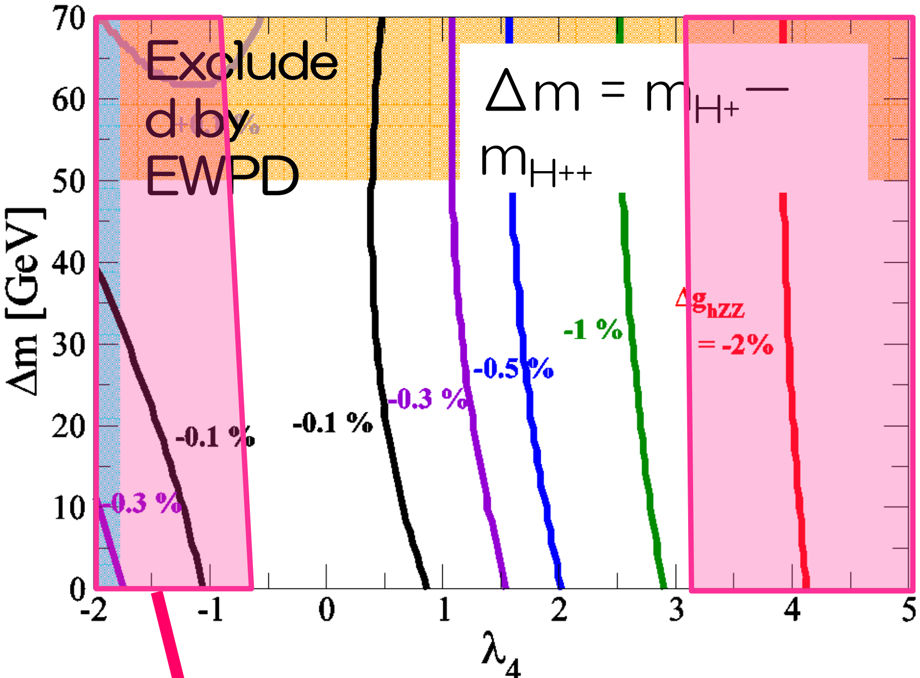
hZZ

- Deviations for hWW from the SM predictions

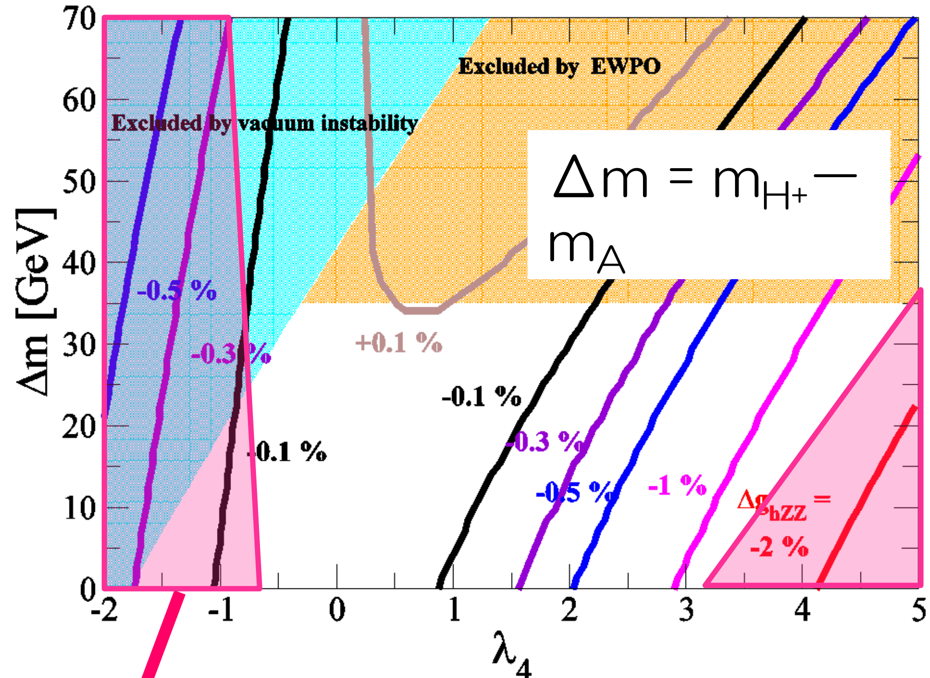
$$\Delta g_{hVV} \equiv \frac{\mathrm{Re}M_1^{hVV} - \mathrm{Re}M_1^{hVV}(\mathrm{SM})}{\mathrm{Re}M_1^{hVV}(\mathrm{SM})}$$

$$\Delta g_{hWW} \simeq -\frac{v^2}{32\pi^2}\left[\frac{{\lambda_4}^2}{6{m_{H++}}^2} + \frac{\left(\lambda_4+\frac{\lambda_5}{2}\right)^2}{6{m_{H+}}^2} + \frac{(\lambda_4+\lambda_5)^2}{6{m_A}^2} + \frac{(\lambda_4+\lambda_5)^2}{6{m_H}^2}\right] + \frac{1}{4\pi^2}\frac{2({c_W}^2-{s_W}^2)}{3{s_W}^2}\left[\frac{(m_{H++}-m_{H+})^2}{v^2} + \frac{(m_{H+}-m_A)^2}{v^2}\right]$$

CaseI m_{H++}=300GeV, v_Δ=1MeV



CaseII m_A=300GeV, v_Δ=1MeV



Excluded region from the LHC（CMS） data（$h\rightarrow\gamma\gamma$）

Δg_{hVV} can be 1%. ➡ Deviations are detectable at ILC !

hhh

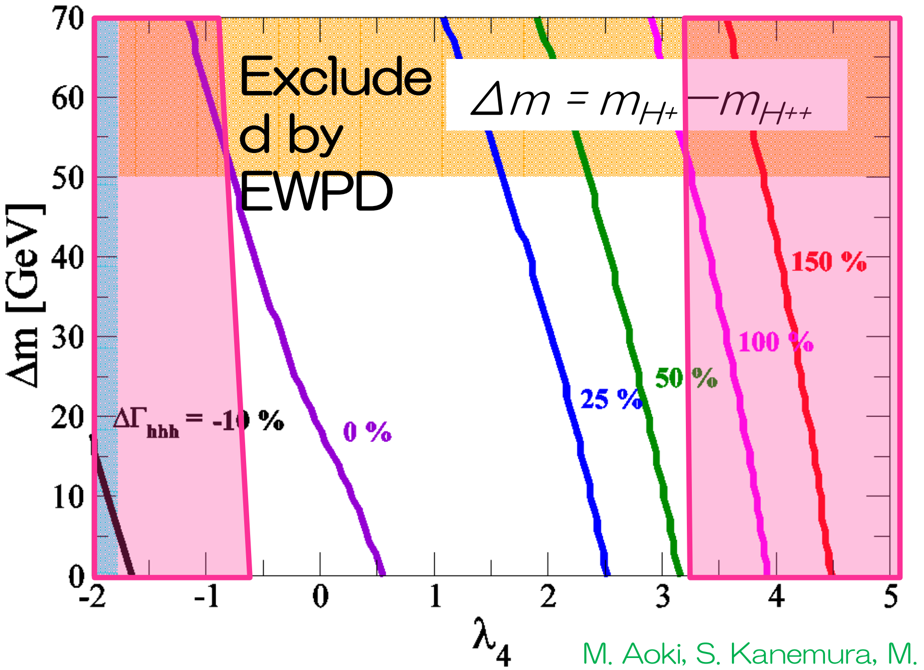
$$\Delta\Gamma_{hhh} \simeq \frac{v}{48\pi^2 m_h^2}\left(\frac{\lambda_{H++H--h}{}^3}{m_{H++}{}^2}+\frac{\lambda_{H+H-h}{}^3}{m_{H+}{}^2}+\frac{4\lambda_{hAA}{}^3}{m_A{}^2}+\frac{4\lambda_{hHH}{}^3}{m_H{}^2}\right)$$

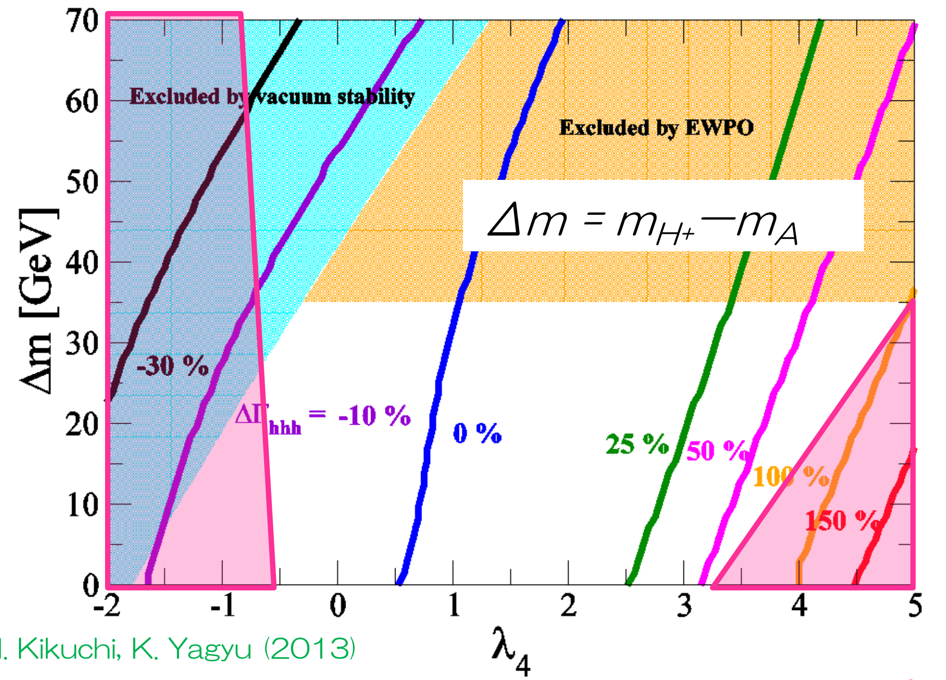
$$\simeq \frac{v^4}{48 m_h^2\pi^2}\left[\frac{\lambda_4{}^3}{m_{H++}{}^2}+\frac{\left(\lambda_4+\frac{\lambda_5}{2}\right)^3}{m_{H+}{}^2}+\frac{(\lambda_4+\lambda_5)^3}{2m_A{}^2}+\frac{(\lambda_4+\lambda_5)^3}{2m_H{}^2}\right]$$

$$\Delta\Gamma_{hhh} \equiv \frac{\mathrm{Re}\Gamma_{hhh} - \mathrm{Re}\Gamma^{SM}_{hhh}}{\mathrm{Re}\Gamma^{SM}_{hhh}}$$

CaseI $\lambda_5 \geq 0$ m_{H++}=300GeV, v_Δ=1MeV

CaseII $\lambda_5 \leq 0$ m_A=300GeV, v_Δ=1MeV





M. Aoki, S. Kanemura, M. Kikuchi, K. Yagyu (2013)

$\Delta\Gamma_{hhh}$ can be 50%. → Deviations are detectable at ILC !